An EDF-based Scheduling Algorithm for Multiprocessor Soft Real-Time Systems*

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Abstract

We consider the use of the earliest-deadline-first (EDF) scheduling algorithm in soft real-time multiprocessor systems. In hard real-time systems, a significant disparity exists between EDF-based schemes and Pfair scheduling (which is the only known way of optimally scheduling recurrent real-time tasks on multiprocessors): on M processors, all known EDF variants have utilization-based schedulability bounds of approximately $M/2$, while Pfair algorithms can fully utilize all processors. This is unfortunate because EDF-based algorithms entail lower scheduling and task-migration overheads. In work on hard real-time systems, it has been shown that this disparity in schedulability can be lessened by placing caps on per-task utilizations. In this paper, we show that it can also be lessened by easing the requirement that all deadlines be met. Our main contribution is a new EDF-based scheme that ensures bounded deadline tardiness. In this scheme, per-task utilizations must be capped, but overall utilization need not be restricted. The required cap is quite liberal. Hence, our scheme should enable a wide range of soft real-time applications to be scheduled with *no constraints on total utilization*. We also propose techniques and heuristics that can be used to reduce tardiness. To the best of our knowledge, this paper is the first to examine multiprocessor EDF scheduling in the context of soft real-time systems.

Keywords: Multiprocessor systems, soft real-time, earliest-deadline-first scheduling.

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1 Introduction

Real-time multiprocessor systems are now commonplace. Designs range from single-chip architectures, with a modest number of processors, to large-scale signal-processing systems, such as synthetic-aperture radar systems. In recent years, scheduling techniques for such systems have received considerable attention. In an effort to catalogue these various techniques, Carpentar *et al.* [4] suggested the categorization shown in Table 1, which pertains to scheduling schemes for *periodic* or *sporadic* tasks systems. In such systems, each task is invoked repeatedly, and each such invocation is called a *job*. The table classifies scheduling schemes along two dimensions:

1. **Complexity of the priority mechanism.** Along this dimension, scheduling disciplines are categorized according to whether task priorities are **(i)** static, **(ii)** dynamic but fixed within a job, or **(iii)** fully-dynamic. Common examples of each type include (i) rate-monotonic (RM) [6], (ii) earliest-deadline-first (EDF) [6], and (iii) least-laxity-first (LLF) [8] scheduling.

2. **Degree of migration allowed.** Along this dimension, disciplines are ranked as follows: **(i)** no migration (*i.e.*, task partitioning), **(ii)** migration allowed, but only at job boundaries (*i.e.*, dynamic partitioning at the job level), and **(iii)** unrestricted migration (*i.e.*, jobs are also allowed to migrate).

The entries in Table 1 give known *schedulable utilization bounds* for each category, assuming that jobs can be preempted and resumed later. If U is a schedulable utilization for an M-processor scheduling algorithm $\mathcal{A}$, then $\mathcal{A}$ can correctly schedule any set of periodic (or sporadic) tasks with total utilization at most U on M processors. The top left entry in the table means that there exists some algorithm in the unrestricted-migration/static-priority class that can correctly schedule every task set with total utilization at most $\frac{M^2}{3M-2}$, and that there exists some task set with total utilization slightly higher than $\frac{M+1}{2}$ that cannot be correctly scheduled by any algorithm in the same class. The other entries in the table have a similar interpretation.

According to Table 1, scheduling algorithms from only one category can schedule tasks correctly with no utilization loss, namely, algorithms that allow full migration and use fully-dynamic priorities (the top right entry). The fact that it is possible for algorithms in this category to incur no utilization loss follows from work on scheduling algorithms that ensure *proportionate fairness* (Pfairness) [3]. Pfair algorithms break tasks into smaller uniform pieces called "subtasks," which are then scheduled. The subtasks of a task may execute on any processor, *i.e.*, tasks may migrate within jobs. Hence, Pfair scheduling algorithms may suffer higher scheduling and migration overheads than other schemes. Thus, the other categories in Table 1 are still of interest.

In four of the other categories, the term α represents a cap on individual task utilizations. Note that, if such a cap is not exploited, then the upper bound on schedulable utilization for *each* of the other categories is

3: full migration	$\frac{M^2}{3M-2} \le U \le \frac{M+1}{2}$	$U \ge M - \alpha(M-1)$, if $\alpha \le \frac{1}{2}$ $\frac{M^2}{2M-1} \le U \le \frac{M+1}{2}$, otherwise	$U = M$
2: restricted migration	$U \le \frac{M+1}{2}$	$U \ge M - \alpha(M-1)$, if $\alpha \le \frac{1}{2}$ $M - \alpha(M-1) \le U \le \frac{M+1}{2}$,otherwise	$U \ge M - \alpha(M-1)$, if $\alpha \le \frac{1}{2}$ $M - \alpha(M-1) \le U \le \frac{M+1}{2}$,otherwise
1: partitioned	$(\sqrt{2}-1)M \le U \le \frac{M+1}{1+2^{\frac{1}{M+1}}}$	$U = \frac{\beta M+1}{\beta+1}$, where $\beta = \left\lfloor \frac{1}{\alpha} \right\rfloor$	$U = \frac{M+1}{2}$
	1: static	**2: job-level dynamic**	**3: fully dynamic**

Table 1: Known lower and upper bounds on schedulable utilization (denoted U) for the different classes of preemptive scheduling algorithms.

approximately $M/2$ or lower. This is no accident: as shown in [4], *no* algorithm in these categories can successfully schedule all task systems with total utilization at most B on M processors, where $(M+1)/2 < B \le M$. Given the scheduling and migration overheads of Pfair algorithms, the disparity in schedulability between Pfair algorithms and those in other categories is somewhat disappointing.

Fortunately, as the table suggests, if individual task utilizations can be capped, then it is sometimes possible to significantly relax restrictions on total utilization. For example, in the entries in the middle column, as α approaches 0, U approaches M. This follows from work on multiprocessor EDF scheduling [1, 2, 7], which shows that an interesting "middle ground" exists between unrestricted EDF-based algorithms (which have upper bounds of approximately $M/2$ on schedulable utilization) and Pfair algorithms (which have a schedulable utilization bound of M). In essence, establishing this middle ground involved addressing the following question: *if per-task utilizations are restricted, and if no deadlines can be missed, then what is the largest overall utilization that can be allowed?* In this paper, we approach this middle ground in a different way by addressing this question: *if per-task utilizations are restricted, but overall utilization is not, then by how much can deadlines be missed?* Our interest in this question stems from the increasing prevalence of applications such as networking, multimedia, and immersive graphics systems (to name a few) that have only soft real-time requirements.

While we do not yet understand how to answer to the question raised above for any EDF-based scheme, we do take a first step towards such an understanding in this paper by presenting one such scheme and by establishing deadline tardiness bounds for it. Our basic scheme adheres to the conditions of the middle entry of Table 1 (restricted migration, job-level dynamic priorities).

The maximum tardiness that any task may experience in our scheme is dependent on the per-task utilization cap assumed—the lower the cap, the lower the tardiness threshold. Even with a cap as high as 0.5 (*half* of the capacity of one processor), reasonable tardiness bounds can be guaranteed for a significant percentage of task systems. (In contrast, if $\alpha = 0.5$ in the middle entry of Table 1, then approximately 50% of the system's overall capacity may be lost.) Hence, our scheme should enable a wide range of soft real-time applications to be scheduled in practice with *no constraints on total utilization*. In addition, when a job misses its deadline, we do *not* require a commensurate delay in the release of the next job of the same task. As a result, each task's required processor share is maintained in the long term. Our scheme has the additional advantage of limiting migration costs, even in comparison to other EDF-based schemes: only up to $M-1$ tasks, where M is the number of processors, *ever* migrate, and those that do, do so only between jobs. As noted in [4], migrations between jobs should not be a serious concern in systems where little per-task state is carried over from one job to the next.

The rest of this paper is organized as follows. In Sec. 2, our system model is presented. In Sec. 3 our proposed algorithm is described and a tardiness bound is derived for it. Techniques and heuristics that can be used to reduce tardiness observed in practice are presented in Sec. 4. In Sec. 5, a simulation-based evaluation of our basic algorithm and proposed heuristics is presented. Finally, we conclude in Sec. 6.

2 System Model

We consider the scheduling of a recurrent (periodic or sporadic) task system τ comprised of N tasks on M identical processors. The k^{th} processor is denoted P_k, where $1 \leq k \leq M$. Each task T_i, where $1 \leq i \leq n$, is characterized by a *period* p_i, an *execution cost* $e_i \leq p_i$, and a *relative deadline* d_i. Each task T_i is invoked at regular intervals, and each invocation is referred to as a *job* of T_i. The k^{th} job of T_i is denoted $T_{i,k}$. The first job may be invoked or *released* at any time at or after time zero and the release times of any two consecutive jobs of T_i should differ by at least p_i time units. If every two consecutive job releases differ by exactly p_i time units, then T_i is said to be a *periodic* task; otherwise, T_i is a *sporadic* task. Every job of T_i has a worst-case execution requirement of e_i time units and an absolute deadline given by the sum of its release time and its relative deadline, d_i. In this paper, we assume that $d_i = p_i$ holds, for all i. We sometimes use the notation $T_i(e_i, p_i)$ to concisely denote the execution cost and period of task T_i.

The *utilization* of task T_i is denoted u_i and is given by e_i/p_i. If $u_i \leq 1/2$, then T_i is called a *light task*. In this paper, we assume that every task to be scheduled is light. Because a light task can consume up to half the capacity of a single processor, we do not expect this to be a restrictive assumption in practice. The *total utilization* of a task system τ is defined as $U_{sum}(\tau) = \sum_{i=1}^{n} u_i$. A task system is said to *fully utilize* the available processing capacity if its total utilization equals the number of processors (M). The maximum utilization of any task in τ is denoted $u_{\max}(\tau)$. A task system is *preemptive* if the execution of its jobs may be interrupted and resumed later.

In this paper, we consider only preemptive scheduling policies. We also place no constraints on total utilization.

The jobs of a *soft* real-time task may occasionally miss their deadlines, if the amount by which a job misses its deadline, referred to as its *tardiness*, is bounded. Formally, the tardiness of a job $T_{i,j}$ in schedule $\mathcal{S}$ is defined as $\mathit{tardiness}(T_{i,j}, \mathcal{S}) = \max(0, t - t_a)$, where t is the time at which $T_{i,j}$ completes executing in $\mathcal{S}$ and t_a is its absolute deadline. The tardiness of a task system τ under scheduling algorithm $\mathcal{A}$, denoted $\mathit{tardiness}(\tau, \mathcal{A})$, is defined as the maximum tardiness of any job in τ under any schedule under $\mathcal{A}$. If κ is the maximum tardiness of any task system under $\mathcal{A}$, then $\mathcal{A}$ is said to *ensure a tardiness bound of* κ. Though tasks in a soft real-time system are allowed to have nonzero tardiness, we assume that *missed deadlines do not delay future job releases*. That is, if a job of a task misses its deadline, then the release time of the next job of that task remains unaltered. Of course, we assume that consecutive jobs of the same task cannot be scheduled in parallel. Thus, a missed deadline effectively reduces the interval over which the next job should be scheduled in order to meet its deadline.

Our goal in this paper is to derive an EDF-based multiprocessor scheduling scheme that ensures bounded tardiness. In a "pure" EDF scheme, jobs with earlier deadlines would (always) be given higher priority. In our scheme, this is usually the case, but (as explained later) certain tasks are treated specially and are prioritized using other rules. Because we do not delay future job releases when a deadline is missed, our scheme ensures (over the long term) that each task receives a processor share approximately equal to its utilization. Thus, it should be useful in settings where maintaining correct share allocations is more important than meeting every deadline. In addition, schemes that ensure bounded tardiness are useful in systems in which a utility function is defined for each task [5]. Such a function specifies the "value" or usefulness of the current job of a task as a function of time; beyond a job's deadline, its usefulness typically decays from a positive value to 0 or below. The amount of time after its deadline beyond which the completion of a job has no value implicitly specifies a tardiness threshold for the corresponding task.

3 Algorithm EDF-fm

In this section, we propose Algorithm EDF-fm (fm denotes that each task is either *fixed* or *migrating*), an EDF-based multiprocessor scheduling algorithm that ensures bounded tardiness for task systems whose per-task utilizations are at most 1/2. EDF-fm does not place any restrictions on the total system utilization. Further, at most $M - 1$ tasks need to be able to migrate, and each such task migrates between two processors, across job boundaries only. This has the benefit of lowering the number of tasks whose states need to be stored on a processor and the number of processors on which each task's state needs to be stored. Also, the runtime context of a job, which can be expected to be larger than that of a task, need not be transferred between processors.

EDF-fm consists of two phases: an *assignment phase* and an *execution phase*. The assignment phase executes offline and consists of sequentially assigning each task to one or two processors. In the execution phase, jobs are scheduled for execution at runtime such that over reasonable intervals (as explained later), each task executes at a rate that is commensurate with its utilization. The two phases are explained in detail below. The following notation shall be used.

$s_{i,j} \stackrel{\text{def}}{=}$ Percentage of P_j's processing capacity (expressed as a fraction) allocated to T_i, $1 \leq i \leq n, 1 \leq j \leq M$. ($T_i$ is said to have a *share* of $s_{i,j}$ on P_j.) (1)

$$f_{i,j} \stackrel{\text{def}}{=} \frac{s_{i,j}}{u_i}, \text{ the fraction of } T_i\text{'s total execution requirement that } P_j \text{ can handle}, 1 \leq i \leq n, 1 \leq j \leq M. \quad (2)$$

3.1 Assignment Phase

The assignment phase represents a mapping of tasks to processors. Each task is assigned to either one or two processors. Tasks assigned to two processors are called *migrating* tasks, while those assigned to only one processor are called *fixed* or *non-migrating* tasks. A fixed task T_i is assigned a *share*, $s_{i,j}$, equal to its utilization u_i on the only processor P_j to which it is assigned. A migrating task has shares on both processors to which it is assigned. The sum of its shares equals its utilization. The assignment phase of EDF-fm also ensures that at most two migrating tasks are assigned to any processor.

In Fig. 1, a task-assignment algorithm, denoted Assign-Tasks, is given that satisfies the following properties for any task set τ with $u_{\max}(\tau) \leq 1/2$ and $U_{sum}(\tau) \leq M$.

(P1) Each task is assigned shares on at most two processors only. A task's total share equals its utilization.

(P2) Each processor is assigned at most two migrating tasks only and may be assigned any number of fixed tasks.

(P3) The sum of the shares allocated to the tasks on any processor is at most one.

In the pseudo-code for this algorithm, the i^{th} element $u[i]$ of the global array u represents the utilization u_i of task T_i, $s[i][j]$ denotes $s_{i,j}$ (as defined in (1)), array $p[i]$ contains the processor(s) to which task i is assigned, and arrays $m[i]$ and $f[i]$ denote the migrating tasks and fixed tasks assigned to processor i, respectively. Note that $p[i]$ and $m[i]$ are each vectors of size two.

Algorithm Assign-Tasks assigns tasks in sequence to processors, starting from the first processor. Tasks and processors are both considered sequentially. Local variables *proc* and *task* denote the current processor and task, respectively. Tasks are assigned to *proc* as long as the processing capacity of *proc* is not exhausted. If the current task *task* cannot receive its full share of u_{task} from *proc*, then part of the processing capacity that it requires is

```
ALGORITHM ASSIGN-TASKS()
    global var
        u : array [1..N] of double initially 0.0;
        s : array [1..N][1..M] of double initially 0.0;
        p : array [1..N][1..2] of 0..M initially 0;
        m: array [1..M][1..2] of 0..N initially 0;
        f: array [1..M][1..N] of 0..N initially 0
    local var
        proc : 1..M initially 1;
        task : 1..N;
        AvailUtil : double;
        mt, ft : integer initially 0
    AvailUtil := 1.0;
    for task := 1 to n do
        if AvailUtil ≥ u[task] then
                s[task][proc] := u[task];
                AvailUtil:= AvailUtil − u[task];
                ft := ft + 1;
                p[task][1] := proc;
                f[proc][ft] := task
        else
                if AvailUtil > 0 then
                    s[task][proc] := AvailUtil;
                    mt := mt + 1;
                    m[proc][mt] := task;
                    p[task][1], p[task][2] := proc, proc + 1;
                    mt, ft := 1, 0;
                    m[proc + 1][mt] := task
                else
                    mt, ft := 0, 1;
                    p[task][1] := proc + 1;
                    f[proc + 1][ft] := task
                fi
                proc := proc + 1;
                s[task][proc] := u[task] − s[task][proc − 1];
                AvailUtil := 1 − s[task][proc]
        fi
    od
```

Figure 1: Algorithm ASSIGN-TASKS.

allocated on the next processor, $proc+1$, such that the sum of the shares allocated to *task* on the two processors equals u_{task}. It is easy to see that assigning tasks to processors following this simple approach satisfies (P1)–(P3).

Example task assignment. Consider a task set τ comprised of nine tasks: $T_1(5, 20)$, $T_2(3, 10)$, $T_3(1, 2)$, $T_4(2, 5)$, $T_5(2, 5)$, $T_6(1, 10)$, $T_7(2, 5)$, $T_8(7, 20)$, and $T_9(3, 10)$. The total utilization of this task set is three. A share assignment produced by ASSIGN-TASKS is shown in Fig. 2. In this assignment, T_3 and T_7 are migrating tasks; the remaining tasks are fixed. T_3 has a share of $\frac{9}{20}$ on processor P_1 and a share of $\frac{1}{20}$ on processor P_2, while T_7 shares of $\frac{1}{20}$ and $\frac{7}{20}$ on processors P_2 and P_3, respectively.

3.2 Execution Phase

Having devised a way of assigning tasks to processors, the next step is to devise an online scheduling algorithm that is easy to analyze and ensures bounded tardiness. For a fixed task, we merely need to decide when to schedule each of its jobs on its (only) assigned processor. For a migrating task, we must decide both *when* and *where* its jobs should execute. Before describing our scheduling algorithm, we discuss some considerations that led to its design.

In order to analyze a scheduling algorithm and for the algorithm to guarantee bounded tardiness, it should be possible to bound the total *demand* for execution time by all tasks on each processor over well-defined time intervals. We first argue that bounding total demand may not be possible if the jobs of migrating tasks are allowed to miss their deadlines.

Because a deadline miss of a job does not lead to a postponement of the release times of subsequent jobs of the

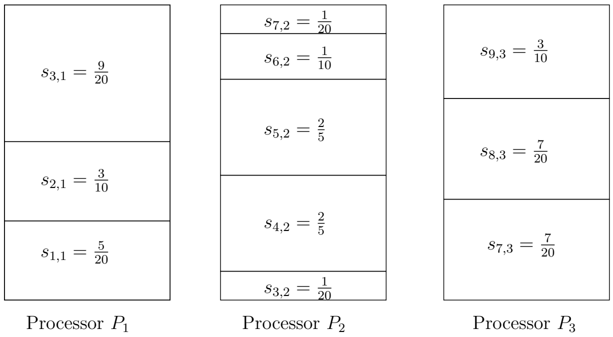


Figure 2: Example task assignment on three processors using Algorithm ASSIGN-TASKS.

same task, and because two jobs of a task may not execute in parallel, the tardiness of a job of a migrating task executing on one processor can affect the tardiness of its successor job, which may otherwise execute in a timely manner on a second processor. In the worst case, the second processor may be forced to idle. The tardiness of the second job may also impact the timeliness of fixed tasks and other migrating tasks assigned to the same processor, which in turn may lead to deadline misses of both fixed and migrating tasks on other processors or unnecessary idling on other processors.

As a result, a set of dependencies is created among the jobs of migrating tasks, resulting in an intricate linkage among processors that complicates scheduling analysis. It is unclear how per-processor demand can be precisely bounded when activities on different processors become interlinked.

Let us look at a concrete example that reveals this linkage among processors. Consider task set τ, introduced earlier, with task assignments and processor shares shown in Fig. 2. For simplicity, assume that the execution of the jobs of a migrating task alternate between the two processors to which the task is assigned. T_3 releases its first job on P_1, while T_7 releases its first job on P_3. (We are assuming such a naïve assignment pattern to illustrate the processor linkage using a short segment of a real schedule. Such a linkage occurs even with an intelligent job-assignment pattern if migrating tasks miss their deadlines.) A complete schedule up to time 27, with the jobs assigned to each processor scheduled using EDF, is shown in Fig. 3.

In Fig. 3, the sixth job of the migrating task T_3 misses its deadline (at time 12) on P_2 and completes executing at time 14. This prevents the next job of T_3 released on P_1 from being scheduled until time 14 and it misses its deadline. Recall that a deadline miss does not cause future job releases to be postponed, thus the seventh job of T_3 is released at time 12 and has a deadline at time 14.

The missed deadlines of the migrating task T_3 impact the execution of the fixed tasks on P_2 also. The deadline misses of the fixed tasks T_4, T_5, and T_6 cause deadline misses of the migrating task T_7 on P_2. As a result, the fourth job of T_7 misses its deadline, which in turn reduces the interval over which the fifth job of the same task can execute on P_3. Thus, a nontrivial linkage is established among the processors that impacts system tardiness.

Per-processor scheduling rules. EDF-fm eliminates this linkage among processors by ensuring that migrating tasks do not miss their deadlines. Jobs of migrating tasks are assigned to processors using static rules that are independent of runtime dynamics. The jobs assigned to a processor are scheduled independently of other processors,

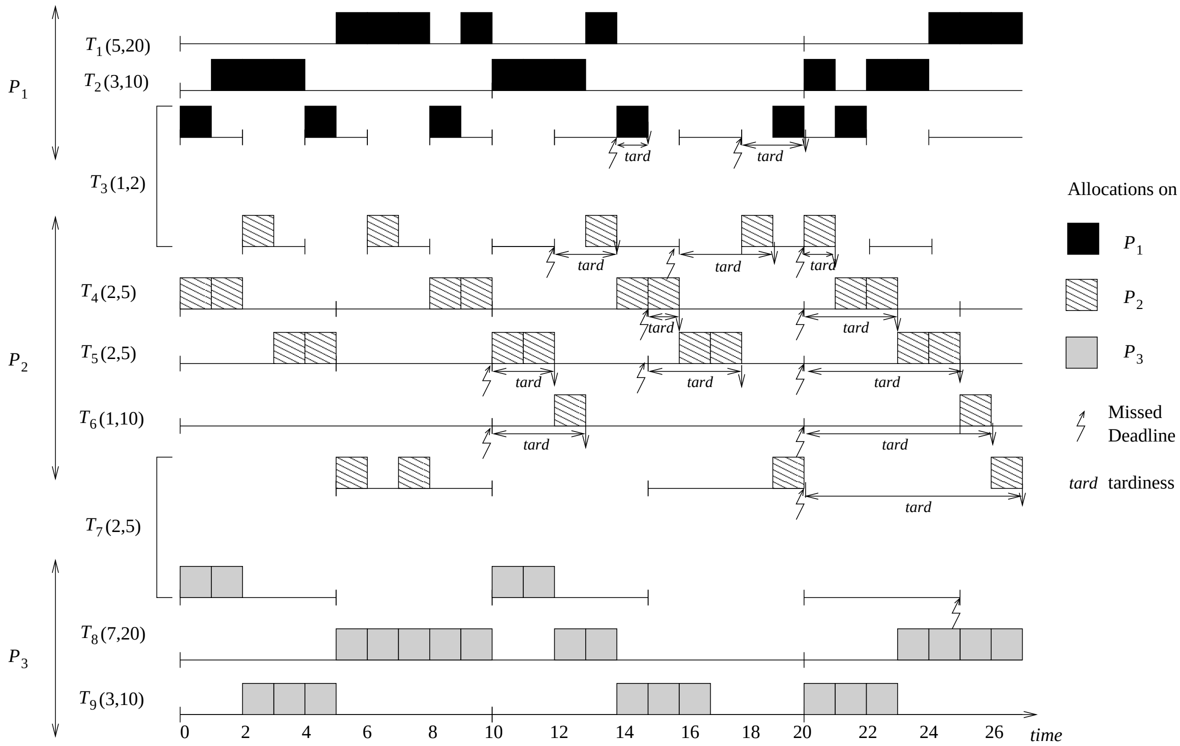


Figure 3: Illustration of processor linkage.

and on each processor, migrating tasks are statically prioritized over fixed tasks. Jobs within each task class are scheduled using EDF, which is optimal on uniprocessors. This priority scheme, together with the restriction that migrating tasks have utilizations at most $1/2$, and the task assignment property (from (P2)) that there are at most two migrating tasks per processor, ensures that migrating tasks never miss their deadlines. Therefore, the jobs of migrating tasks executing on different processors do not impact one another, and each processor can be analyzed independently. Thus, the multiprocessor scheduling analysis problem at hand is transformed into a simpler uniprocessor one.

In the description of EDF-fm, we are left with defining rules that map jobs of migrating tasks to processors. A naïve assignment of the jobs of a migrating task to its processors can cause an over-utilization on one of its assigned processors. EDF-fm follows a job assignment pattern that prevents over-utilization in the long run by ensuring that over well-defined time intervals (explained later), the demand due to a migrating task on each processor is in accordance with its allocated share on that processor.

For example, consider the migrating task $T_7(2,5)$ in the example above. T_7 has a share of $s_{7,2} = \frac{1}{20}$ on P_2 and $s_{7,3} = \frac{7}{20}$ on P_3. Also, $f_{7,2} = \frac{s_{7,2}}{u_7} = \frac{1}{8}$ and $f_{7,3} = \frac{s_{7,3}}{u_7} = \frac{7}{8}$, which imply that P_2 and P_3 should be capable of executing $\frac{1}{8}$ and $\frac{7}{8}$ of the workload of T_7, respectively. Our goal is to devise a job assignment pattern that would ensure that in the long run, the fraction of a migrating task T_i's workload executed on P_j is close to $f_{i,j}$. One

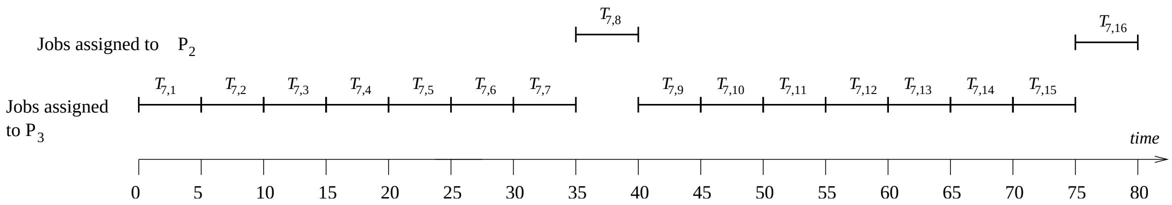


Figure 4: Assignment of periodically released jobs of migrating task T_7 to processors P_2 and P_3.

such job assignment pattern for T_7 over interval $[0, 80)$ is shown in Fig. 4. Assuming that T_7 is a periodic task, the pattern in $[0, 40)$ would repeat every 40 time units.

