Abstract—Many computer-vision (CV) applications used in autonomous vehicles rely on historical results, which introduce cycles in processing graphs. However, existing response-time analysis breaks down in the presence of cycles, either by failing completely or by drastically sacrificing parallelism or CV accuracy. To address this situation, this paper presents a new graph-based task model, based on the recently ratified OpenVX standard, that includes historical requirements and their induced cycles as first-class concepts. Using this model, response-time bounds for graphs that may contain cycles are derived. These bounds expose a tradeoff between responsiveness and CV accuracy that hinges on the extent of allowed parallelism. This tradeoff is illustrated via a CV case study involving pedestrian tracking. In this case study, the methods proposed in this paper enabled significant improvements in both analytical and observed response times, with acceptable CV accuracy, compared to prior methods.

I. INTRODUCTION

In semi- or fully autonomous advanced driver-assist systems (ADASs), computer-vision (CV) algorithms are often used to provide much of the safety-critical sensor-based processing. To facilitate the development of these algorithms, the OpenVX standard was ratified in 2014 [37]. OpenVX, which specifically targets heterogeneous embedded hardware, allows programmers to specify CV algorithms as dataflow graphs by interconnecting high-level CV primitives. While such an approach eases the design of CV algorithms, the OpenVX API has a glaring omission: it completely ignores real-time concerns. This omission led to recent work directed at applying real-time scheduling principles to OpenVX graphs and producing response-time bounds for such graphs [14,42–44].

Unfortunately, prior OpenVX response-time analysis breaks down in the presence of cycles, either by failing completely or by drastically sacrificing parallelism or CV accuracy. This is a critical shortcoming, because actual ADAS CV processing graphs often have cycles due to historical dependencies. For example, pedestrian tracking entails predicting future pedestrian positions from their prior trajectories. In order to be able to certify CV applications as used in ADASs, response-time analysis for cyclic OpenVX graphs is needed. If this problem is not addressed, these workloads cannot be certified.

In this paper, we address this problem by presenting the first ever response-time analysis for cyclic OpenVX graphs that does not require conservative methods that obviate cycles in simplistic ways at the price of degrading CV accuracy or schedulability. Our work specifically targets multicore platforms augmented with graphics processing units (GPUs)—arguably the most commonly considered hardware platform type in work involving OpenVX. To understand what we mean by “conservative methods,” and how we avoid them, an overview of prior work is needed.

Prior work. A number of methods exist for modeling dataflow applications [5, 7, 8, 21, 22, 36, 40]. Generally, these methods specify computations as processing graphs, with tasks corresponding to graph nodes, and edges indicating precedence relationships between tasks. The real-time scheduling and analysis of such graphs, both on uniprocessor and multiprocessors, has been extensively studied; representative publications include [1–4, 11, 13, 14, 17–20, 25–29, 31–35, 39, 41–44].

Of the just-cited papers, three [14, 42, 44] warrant further scrutiny: they are the only ones to consider OpenVX graphs, and one of them [42] is the only prior work to consider response-time bounds for cyclic multicore graphs. Two of these papers, by Elliott et al. [14] and by K. Yang et al. [42], are companion papers, focusing on implementation and analysis, respectively. K. Yang et al. proposed two techniques for breaking cycles. First, they noted that any back edge in a graph that feeds history information to its target task that is so “old” that real-time scheduling ensures the precedence constraint can simply be removed. However, in any CV algorithm that provides reasonable accuracy, such “old” history information would likely be of little use. Second, they showed that a given cycle can be broken by combining all of its nodes into a single sequential supernode. This technique can be applied to convert any OpenVX graph into a DAG. K. Yang et al. showed that a response-time bound can be computed for such a DAG by transforming it to an “equivalent” set of independent sporadic tasks, as done in earlier work by Liu and Anderson on DAGs generally [25]. However, this transformation process requires the utilization of each node (i.e., task) to be at most 1.0, a restriction that can be easily violated by a supernode.

More recently, M. Yang et al. [44] proposed altering the transformation process above by converting to a sporadic task set that allows intra-task parallelism (i.e., multiple jobs of the same task may execute concurrently), as done in earlier work by K. Yang et al. [43] in work not pertaining to OpenVX. M. Yang et al. showed that such parallelism enables much lower response-time bounds for OpenVX graphs [15]. However, parallel node execution breaks the supernode idea, so they expressly considered cycles to be out of scope [44].

Contributions. We extend the prior transformation-based...
methods discussed above [14, 42, 44] to enable the real-time certification of arbitrary OpenVX graphs on multicore+GPU platforms. We make three key contributions.

First, we extend the transformation process of M. Yang et al. [44] to deal with cyclic graphs. Our key insight here is based upon a property of the sporadic task model with intra-task parallelism: under this model, per-task response-time bounds can be computed without requiring task utilizations to be at most 1.0. This fact suggests a way forward for handling arbitrary supernodes. While (as noted earlier) parallelism breaks the supernode idea, we show that it can be allowed if back edges can supply slightly older history information. In fact, we will show that, for any schedulable system of graphs, the degree of parallelism that can be allowed, the age of history information, and the response-time bounds that can be guaranteed are all closely linked. Loosely speaking, older history information allows for increased parallelism and lower response-time bounds; insisting on the most recent possible history information can kill parallelism and result in an unschedulable graph. The designers of CV algorithms should be aware of these tradeoffs when constructing OpenVX graphs. In particular, they should set history age requirements so that both CV accuracy and response-time bounds are acceptable.

From a schedulability point of view, setting history age requirements equates to specifying an allowed degree of parallelism in processing a cycle. Thus, we need as the end point of our transformation process a sporadic task model wherein the allowed intra-task parallelism is a per-task settable parameter. Our second key contribution involves defining such a task model, namely the rp-sporadic task model (restricted parallelism), and presenting response-time analysis for it.

Though analytically interesting, it remains to be seen whether the parallelism/accuracy/response-time tradeoffs enabled by our work are worthwhile to consider from the perspective of a CV algorithm designer. As a final contribution, we present an assessment of this issue via a case study involving pedestrian tracking. In this study, we consider an OpenVX graph that is actually unschedulable as originally specified and show the effects of increasing parallelism. We found that we were able to bound response times for this graph if intra-task parallelism is enabled, with only a minor accuracy drop compared to the original unschedulable graph (which has the highest accuracy but unbounded response times).

**Generality.** Although we focus on OpenVX as our motivation, the rp-sporadic task model and the derived response-time analysis are applicable to any application that can be specified as a sporadic task graph containing cycles, as Fig. 1 (discussed in Sec. II) implies. Such graphs may arise in many contexts, such as control, motion-planning, and recurrent neural networks; if the utilization of a cycle is greater than 1.0, prior work cannot provide response-time bounds for these graphs.

**Organization.** In the rest of this paper, we present our new transformation process (Sec. II), present the rp-sporadic task model (Sec. III) and response-time analysis under it (Sec. IV), present our case study (Sec. V), and conclude (Sec. VII).

### II. Transformation Process

Prior work has shown how to transform an OpenVX graph into an “equivalent” set of independent sporadic tasks [32, 42, 44], for which response-time analysis exists [10, 15, 16, 23–25]. This process is depicted in Fig. 1. However, Step 3, as originally proposed [42], requires that the utilization of each cycle is at most 1.0.

In this section, we illustrate the existing transformation steps and discuss the implications of full or no intra-task parallelism (choices (a) and (b) in Fig. 1). We then describe how we augment Steps 2-4 to allow restricted parallelism, enabling this approach for graphs containing cycles of any utilization.

#### A. OpenVX

In OpenVX, primitives and the data objects upon which they operate comprise a bipartite graph [38]. An OpenVX graph $G^i$ contains data objects $D^i_1, \ldots, D^i_{u_i}$ and nodes $N^i_1, \ldots, N^i_{2i}$. An edge $(N^i_u, D^i_w)$ corresponds to a data object $D^i_w$ that is written by node $N^i_u$, and $(D^i_{w'}, N^i_k)$ corresponds to a data object read by node $N^i_k$. Data objects can optionally be delay objects,
indicating that the data from prior time steps must be buffered for later use. Associated with each delay object is a value indicating the age, in time steps, of the data.

To simplify analysis, we assume that each graph has a single source node and a single sink node. (If this is not the case, a single “virtual” source and/or sink can be added.) For all graphs we consider, we assume that the first indexed node \( (N_1^i) \) for an OpenVX graph \( G^i \) is the source.

Ex. 1. An example OpenVX graph \( G^1 \) is shown in Fig. 2. In this figure, rectangles correspond to data objects, \( D_1^1, \ldots, D_3^1 \), and round nodes indicate primitives, \( N_1^1, \ldots, N_4^1 \), that act on there. There are three delay objects, \( D_3^1, D_5^1, \) and \( D_6^1 \), with delay values 1, 3, and 2, respectively.

The OpenVX standard specifies a series of rules for processing graphs [38]. The rules relevant to our work are:

1) **Single Writer**: Every data object has at most one incoming edge.

2) **Broken Cycles**: Every cycle in \( G^i \) must contain at least one input edge \( (D_u^i, N_v^i) \) where \( D_u^i \) is a delay object.

Ex. 1 (cont’d). In \( G^1 \), every data object has a single incoming edge (although \( D_2^1 \) has two outgoing edges). Additionally, there are two cycles, containing edges from delay objects \( D_1^1 \) and \( D_6^1 \) to node \( N_3^1 \).

**B. Transforming OpenVX Graphs to Sporadic Task Sets**

The transformation process depicted in Fig. 1 must be performed for each OpenVX graph \( G^i \) in a system. We now illustrate each step in detail.

**Step 1: From a coarse- to a fine-grained OpenVX graph.**

The OpenVX standard specifies little about the concurrent execution of primitives within a graph. M. Yang et al. [44] showed that treating each primitive as a schedulable entity is often too coarse-grained to guarantee bounded response times. Rather, primitives should be split into multiple nodes, with each executing on either a CPU or a GPU.

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We assume the mapping of primitive to processor type is decided by the application designer.

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**Step 2: From a fine-grained OpenVX graph to a sporadic task graph.**

A sporadic task graph \( \Gamma^i \) is comprised of \( z_i \) nodes, \( \tau_1^i, \ldots, \tau_z^i \), with each node corresponding to a task. A task \( \tau_u^i \) releases a potentially infinite series of jobs \( J_u^1, J_u^2, \ldots \).

Edges in \( \Gamma^i \) indicate producer/consumer relationships between tasks: a job must wait to begin execution until the corresponding job of each task from which it consumes data (i.e., for each edge for which it is a consumer) has completed.

Given a fine-grained OpenVX graph \( G^i \), we can perform a simple transformation to obtain a sporadic task graph \( \Gamma^i \):

- Each node \( \tau_u^i \) in \( G^i \) becomes a node \( \tau_u^i \) in \( \Gamma^i \).
- Each input edge \( (D_u^i, N_v^i) \) (other than that into the source \( \tau_1^i \)) becomes a directed edge \( (\tau_u^i, \tau_v^i) \), where \( \tau_u^i \) is the single writer of data object \( D_u^i \).
- An edge is a delay edge if its corresponding data object \( D_u^i \) is a delay object, and a regular edge otherwise.

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Fig. 3: A fine-grained OpenVX graph \( G^2 \) corresponding to the coarse-grained graph in Fig. 2. \( G^2 \) contains six nodes (four CPU nodes and two GPU nodes) and ten data objects. \( N_3^1 \) and \( N_4^1 \) have each been expanded to separate CPU and GPU nodes, and new data objects have been added.

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Fig. 4: A sporadic task graph \( \Gamma^2 \) derived from the fine-grained OpenVX graph from Fig. 3.
Note that delay edges can be either forward or backward edges, depending on whether they result in a cycle in the graph. For each delay edge \((\tau_i, \tau_j)\), we include a range \([p, q]\), \(p \leq q\), corresponding to the range of delay values for that edge. Thus, a delay edge \((\tau_i, \tau_j)\) with range \([p, q]\) indicates that a job \(J_{i,j}\) relies on the outputs of \(\{J_{i,j-p}, \ldots, J_{i,j-q}\}\).

Ex. 2 (cont’d). Fig. 4 shows the sporadic task graph \(\Gamma^2\) corresponding to the fine-grained OpenVX graph from Fig. 3. The three delay objects are represented here as two delay edges, one forward and one backward. The delay values are encapsulated in the \(p\) and \(q\) values for the delay edges. \(\diamond\)

Step 3: From a sporadic task graph to a sporadic task DAG. K. Yang et al. [42] provided a series of rules for removing delay edges from graphs, resulting in a DAG. They showed that forward delay edges can simply be replaced by regular edges, and they proposed to break cycles by combining all nodes in a given cycle in a graph into a single supernode.

Ex. 2 (cont’d). Fig. 5 shows the DAG \(\tau^2\) derived from the cyclic graph \(\Gamma^2\) in Fig. 4. The forward delay edge from \(\tau_1^2\) to \(\tau_2^2\) has been removed because a regular edge between these nodes already exists, and nodes \(\tau_2^2\), \(\tau_3^2\), and \(\tau_6^2\) comprising the cycle have been combined into a single supernode \(\tau_{456}^2\). \(\diamond\)

Step 4: From a sporadic task DAG to a sporadic task set. Given a sporadic task DAG \(\tau^i\), it is straightforward to consider each node as an independent sporadic task. Each task \(\tau_i^j\) has a worst-case execution time given by \(C_i^j\) and a relative deadline given by \(D_i^j\). All tasks belonging to \(\tau^i\) share a period \(T^i\). Jobs of the source task \(\tau_1^i\) are assumed to be released sporadically, at least \(T^i\) time units apart. For non-source tasks, Liu et al. [32] showed how response-time bounds \(R_u^i\) (explained in detail later) of tasks \(\tau_u^i\) that produce data consumed by \(\tau_v^i\) can be used to set an offset \(\Phi_v^i\). This offset specifies the release time of a job \(J_{i,j}\) relative to that of its graph’s corresponding source job \(J_{i,j}^1\). Note that task deadlines are used here to define priorities rather than strict (hard) timing constraints, so \(R_u^i\) may exceed \(D_i^j\), i.e., jobs may complete after their deadlines.

Ex. 2 (cont’d). Fig. 6 depicts an example schedule for the task set derived from the sporadic task DAG \(\tau^2\) in Fig. 5. In this example, we assume response-time bounds of the four DAG tasks have been computed to be \(R_1^2 = 9, R_2^2 = 7, R_3^2 = 5,\) and \(R_{456}^2 = 9\). As described in [32], \(\Phi_1^2 = 0,\) \(\Phi_2^2 = \Phi_3^2 = R_2^2 = 9,\) and \(\Phi_{456}^2 = \max\{\Phi_2^2 + R_2^2, \Phi_3^2 + R_3^2\} = 16\). \(\diamond\)

We define the utilization of \(\tau_i^j\) to be \(u_v^i = C_i^j/T^i\). The utilization of the entire system is given by \(U = \sum_{\tau_i \in \Gamma} \sum_{\tau_v \in \tau_i} u_v^i\). We can define the utilization of a cycle similarly: \(\sum_{\tau_v \in \tau} u_v^i\), where \(\tau^i\) is the set of tasks in the cycle.

C. Response-Time Analysis

For a job \(J_{i,j}\) of the task \(\tau_v^i\), let \(r_{u,j}^i\) denote its release time, let \(f_{u,j}^i\) denote its completion time (or finish time), and let its deadline be \(d_{u,j}^i = r_{u,j}^i + T^i\). We define \(J_{i,j}^1\)’s response time as \(f_{i,j}^1 = r_{i,j}^1 + T^i\) and the end-to-end response time of a sporadic task graph \(\Gamma^i\) as \(\max_j\{f_{i,j}^1 - r_{i,j}^1\}\).

We seek to calculate a response-time bound \(R_v^i\) for each task \(\tau_v^i\). Such bounds can be propagated back to the original graph(s) to give end-to-end response-time bounds of all graphs. The available response-time analysis depends upon the choice of parallelism in the sporadic task model.

Existing sporadic task models. The conventional sporadic task model requires jobs of the same task to execute sequentially, i.e., a job \(J_{i,j}, j \geq 2,\) is not ready unless \(J_{i,j-1}\) has completed execution. This model has been the subject of much prior work on response-time analysis under global schedulers [10, 16, 23, 24], which will be our focus here.

Ex. 3. Fig. 7 depicts a possible schedule for jobs of \(\tau_{456}^2\) from Fig. 5 on a platform with four CPUs and one GPU, assuming \(T^2 = 5, C_{456}^2 = 6,\) and \(R_{456}^2 = 21\). The schedule begins at time 100, when job \(J_{456,21}^2\) is released.

In schedule (a), the jobs execute sequentially. Due to jobs of other tasks (not shown), \(J_{456,21}^2\) is not scheduled until time 114. This postponement impacts the subsequent jobs; \(J_{456,24}^2\) has a response time of 7.4. However, the \(p = 2\) requirement is met, e.g., \(J_{456,21}^2\) completes before \(J_{456,23}^2\) begins. \(\diamond\)
Later work considered a model that allows full intra-task parallelism, i.e., any number of unfinished jobs of the same task may execute concurrently. This model enables much smaller response-time bounds to be ensured [15].

Ex. 3 (cont’d). Schedule (b) in Fig. 7 shows the result of allowing full intra-task parallelism. We assume GPU computations are FIFO scheduled, which causes three of the four jobs’ execution times to increase. However, the response time of $J_{456,24}$ is reduced to 3.2 time units.

Unfortunately, unrestricted intra-task parallelism creates two problems. First, the jobs of a task can complete out of order; but this can be simply resolved by buffering job outputs, as discussed in [14]. Second, and more importantly, such parallelism can violate the dependencies required by backward delay edges. In fact, sequential execution was originally assumed for the transformation to a DAG (Step 3) [42]. Theorem 3 in [42] showed that if $p = 1$ for some backward delay edge, then no two jobs of any task in that cycle can execute in parallel. This proof can be generalized to show that if more than $p$ jobs of a task in a cycle execute concurrently, then a precedence constraint must be violated.

Ex. 3 (cont’d). The supernode $\tau^2_{456}$ was created from a cycle with $p = 2$. Thus, job $J^2_{456,22}$ requires output from job $J^2_{456,21}$. However, in schedule (b) of Fig. 7, jobs $J^2_{456,21}$ and $J^2_{456,23}$ execute concurrently, violating this precedence constraint.

The troublesome history. Response-time analysis for sequential sporadic tasks requires $u^i_v \leq 1.0$ for all tasks. This requirement extends to supernodes in [42]: the utilization of each cycle must be at most 1.0. However, if smaller bounds are desired or if the cycle has higher utilization, no existing analysis can be applied. Furthermore, cycles with utilization exceeding 1.0 can easily occur in actual CV graphs. When full intra-task parallelism is enabled, $u^i_v \leq 1.0$ is no longer required, but historical requirements may not be met.

Ex. 3 (cont’d). If jobs execute sequentially as in Fig. 7(a), response times can be unbounded for $\tau^2_{456}$, as $u^2_{456} = 6/5$.

D. A New Hybrid Approach

Our work bridges this parallelism divide, resulting in response-time bounds for sporadic task graphs (and thus OpenVX graphs) that prior work deemed infeasible. We provide a new restricted-parallelism sporadic task model that specifies intra-task parallelism on a per-task basis. A key feature of our approach is that per-task utilizations are allowed to exceed 1.0, yet parallelism (and hence accuracy) is controlled.

Ex. 3 (cont’d). Restricted intra-task parallelism is shown in schedule (c) of Fig. 7. The response time of $J^2_{156,24}$ is increased to 4.0, but the history requirements are respected, as only $p = 2$ jobs of $\tau^2_{456}$ execute concurrently.

Abstracting GPU computations. Although M. Yang et al. [44] suggested considering CPU and GPU tasks separately in response-time analysis, their results hold only for DAGs. Instead, as in K. Yang et al. [42], we arbitrate access to the GPU with a locking protocol, such as GPUSync [12]. Thus, we henceforth assume that all graph nodes are CPU nodes, with their worst-case execution times inflated to include GPU blocking and execution time, and that tasks can contain non-preemptive regions due to said locking protocol.

Transforming cycles, revisited. We leverage the supernode concept from [42] to transform a sporadic task graph $\Gamma^i$ into a sporadic task DAG $\tau^i$. We supplement each node $\tau^i_v$ of the DAG with a value $P^i_v$ indicating the allowed intra-task parallelism for the jobs of that task. All tasks within a cycle are combined into a single supernode $\tau^i_v$ with $P^i_v$ defined to be the smallest $p$ of any forward or backward delay edge contained in the cycle (we do not use $q$, as it is does not limit the parallelism of the cycle). A task $\tau^i_v$ that is not part of any cycle has $P^i_v = m$, the number of CPU processors, i.e., unrestricted intra-task parallelism, as in [44].

Ex. 4. Fig. 8 depicts the DAG that results from our parallelism-aware supernode transformation. The nodes correspond to those in Fig. 5, and are labeled with their intra-task parallelism values $P^i_v$. For tasks that are not supernodes, the intra-task parallelism is $m$. Task $\tau^2_{456}$ is a supernode derived from a cycle with $p = 2$ in Fig. 4, so it has $P^2_{456} = 2$.

Offset computation for forward delay edges. In prior work, forward delay edges were either deemed as out of scope [44],

\[\text{Fig. 8: Per-task intra-task parallelism for nodes of } \tau^2 \text{ from Fig. 5.}\]
or supported assuming only sequential task execution [13, 42]. We propose a different method for handling such edges here.

Consider such a forward delay edge \((\tau_i^u, \tau_i^v)\) with a delay \(p\). Denote the offset of \(\tau_i^u\) computed in a DAG without the delay edge as \(\Phi_i^u\). The forward delay edge adds the requirement that \(\tau_i^v\) not start earlier than the completion of node \(\tau_i^u\) \(p\) DAG periods ago. Thus, we require \(\Phi_i^u \geq \Phi_i^v + R_i - p \cdot T_i\). At the same time, we require \(\Phi_i^u \geq \Phi_i^v\). Combining both expressions, we have \(\Phi_i^v = \max(\Phi_i^u, \Phi_i^v + R_i - p \cdot T_i)\).

Note that, because offsets are determined from source to sink [32], by the definition of a forward delay, \(\Phi_i^u\) is available when \(\Phi_i^v\) is determined. Note also that the method above can be generalized for the case wherein forward delay edges are directed from several nodes to the node \(\tau_i^v\).

K. Yang et al. [42] proposed instead to replace each forward delay edge with a regular forward edge. Effectively, such a replacement is equivalent to the computation of \(\Phi_i^u\) with \(p = 0\), so our approach generalizes theirs.

To this point, we have explained how to adapt prior work to transform a coarse-grained OpenVX graph into an “equivalent” sporadic task set with restricted parallelism. What remains is to formally define this sporadic task model variant and to derive response-time bounds under it. This we do next in Secs. III and IV, respectively.

III. THE RP-SPORADIC TASK MODEL

We now introduce the rp-sporadic task model, which permits per-task allowed parallelism to be specified. Under this model, the \(i^{th}\) task is specified as \(\tau_i = (\Phi_i, T_i, C_i, P_i)\), where \(\Phi_i, T_i, C_i,\) and \(P_i\) are as defined in Sec. II (but omitting the graph index, as it is not relevant to us here). We assume that tasks have implicit deadlines, i.e., \(D_i = T_i\). We denote \(\tau_i\)’s utilization as \(u_i = C_i/T_i\); total utilization as \(U\), the \(j^{th}\) job of \(\tau_i\) as \(J_{i,j}\), its release time as \(r_{i,j}\), the maximal length of a single non-preemptive section as \(B_{\text{max}}\) (recall our earlier discussion about using locking protocols to arbitrate GPU access), and the maximal worst-case execution time (WCET) of any task as \(C_{\text{max}}\).

Scheduler. We consider a platform with \(m\) CPUs (recall that, with GPU access arbitrated using locking protocols, we can focus on a CPU-only system in our analysis). Global earliest-deadline-first (G-EDF) scheduling guarantees bounded response times [10, 15], so we assume G-EDF scheduling with deadline ties broken arbitrarily but consistently (e.g., by task index). We let \(J_{i,j} \prec J_{k,l}\) denote that job \(J_{i,j}\) has higher priority than job \(J_{k,l}\).

Feasibility conditions. As in existing response-time analysis, we require \(U \leq m\), or the entire system can become overutilized, with response times being unbounded. Additionally, at most \(P_i\) jobs of a task \(\tau_i\) can execute at once, so we require

\[
\forall i : \quad u_i \leq P_i. \tag{1}
\]

In particular, with \(\tau_i\) restricted to execute on at most \(P_i\) processors at any time, if \(u_i > P_i\) and \(\tau_i\) releases jobs as early as possible, its response times will grow without bound.

IV. RESPONSE-TIME BOUNDS

In this section, we prove that every task of a feasible rp-sporadic task set \(\tau\) has bounded response times under G-EDF. In proving this result, we assume time to be continuous.

A. Basic Bound

Throughout this section, we consider a job of interest; as the proven response-time bound holds for any job of interest, it inductively applies to all jobs of all tasks in the task system. We consider an analysis window, and bound the amount of work that conflicts with the job of interest within this window. Initially, we consider a simpler edge case (Lemma 2). For the more complex case, we first show that non-preemptive sections of lower-priority jobs can affect the execution of higher-priority jobs only if such sections are scheduled at the start of the analysis window (Lemma 3). To bound the response time for the job of interest, we first bound the total workload of high-priority jobs given their maximal response times (Lemma 4). Then, we show that the inductively assumed response-time bounds of high-priority jobs ensure the same bound for the job of interest if it is big enough (Lemma 5). Finally, we present our full response-time theorem (Theorem 1) and its closed-form version (Corollary 1).

Def. 1. At a time instant \(t\), job \(J_{i,j}\) is unreleased if \(t < r_{i,j}\) and released otherwise; \(J_{i,j}\) is complete if it is completed by \(t\); and \(J_{i,j}\) is ready if it is released, and the job \(J_{i,j-P_i}\) is complete (i.e., \(J_{i,j}\) can be scheduled at \(t\)).

Job of interest. We consider an arbitrary job \(J_{k,l}\) of a task \(\tau_k \in \tau\). Let \(d\) be the absolute deadline of \(J_{k,l}\), i.e., \(d = r_{k,l} + T_k\). Let \(f\) be the completion time of \(J_{k,l}\). We assume \(d < f\), for otherwise the response time of \(J_{k,l}\) is less than \(T_k\). We will show inductively with respect to \(\prec\) that the response time of \(\tau_k\) is bounded by \(x + T_k + C_k\) for any positive \(x\) that is large enough (as formalized later in (9)).

Def. 2. We let \(\Psi\) (resp., \(\overline{\Psi}\)) denote the job set consisting of all jobs that have higher (resp., lower) priority than \(J_{k,l}\).

Def. 3. We say that a time instant \(t\) is busy if \(m\) jobs of \(\Psi \cup \{J_{k,l}\}\) are scheduled, or there is a ready job in \(\Psi \cup \{J_{k,l}\}\) that is not scheduled at \(t\), and non-busy otherwise. Note that both conditions imply that every processor executes a job. We say that a time interval \([t', t']\) is busy if all instants in it are busy.

Lemma 1. For any task \(\tau_i\), the number of its ready jobs in \(\Psi \cup \{J_{k,l}\}\) does not increase after \(t_d\).

Proof. All jobs in \(\Psi \cup \{J_{k,l}\}\) are released within \([0, t_d]\). A job \(J_{i,j}\) becomes ready at \(t\) if and only if \(J_{i,j-P_i}\) is completed at \(t\). