A Note on the Interpretation of σ_k

Clara Hobbs and James H. Anderson
Department of Computer Science, University of North Carolina at Chapel Hill

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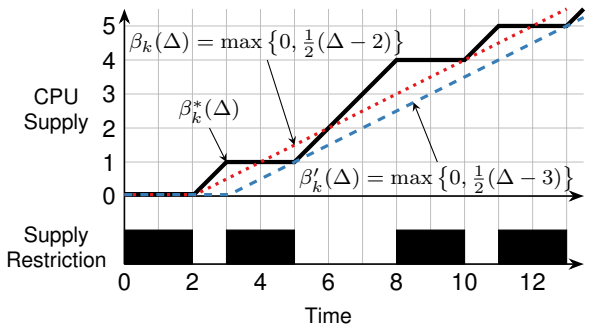

Figure 1: Service functions discussed in Example 1.

1 Problem

In several works pertaining to restricted processor supply [3, 4, 5, 6, 7, 8], a piecewise-linear lower bound called a *service function* [1] is given for the available processor time to tasks in some task set τ on processor k over any interval of length $\Delta \geq 0$. This bound is of the form

$$\beta_k(\Delta) = \max\{0, \widehat{u_k} \cdot (\Delta - \sigma_k)\},$$

and accompanied by an interpretation like the following.

> $\widehat{u_k}$ is the total long-term utilization available to the tasks in τ on processor k and σ_k is the maximum length of time when the processor can be unavailable. [3]

This interpretation of σ_k is flawed, and may result in the construction of a function $\beta_k(\Delta)$ that is not actually a lower bound to the available processor time on processor k. To see this, consider the following example.

Example 1. *Figure 1 depicts a processor k with a pattern of supply restriction that repeats every 8 time units. The intervals over which the processor is unavailable are shown in black at the bottom of the figure, and the total supply is given by the curve $\beta_k^*(\Delta)$. The long-term utilization available on processor k is $1/2$. According to the interpretation above, the value for σ_k should be 2, giving the function β_k shown in the figure. Clearly, β_k is not a lower-bound of $\beta_k^*(\Delta)$ (see e.g. the interval $(4, 6)$).*

2 Correct Explanation

A correct interpretation of the constants σ_k and $\widehat{u_k}$ in the function $\beta_k(\Delta)$ is given by Erickson and Anderson [2], reproduced here.

> σ_k is set to the x-intercept necessary in order for $\beta_k(\Delta)$ to lower-bound the actual supply, when the slope of $\beta_k(\Delta)$ is $\widehat{u_k}$. [2]

Example 1 (continued)**.** *Referring back to Figure 1, choosing a value of 3 for σ_k yields the function $\beta'_k(\Delta)$, which is a tight lower-bound of the actual supply $\beta_k^*(\Delta)$.*

References

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[2] Jeremy P Erickson and James H Anderson. Soft real-time scheduling. In Yu-Chu Tian and David C Levy, editors, *Handbook of Real-Time Computing*. Springer, Singapore, 2019.

[3] Hennadiy Leontyev. *Compositional analysis techniques for multiprocessor soft real-time scheduling*. PhD thesis, The University of North Carolina at Chapel Hill, 2010.

[4] Hennadiy Leontyev and James H Anderson. A hierarchical multiprocessor bandwidth reservation scheme with timing guarantees. In *Euromicro Conference on Real-Time Systems*, pages 191–200, July 2008.

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