In the job assignment of Fig. 4, exactly one job out of every eight consecutive jobs of T_7 released in the interval $[5k, 5(k+8))$, where $k \geq 0$, is assigned to P_2. Because $e_7 = 2$, T_7 executes for two units of time, *i.e.*, consumes $1/20$ of P_2 in $[5k, 5(k+8))$. Note that T_7 is allocated a share of $s_{7,2} = 1/20$ on P_2, thus this job assignment pattern ensures that in the long run T_7 does not overload P_2. However, the demand due to T_7 on P_2 over short intervals may exceed or fall below the share allocated to it. For instance, T_7 consumes $2/5$ of P_2 in the interval $[40k + 35, 40(k+1))$, and produces no demand in the interval $[40k, 40k + 35)$.

Similarly, exactly seven out of every eight consecutive jobs of T_7 are assigned to P_3. Thus, T_7 executes for 14 units of time, or $7/20$ of the time, in $[5k, 5(k+8))$, which is what is desired.

The above job assignment pattern ensures, over the long term, that the demand of each migrating task on each processor is in accordance with the share allocated to it. However, as illustrated above, this assignment pattern can result in a migrating task overloading a processor over short time intervals, leading to deadline misses for fixed tasks. Nevertheless, because a deadline miss of a job does not delay the next job release of the same task, this scheme also ensures, over the long term, that each fixed task executes at its prescribed rate (given by its utilization). Later in this section, we also show that the amount by which fixed tasks can miss their deadlines due to the transient overload of migrating tasks is bounded.

A job assignment pattern similar to the one in Fig. 4 can be defined for any migrating task. We draw upon some concepts of Pfair scheduling to derive formulas that can be used to determine such a pattern at run-time. Hence, before proceeding further, a brief digression on Pfair scheduling that reviews needed concepts is in order. (We stress that we are *not* using Pfair algorithms in our scheduling approach. We merely wish to borrow some relevant formulas from the Pfair scheduling literature.)

3.2.1 Digression — Basics of Pfair Scheduling

Currently, Pfair scheduling [3] is the only known way of *optimally* scheduling recurrent real-time task systems on multiprocessors. In Pfair scheduling terminology, each task T has an integer execution cost $T.e$ and an integer period $T.p \geq T.e$. The utilization of T, $T.e/T.p$, is also referred to as the *weight* of T and is denoted $wt(T)$. (Note that in the context of Pfair scheduling, tasks are denoted using upper-case letters without subscripts.)

Pfair algorithms allocate processor time in discrete quanta that are uniform in size. Assuming that a quantum is one time unit in duration, the interval $[t, t+1)$, where t is a non-negative integer, is referred to as *slot* t. At most one task may execute on each processor in each slot, and each task may execute on at most one processor only in every slot. The sequence of allocation decisions over time slots defines a *schedule* $\mathcal{S}$. Formally, $\mathcal{S} : \tau \times \mathbb{N} \mapsto \{0, 1\}$. $\mathcal{S}(T, t) = 1$ iff T is scheduled in slot t.

The notion of a Pfair schedule for a periodic task T is defined by comparing such a schedule to an ideal fluid schedule, which allocates $wt(T)$ processor time to T in each slot. Deviation from the allocation in a fluid schedule is formally captured by the concept of *lag*. Formally, the *lag of task T at time t* in schedule $\mathcal{S}$ is the difference between the total allocations to T in a fluid schedule and $\mathcal{S}$ in the interval $[0, t)$, *i.e.*,

$$lag(T, t, \mathcal{S}) = wt(T) \cdot t - \sum_{u=0}^{t-1} \mathcal{S}(T, u). \tag{3}$$

A schedule $\mathcal{S}$ is said to be *Pfair* iff

$$(\forall T, t :: -1 < lag(T, t, \mathcal{S}) < 1) \tag{4}$$

holds. Informally, the allocation error associated with each task must always be less than one quantum.

The above constraints on lag have the effect of breaking task T into a potentially infinite sequence of quantum-length *subtasks*. The i^{th} subtask of T is denoted T_i, where $i \geq 1$. (In the context of Pfair scheduling, T_i does not denote the i^{th} task, but the i^{th} subtask of task T.)

Each subtask T_i is associated with a *pseudo-release* $r(T_i)$ and a *pseudo-deadline* $d(T_i)$ defined as follows:

$$r(T_i) = \left\lfloor \frac{i-1}{wt(T)} \right\rfloor \tag{5}$$

$$d(T_i) = \left\lceil \frac{i}{wt(T)} \right\rceil \tag{6}$$

To satisfy (4), T_i must be scheduled in the interval $w(T_i) = [r(T_i), d(T_i))$, termed its *window*. Fig. 5(a) shows the windows of the first job of a periodic task with weight $3/7$. In this example, $r(T_1) = 0$, $d(T_1) = 3$, and $w(T_1) = [0, 3)$ hold for subtask T_1.

We next define the notion of a *complementary task*, which is used to guide the sequence in which the jobs of a migratory task are assigned to its processors.

Definition 1: Tasks T and U are *complementary* iff $wt(U) = 1 - wt(T)$.

Tasks T and U shown in Fig. 5(b) are complementary to one another. A partial Pfair schedule for these two tasks on one processor, in which the subtasks of T are always scheduled in the last slot of their windows and those of U in the first slot, is also shown. We call such a schedule a *complementary schedule*. It is easy to show that such a schedule is always possible for two complementary periodic tasks.

With the above introduction to Pfair scheduling, we are now ready to present the details of distributing the jobs of a migrating task between its processors.

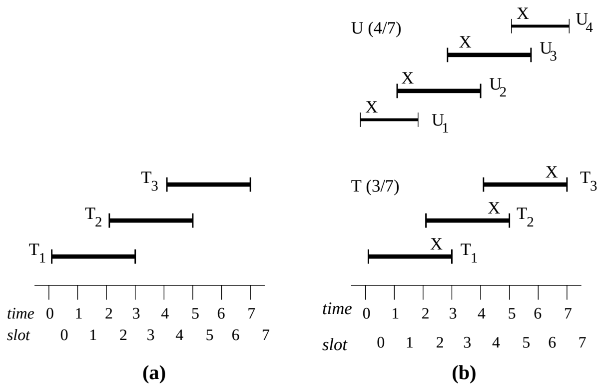


Figure 5: **(a)** Windows of the first job of a periodic task T with weight 3/7. This job consists of subtasks T_1, T_2, and T_3, each of which must be scheduled within its window. (This pattern repeats for every job.) **(b)** A partial complementary Pfair schedule for a pair of complementary tasks, T and U, on one processor. The slot in which a subtask is scheduled is indicated by an "X". In this schedule, every subtask of U is scheduled in the first slot of its window, while every subtask of T is scheduled in the last slot.

3.2.2 Assignment Rules for Jobs of Migrating Tasks

Let T_i be any migrating periodic task (we later relax the assumption that T_i is periodic) that is assigned shares $s_{i,j}$ and $s_{i,j+1}$ on processors P_j and P_{j+1}, respectively. (Note that every migrating task is assigned shares on two consecutive processors by Assign-Tasks.) As explained earlier, $f_{i,j}$ and $f_{i,j+1}$ (given by (2)) denote the fraction of the workload (*i.e.*, the total execution requirement) of T that should be executed on P_j and P_{j+1}, respectively, in the long run. By (P1), the total share allocated to T_i on P_j and P_{j+1} is u_i. Hence, by (2), it follows that

$$f_{i,j} + f_{i,j+1} = 1. \tag{7}$$

Assuming that the execution cost and period of every task are rational numbers (that can be expressed as a ratio of two integers), u_i, $s_{i,j}$, and hence, $f_{i,j}$ and $f_{i,j+1}$ are also rational numbers. Let $f_{i,j} = \frac{x_{i,j}}{y_i}$, where $x_{i,j}$ and y_i are positive integers that are relatively prime. Then, by (7), it follows that $f_{i,j+1} = \frac{y_i - x_{i,j}}{y_i}$. Therefore, one way of distributing the workload of T_i between P_j and P_{j+1} that is commensurate with the shares of T_i on the two processors would be to assign $x_{i,j}$ out of every y_i jobs to P_j and the remaining jobs to P_{j+1}.

We borrow from the aforementioned concepts of Pfair scheduling to guide in the distribution of jobs. If we let $f_{i,j}$ and $f_{i,j+1}$ denote the weights of two fictitious Pfair tasks, V and W, and let a quantum span p_i time units, then the following analogy can be made between the jobs of the migrating task T_i and the subtasks of the fictitious tasks V and W. First, slot s represents the interval in which the $(s+1)^{st}$ job of T_i, which is released at the beginning of slot s, needs to be scheduled. (Recall that slots are numbered starting from 0.) Next, subtask V_g represents the

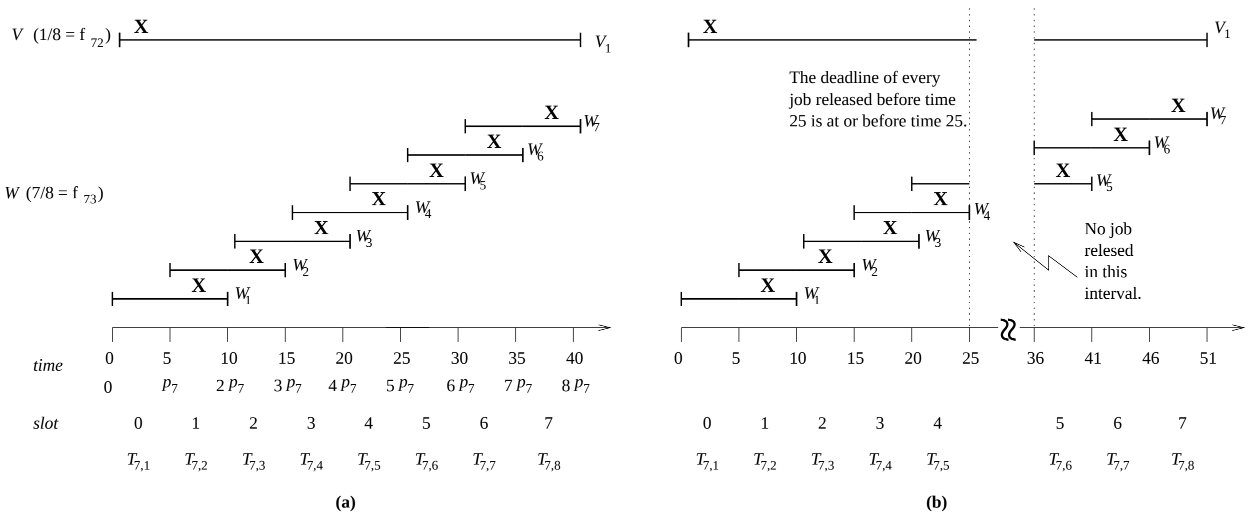


Figure 6: Complementary Pfair schedule for tasks with weights $f_{7,2} = 1/8$ and $f_{7,3} = 7/8$ that guides the assignment of jobs of task $T_7(2, 5)$ to processors P_2 and P_3. Slot k corresponds to job $k+1$ of T_7. The slot in which a subtask is scheduled is indicated by an "X." **(a)** The jobs of T_7 are released periodically. **(b)** The sixth job of T_7 is delayed by 11 time units.

g^{th} job of the jobs of T_i assigned to P_j; similarly, subtask W_h represents the h^{th} job of the jobs of T_i assigned to P_{j+1}.

By Def. 1 and (7), Pfair tasks V and W are complementary. Therefore, a complementary schedule for V and W in which the subtasks of V are scheduled in the first slot of their windows and those of W in the last slot of their windows is feasible. Accordingly, we consider a job assignment policy in which the job of T_i corresponding to the first slot in the window of subtask V_g is assigned as the g^{th} job of T_i to P_j and the job of T_i corresponding to the last slot in the window of subtask W_h is assigned as the h^{th} job of T_i to P_{j+1}, for all g and h. This policy satisfies the following property.

(A) Each job of T_i is assigned to exactly one of P_j and P_{j+1}.

Fig. 6(a) shows a complementary schedule for the Pfair tasks that represent the rates at which the jobs of task T_7 in the example we have been considering should be assigned to P_2 and P_3. Here, tasks V and W are of weights $f_{7,2} = 1/8$ and $f_{7,3} = 7/8$, respectively. A job assignment based on this schedule will assign the first of jobs $8k+1$ through $8(k+1)$ to P_2 and the remaining seven jobs to P_3, for all k.

More generally, can we use the formula for the release time of a subtask given by (5) for job assignments. Let job_i denote the total number of jobs released by task T_i up to some time that is just before t and let $job_{i,j}$ denote the total number of jobs of T_i that have been assigned to P_j up to just before t. Let $p_{i,\ell}$ denote the processor to which job ℓ of task T_i is assigned. Then, the processor to which job $job_i + 1$, released at or after time t, is assigned is determined as follows.

$$p_{i,job_i+1} = \begin{cases} j, & \text{if } job_i = \left\lfloor \dfrac{job_{i,j}}{f_{i,j}} \right\rfloor \\ j+1, & \text{otherwise} \end{cases} \tag{8}$$

As before, let $f_{i,j}$ and $f_{i,j+1}$ be the weights of two fictitious Pfair tasks V and W, respectively. Then, by (5), $t_r = \left\lfloor \frac{job_{i,j}}{f_{i,j}} \right\rfloor$ denotes the release time of subtask $V_{job_{i,j}+1}$ of task V. Thus, (8) assigns to P_j, the job that corresponds to the first slot in the window of subtask V_g as the g^{th} job of T_i on P_j, for all g. (Recall that the index of the job of the migrating periodic task T_i that is released in slot t_r is given by $t_r + 1$.) The job that corresponds to the last slot in the window of subtask W_h is assigned as the h^{th} job of T_i on P_{j+1}, for all h.

Thus far in our discussion, in order to simplify the exposition, we assumed that the job releases of task T_i are periodic. However, note that the job assignment given by (8) is independent of "real" time and is based on the job number only. Hence, assigning jobs using (8) should be sufficient to ensure (A) even when T_i is sporadic. This is illustrated in Fig. 6(b). Here, we assume that T_7 is a sporadic task, whose sixth job release is delayed by 11 time units to time 36 from time 25. As far as T_7 is concerned, the interval $[25, 36)$ is "frozen" and the job assignment resumes at time 36. As indicated in the figure, in any such interval in which activity is suspended for a migrating task T_i, no jobs of T_i are released. Furthermore, the deadlines of all jobs of T_i released before the frozen interval fall at or before the beginning of the interval.

We next prove a property that bounds from above the number of jobs of a migrating task assigned to each of its processors by the job assignment rule given by (8).

Lemma 1 *Let T_i be a migrating task that is assigned to processors P_j and P_{j+1}. The number of jobs out of any consecutive $\ell \geq 0$ jobs of T_i that are assigned to P_j and P_{j+1} is at most $\lceil \ell \cdot f_{i,j} \rceil$ and $\lceil \ell \cdot f_{i,j+1} \rceil$, respectively.*

Proof: We prove the lemma for the number of jobs assigned to P_j. The proof for P_{j+1} is similar. We first claim the following.

(J) Exactly $\lceil \ell_0 \cdot f_{i,j} \rceil$ of the first ℓ_0 jobs of T_i are assigned to P_j.

(J) holds trivially when $\ell_0 = 0$. Therefore, assume $\ell_0 \geq 1$. Let q denote the total number of jobs of the first ℓ_0 jobs of T_i that are assigned to P_j. (By (8), the first job of T_i is assigned to P_j, hence, $q \geq 1$ holds.) Then, there exists an $\ell' \leq \ell_0$ such that job ℓ' of T_i is the q^{th} job of T_i assigned to P_i. Therefore, by (8),

$$\ell' - 1 = \left\lfloor \frac{q-1}{f_{i,j}} \right\rfloor \tag{9}$$

holds. ℓ, ℓ', and q denote job numbers or counts, and hence are all non-negative integers. By (9), we have

$$\frac{q-1}{f_{i,j}} \;\geq\; \ell' - 1 \quad\Rightarrow\quad q - 1 \;\geq\; (\ell' - 1)\cdot f_{i,j} \quad\Rightarrow\quad q \;\geq\; \ell' \cdot f_{i,j} \qquad \{\text{because } f_{i,j} < 1\}, \tag{10}$$

and

$$\frac{q-1}{f_{i,j}} \;<\; \ell' \quad\Rightarrow\quad q - 1 \;<\; \ell' \cdot f_{i,j} \quad\Rightarrow\quad q \;<\; \ell' \cdot f_{i,j} + 1. \tag{11}$$

Because q is an integer, by (10) and (11), we have

$$q = \lceil \ell' \cdot f_{i,j} \rceil. \tag{12}$$

If $\ell' = \ell_0$ holds, then (J) follows from (12) and our definition of q. On the other hand, to show that (J) holds when $\ell' < \ell_0$, we must show that $q = \lceil \hat{\ell} \cdot f_{i,j} \rceil$ holds for all $\hat{\ell}$, where $\ell' < \hat{\ell} \leq \ell_0$. (Note that $\hat{\ell}$ is an integer.) By the definitions of q, ℓ', and ℓ_0, q of the first ℓ' jobs of T_i are assigned to P_j, and none of jobs $\ell' + 1$ through ℓ_0 are assigned to P_j. Therefore, by (8), it follows that $\hat{\ell} - 1 < \left\lfloor \frac{q}{f_{i,j}} \right\rfloor$ holds for all $\hat{\ell}$, where $\ell' < \hat{\ell} \leq \ell_0$. Thus, we have the following, for all $\hat{\ell}$, where $\ell' < \hat{\ell} \leq \ell_0$.

$$\left\lfloor \frac{q}{f_{i,j}} \right\rfloor > \hat{\ell} - 1 \;\Rightarrow\; \left\lfloor \frac{q}{f_{i,j}} \right\rfloor \geq \hat{\ell} \;\Rightarrow\; \frac{q}{f_{i,j}} \geq \hat{\ell} \;\Rightarrow\; q \geq \hat{\ell} \cdot f_{i,j} \;\Rightarrow\; q \geq \lceil \hat{\ell} \cdot f_{i,j} \rceil \quad \{\text{because } q \text{ is an integer}\} \tag{13}$$

By (12) and because $\hat{\ell} > \ell'$ holds, (13) implies that $\lceil \hat{\ell} \cdot f_{i,j} \rceil = \lceil \ell' \cdot f_{i,j} \rceil = q$ holds.

Finally, we are left with showing that at most $\lceil \ell \cdot f_{i,j} \rceil$ of any consecutive ℓ jobs of T_i are assigned to P_j. Let $\mathcal{J}$ represent jobs $\ell_0 + 1$ to $\ell_0 + \ell$ of T_i, where $\ell_0 \geq 0$. Then, by (J), exactly $\lceil \ell_0 \cdot f_{i,j} \rceil$ of the first ℓ_0 jobs and $\lceil (\ell_0 + \ell) \cdot f_{i,j} \rceil$ of the first $\ell_0 + \ell$ jobs of T_i are assigned to P_j. Therefore, the number of jobs belonging to $\mathcal{J}$ that are assigned to P_j, denoted $Jobs(\mathcal{J}, j)$, is given by

$$Jobs(\mathcal{J}, j) = \lceil (\ell_0 + \ell) \cdot f_{i,j} \rceil - \lceil \ell_0 \cdot f_{i,j} \rceil \leq \lceil \ell_0 \cdot f_{i,j} \rceil + \lceil \ell \cdot f_{i,j} \rceil - \lceil \ell_0 \cdot f_{i,j} \rceil = \lceil \ell \cdot f_{i,j} \rceil,$$

which proves the lemma. (The second step in the above derivation follows from $\lceil x + y \rceil \leq \lceil x \rceil + \lceil y \rceil$.) ■

We are now ready to derive a tardiness bound for EDF-fm.

3.3 Tardiness Bound for EDF-fm

As discussed earlier, jobs of migrating tasks do not miss their deadlines under EDF-fm. Also, if no migrating task is assigned to processor P_k, then the fixed tasks on P_k do not miss their deadlines. Hence, our analysis is reduced to determining the maximum amount by which a job of a fixed task may miss its deadline on each processor P_k, in the presence of migrating jobs. We assume that two migrating tasks, denoted T_i and T_j, are assigned to P_k. (A tardiness bound with only one migrating task can be derived from that obtained with two migrating tasks.) We prove the following.

(L) The tardiness of every fixed task of P_k is at most $\Delta = \frac{e_i(f_{i,k}+1)+e_j(f_{j,k}+1)}{1-s_{i,k}-s_{j,k}}$.

We prove (L) by contradiction. Contrary to (L), assume that job $T_{q,\ell}$ of a fixed task T_q assigned to P_k has a tardiness exceeding Δ. We use the following notation to assist with our analysis.

$$t_d \stackrel{\text{def}}{=} \text{absolute deadline of job } T_{q,\ell} \quad (14)$$

$$t_c \stackrel{\text{def}}{=} t_d + \Delta \quad (15)$$

$$t_0 \stackrel{\text{def}}{=} \text{latest instance before } t_c \text{ that } P_k \text{ was either idle or was executing a job of a fixed task with a deadline later than } t_d \quad (16)$$

By our assumption that job $T_{q,\ell}$ with absolute deadline at t_d has a tardiness exceeding Δ, it follows that $T_{q,\ell}$ does not complete execution at or before $t_c = t_d + \Delta$.

Let $\tau_{k,f}$ and $\tau_{k,m}$ denote the sets of all fixed and migrating tasks, respectively, that are assigned to P_k. (Note that $\tau_{k,m} = \{T_i, T_j\}$.) Let $demand(\tau, t_0, t_c)$ denote the maximum time that jobs of tasks in τ could execute in the interval $[t_0, t_c)$ on processor P_k (under the assumption that $T_{q,\ell}$ does not complete executing at t_c). We first determine $demand(\tau_{k,m}, t_0, t_c)$ and $demand(\tau_{k,f}, t_0, t_c)$.

By (16) and because migrating tasks have higher priority than fixed tasks under EDF-fm, jobs of T_i and T_j that are released before t_0 and assigned to P_k complete executing at or before t_0. Thus, every job of T_i or T_j that executes in $[t_0, t_c)$ on P_k is released in $[t_0, t_c)$. Also, every job released in $[t_0, t_c)$ and assigned to P_k places a demand for execution in $[t_0, t_c)$. The number of jobs of T_i that are released in $[t_0, t_c)$ is at most $\left\lceil \frac{t_c - t_0}{p_i} \right\rceil$. By Lemma 1, at most $\left\lceil f_{i,k} \left\lceil \frac{t_c - t_0}{p_i} \right\rceil \right\rceil \leq f_{i,k} \left(\frac{t_c - t_0}{p_i} + 1 \right) + 1$ of all the jobs of T_i released in $[t_0, t_c)$ are assigned to P_k. Similarly, the number of jobs of T_j that are assigned to P_k of all jobs of T_i released in $[t_0, t_c)$ is at most $f_{j,k} \left(\frac{t_c - t_0}{p_j} + 1 \right) + 1$. Each job of T_i executes for at most e_i time units and that of T_j for e_j time units. Therefore,

$$\begin{aligned} demand(\tau_{k,m}, t_0, t_c) &\leq \left(f_{i,k} \left(\frac{t_c - t_0}{p_i} + 1 \right) + 1 \right) \cdot e_i + \left(f_{j,k} \left(\frac{t_c - t_0}{p_j} + 1 \right) + 1 \right) \cdot e_j \\ &= s_{i,k}(t_c - t_0) + e_i(f_{i,k} + 1) + s_{j,k}(t_c - t_0) + e_j(f_{j,k} + 1) \qquad (17) \\ & \qquad \{\text{by (2) and simplification}\}. \end{aligned}$$

By (14)–(16), and our assumption that the tardiness of $T_{q,\ell}$ exceeds Δ, any job of a fixed task that executes on P_k in $[t_0, t_c)$ is released at or after t_0 and has a deadline at or before t_d. The number of such jobs of a fixed task T_f is at most $\left\lfloor \frac{t_d - t_0}{p_f} \right\rfloor$. Therefore,

$$\begin{aligned}
demand(\tau_{k,f}, t_0, t_c) &\leq \sum_{T_f \in \tau_{k,f}} \left\lfloor \frac{t_d - t_0}{p_f} \right\rfloor \cdot e_f \\
&\leq (t_d - t_0) \sum_{T_f \in \tau_{k,f}} \frac{e_f}{p_f} \\
&\leq (t_d - t_0)(1 - s_{i,k} - s_{j,k}) \quad \{\text{by (P3)}\}.
\end{aligned} \tag{18}$$

By (17) and (18), we have the following.

$$\begin{aligned}
demand(\tau_{k,f} \cup \tau_{k,m}, t_0, t_c) &\leq s_{i,k}(t_c - t_0) + e_i(f_{i,k} + 1) + s_{j,k}(t_c - t_0) + e_j(f_{j,k} + 1) + (t_d - t_0)(1 - s_{i,k} - s_{j,k}) \\
&= (s_{i,k} + s_{j,k})(t_c - t_d) + (s_{i,k} + s_{j,k})(t_d - t_0) + e_i(f_{i,k} + 1) \\
&\quad + e_j(f_{j,k} + 1) + (t_d - t_0)(1 - s_{i,k} - s_{j,k}) \\
&= (s_{i,k} + s_{j,k})(t_c - t_d) + e_i(f_{i,k} + 1) + e_j(f_{j,k} + 1) + (t_d - t_0)
\end{aligned}$$

Because $T_{q,\ell}$ does not complete executing by time t_c, it follows that the total processor time available in the interval $[t_0, t_c] = t_c - t_0 < demand(\tau_{k,f} \cup \tau_{k,m}, t_0, t_c)$, *i.e.*,

$$\begin{aligned}
& t_c - t_0 < (s_{i,k} + s_{j,k})(t_c - t_d) + e_i(f_{i,k} + 1) + e_j(f_{j,k} + 1) + (t_d - t_0) \\
\Rightarrow\ & t_c - t_d < (s_{i,k} + s_{j,k})(t_c - t_d) + e_i(f_{i,k} + 1) + e_j(f_{j,k} + 1) \\
\Rightarrow\ & t_c - t_d < \frac{e_i(f_{i,k} + 1) + e_j(f_{j,k} + 1)}{1 - s_{i,k} - s_{j,k}} = \Delta.
\end{aligned} \tag{19}$$

The above contradicts (15), and hence our assumption that the tardiness of $T_{q,\ell}$ exceeds Δ is incorrect. Therefore, (L) follows.

If only one migrating task T_i is assigned to P_k, then e_j and $s_{j,k}$ are zero. Hence, a tardiness bound for any fixed task on P_k is given by

$$\frac{e_i(f_{i,k} + 1)}{1 - s_{i,k}}. \tag{20}$$

If we let $m_{k,\ell}$, where $1 \leq \ell \leq 2$ denote the indices of the migrating tasks assigned to P_k, then by (L), a tardiness bound for EDF-fm is given by the following theorem.

Theorem 1 *On M processors, Algorithm* EDF-fm *ensures a tardiness of at most*

$$\max_{1 \leq k \leq M} \frac{e_{m_{k,1}}(f_{m_{k,1},k} + 1) + e_{m_{k,2}}(f_{m_{k,2},k} + 1)}{1 - s_{m_{k,1},k} - s_{m_{k,2},k}} \tag{21}$$

for every task set τ with $U_{sum}(\tau) \leq M$ and $u_{\max}(\tau) \leq 1/2$.

The tardiness bound given by Theorem 1 is directly proportional to the execution costs of the migrating tasks and the shares assigned to them. This bound could be high if the share of each migrating task is close to 1/2. However, because all tasks are light, in practice the sum of the shares of the migrating tasks assigned to a processor can be expected to be less than 1/2. Theorem 1 also suggests that the tardiness that results in practice could be reduced by choosing the set of migrating tasks carefully. Tardiness can also be reduced by distributing smaller pieces of works of migrating tasks than entire jobs. Some such techniques are discussed in the next section.

4 Tardiness Reduction Techniques for EDF-fm

The problem of assigning tasks to processors such that the tardiness bound given by (21) is minimized is a combinatorial optimization problem with complexity that is exponential in the number of tasks. Hence, in this section, we propose methods and heuristics that can lower tardiness. We consider the technique of *period transformation* [9] as a way of distributing the execution of jobs of migrating tasks more evenly over their periods in order to reduce the tardiness of fixed tasks. We also propose task assignment heuristics that can reduce the fraction of a processor's capacity consumed by migrating tasks.

Job-slicing approach. The tardiness bound of EDF-fm given by Theorem 1 is in multiples of the execution costs of migrating tasks. This is a direct consequence of statically prioritizing migrating tasks over fixed tasks and the overload (in terms of the number of jobs) that a migrating task may place on a processor over short intervals. The deleterious effect of this approach on jobs of fixed tasks can be mitigated by "slicing" each job of a migrating task into *sub-jobs* that have lower execution costs, assigning appropriate deadlines to the sub-jobs, and distributing and scheduling sub-jobs in the place of whole jobs. For example, every job of a task with an execution cost of 4 time units and relative deadline of 10 time units can be sliced into two sub-jobs with execution cost and relative deadline of 2 and 5, respectively, per sub-job, or four sub-jobs with an execution cost of 1 and relative deadline of 2.5, per sub-job. Such a job-slicing approach, termed *period transformation*, was proposed by Sha and Goodman [9] in the context of RM scheduling on uniprocessors. Their purpose was to boost the priority of tasks that have larger periods, but are more important than some other tasks with shorter periods, and thus ensure that the more important tasks do not miss deadlines under overloads. However, with the job-slicing approach under EDF-fm, it may be necessary to migrate a job between its processors, and EDF-fm loses the property that a task that migrates does so only across job boundaries. Thus, this approach presents a trade-off between tardiness and migration overhead.

Task-assignment heuristics. Another way of lowering the actual tardiness observed in practice would be to lower the total share $s_{m_{k,1},k} + s_{m_{k,2},k}$ assigned to the migrating tasks on any processor P_k. In the task assignment algorithm ASSIGN-TASKS of Fig. 1, if a low utilization-task is ordered between two high-utilization tasks, then it is possible that $s_{m_{k,1},k} + s_{m_{k,2},k}$ is arbitrarily close to one. For example, consider tasks T_{i-1}, T_i, and T_{i+1} with utilizations $\frac{1-\epsilon}{2}$, 2ϵ, and $\frac{1-\epsilon}{2}$, respectively, and a task assignment wherein T_{i-1} and T_{i+1} are the migrating tasks of P_k with shares of $\frac{1-2\epsilon}{2}$ each, and T_i is the only fixed task on P_k. Such an assignment, which can delay T_i excessively if the periods of T_{i-1} and T_{i+1} are large, can be easily avoided by ordering tasks by (monotonically) decreasing utilization prior to the assignment phase. Note that with tasks ordered by decreasing utilization, of all the tasks not yet assigned to processors, the one with the highest utilization is always chosen as the next migrating task. Hence, we call this assignment scheme *highest utilization first*, or HUF. An alternative *lowest utilization first*, or LUF, scheme can be defined that assigns fixed tasks in the order of (monotonically) decreasing utilization, but chooses the task with the lowest utilization of all the unassigned tasks as the next migrating task. Such an assignment can be accomplished using the following procedure when a migrating task needs to be chosen: traverse the unassigned task array in reverse order starting from the task with the lowest utilization and choose the first task whose utilization is at least the capacity available in the current processor. In general, this scheme can be expected to lower the shares of migrating tasks. However, because the unassigned tasks have to be scanned each time a migrating task is chosen, the time complexity of this scheme increases to $\mathcal{O}(NM)$ (from $\mathcal{O}(N)$). This complexity can be reduced to $\mathcal{O}(M \log N)$ by adopting a binary search strategy.

A third task-assignment heuristic, called *lowest execution-cost first*, or LEF, that is similar to LUF, can be defined by ordering tasks by execution costs, as opposed to utilizations. Fixed tasks are chosen in non-increasing order of execution costs; the unassigned task with the lowest execution cost, whose utilization is at least that of the available capacity in the current processor, is chosen as the next migrating task. The experiments reported in the next section show that LEF actually performs the best of these three task-assignment heuristics and that when combined with the job-slicing approach, can reduce tardiness dramatically in practice.

Including non-light tasks. The primary reason for restricting all tasks to be light is to prevent the total utilization $u_i + u_j$ of the two migrating tasks T_i and T_j assigned to a processor from exceeding one. (As already noted, ensuring that migrating tasks do not miss their deadlines may not be possible otherwise.) However, if the number of non-light tasks is small in comparison to the number of light tasks, then it may be possible to assign tasks to processors without assigning two migrating tasks with total utilization exceeding one to the same processor. In the simulation experiments discussed in Sec. 5, with no restrictions on per-task utilizations, the LUF approach could successfully assign approximately 78% of the one million randomly-generated task sets on 4 processors. The success ratio dropped to approximately one-half when the number of processors increased to 16.

Heuristic for processors with one migrating task. If the number of migrating tasks assigned to a processor P_k is one, then the commencement of the execution of a job $T_{i,j}$ of the only migrating task T_i of P_k can be postponed to time $d(T_{i,j}) - e_i$, where $d(T_{i,j})$ is the absolute deadline of job $T_{i,j}$ (instead of beginning its execution immediately upon its arrival). This would reduce the maximum tardiness of the fixed tasks on P_k to $e_i/(1 - s_{i,k})$ (from the value given by (20)). This technique will be particularly effective on two-processor systems, where each processor would be assigned at most one migrating task only under EDF-fm, and on three-processor systems, where at most one processor would be assigned two migrating tasks.

5 Experimental Evaluation

In this section, we describe the results of three sets of simulation experiments conducted using randomly-generated task sets to evaluate EDF-fm and the heuristics described in Sec. 4.

The experiments in the first set evaluate the various task assignment heuristics for varying numbers of processors, M, and varying maximum values of per-task utilization, $u_{\max}$. For each M and $u_{\max}$, $1,000,000$ task sets were

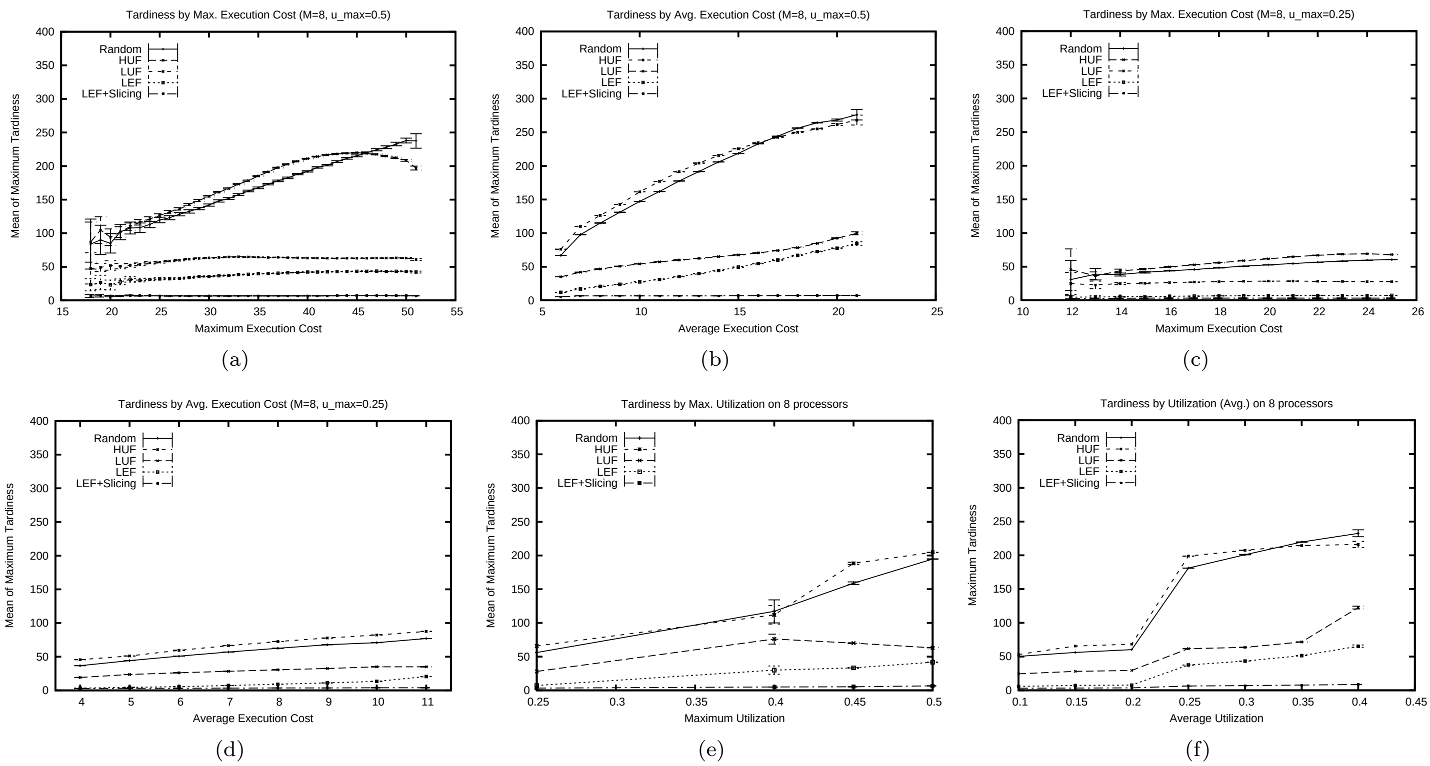
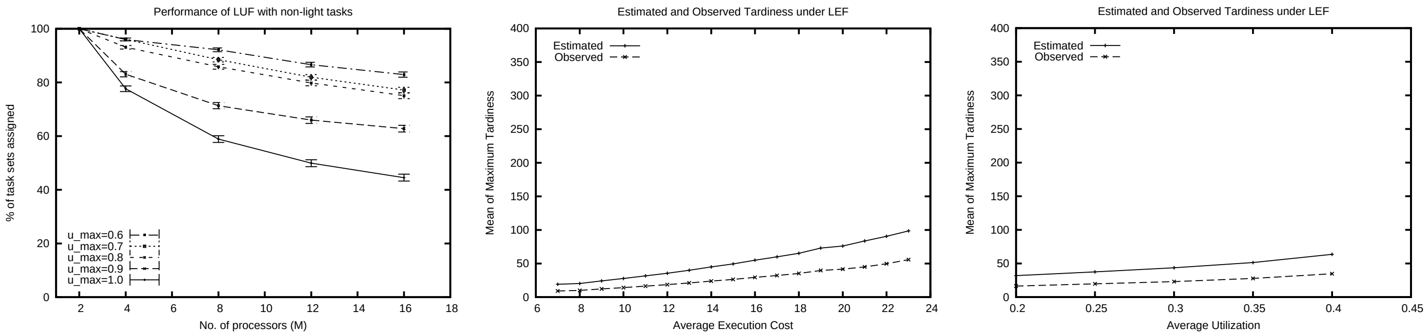


Figure 7: Comparison of different task assignment heuristics. Tardiness for $M = 8$ and $u_{\max} = 0.5$ by **(a)** $e_{\max}$ and **(b)** e_{avg}. Tardiness for $M = 8$ and $u_{\max} = 0.25$ by **(c)** $e_{\max}$ and **(d)** e_{avg}. Tardiness for $M = 8$ by **(e)** $u_{\max}$ and **(f)** u_{avg}.

generated. Each task set τ was generated as follows: New tasks were added to τ as long as the total utilization of τ was less than M. For each new task T_i, first, its period p_i was generated as a uniform random number in the range $[1, 100]$; then, its execution cost was chosen randomly in the range $[1/u_{max}, u_{\max} \cdot p_i]$. The last task was generated such that the total utilization of τ exactly equaled M. The generated task sets were classified in four different ways: **(i)** by the maximum execution cost of any task in a task set, $e_{\max}$, **(ii)** by the average execution cost of a task set, e_{avg} **(iii)** by the maximum utilization of any task in a task set, $u_{\max}$, and **(iv)** by the average utilization of a task set, u_{avg}. The tardiness bound given by (21) was computed for each task set under a random task assignment and also under heuristics HUF, LUF, and LEF. The average value of the tardiness bound for task sets in each group under each classification and heuristic was then computed. The results for the groups classified by $e_{\max}$ and e_{avg} for $M = 8$ and $u_{\max} = 0.5$ are shown in insets (a) and (b), respectively, of Fig. 7. Insets (c) and (d) contain the results under the same classifications for $M = 8$ and $u_{\max} = 0.25$. (99% confidence intervals are also shown in these plots.) Results for classification by $u_{\max}$ and u_{avg} are given in insets (e) and (f), respectively. (It is somewhat difficult to distinguish the plots in the figures. Mostly, the orders in the legend and the plots coincide.)

The plots show that LEF guarantees the minimum tardiness of the four task-assignment approaches. Tardiness is quite low (approximately 8 time units mostly) under LEF for $u_{\max} = 0.25$ (insets (c), (d), and (e)), which suggests that LEF may be a reasonable strategy for such task systems. Tardiness increases with increasing $u_{\max}$, but is still a reasonable value of 25 time units only for $e_{avg} \leq 10$ when $u_{\max} = 0.5$. However, for $e_{avg} = 20$, tardiness exceeds 75 time units, which may not be acceptable. For such systems, tardiness can be reduced by using



(a) (b) (c)

Figure 8: **(a)**Percentage of randomly-generated task sets with non-light tasks successfully assigned by the LUF heuristic. **(b)** & **(c)** Comparison of estimated and observed tardiness under EDF-fm-LEF by **(b)** average execution cost and **(c)** average utilization.

the job-slicing approach, at the cost of increased migration overhead. Therefore, in an attempt to determine the reduction possible with the job-slicing approach, we also computed the tardiness bound under LEF assuming that each job of a migrating task is sliced into sub-jobs with execution costs in the range $[1, 2)$. This bound is also plotted in the figures referred to above. For $M > 4$ and $u_{\max} = 0.5$, we found the bound to settle to approximately 7–8 time units, regardless of the execution costs and individual task utilizations. (When $u_{\max} = 0.25$, tardiness is 1–2 time units only under LEF with job slicing.) In our experiments, on average, a seven-fold decrease in tardiness was observed with job slicing with a granularity of one to two time units per sub-job. However, a commensurate increase in the number of migrations is also inevitable.

The second set of experiments evaluates the different heuristics in their ability to successfully assign task sets that contain non-light tasks also. Task sets were generated using the same procedure as that described for the first set of experiments above, except that $u_{\max}$ was varied between 0.6 and 1.0 in steps of 0.1. All of the four approaches could assign 100% of the task sets generated for $M = 2$, as expected. However, for higher values of M, the success ratio plummeted for all but the LUF approach. The percentage of task sets that LUF could successfully assign for varying M and $u_{\max}$ is shown in Fig. 8(a).

The third set of experiments was designed to evaluate the pessimism in the tardiness bound of (21). 300,000 task sets were generated with $u_{\max} = 0.5$ and $U_{sum} = 8$. The tardiness bound estimated by (21) under the LEF task assignment heuristic was computed for each task set. A schedule under EDF-fm-LEF for 100,000 time units was also generated for each task set and the actual maximum tardiness observed was noted. (The time limit of 100,000 was determined by trial-and-error as an upper bound on the time within which tardiness converged for the tasks sets generated.) Plots of the average of the estimated and observed values for tasks grouped by e_{avg} and u_{avg} are shown in insets (b) and (c) of Fig. 8, respectively. In general, we found that actual tardiness is only approximately half of the estimated value.

6 Concluding Remarks

We have proposed a new algorithm, EDF-fm, which is based on EDF, for scheduling recurrent soft real-time task systems on multiprocessors, and have derived a tardiness bound that can be guaranteed under it. Our algorithm places no restrictions on the total system utilization, but requires per-task utilizations to be at most one-half of a processor's capacity. This restriction is very liberal, and hence, our algorithm can be expected to be sufficient for scheduling a large percentage of soft real-time applications. Furthermore, under EDF-fm, only a bounded number of tasks need migrate, and each migrating task will execute on exactly two processors only. Thus, task migrations are restricted and the migration overhead of EDF-fm is limited. We have also proposed the use of the job-slicing technique, which can reduce the actual tardiness observed in practice, significantly.

Though a global EDF algorithm, with no restrictions on migration, would appear to be capable of guaranteeing a lower tardiness bound than EDF-fm, we have so far not been able to derive a non-trivial bound under it. In fact, we conjecture that a severe restriction on total utilization may be necessary, in addition to per-task utilization restrictions, to guarantee a non-trivial tardiness bound under unrestricted EDF.

We have only taken a first step towards understanding tardiness under EDF-based algorithms on multiprocessors and have not addressed all practical issues concerned. Foremost, the migration overhead of job slicing would translate into inflated execution costs for migrating tasks, and to an eventual loss of schedulable utilization. Hence, an iterative procedure for slicing jobs optimally may be essential. Next, our assumption that arbitrary task assignments are possible may not be true if tasks are not independent. Therefore, given a system specification that includes dependencies among tasks and tardiness that may be tolerated by the different tasks, a framework that determines whether a task assignment that meets the system requirements is feasible, is required. Finally, our algorithm, like every partitioning-based scheme, suffers from the drawback of not being capable of supporting dynamic task systems in which the set of tasks and task parameters can change at runtime. We defer addressing these issues to future work.

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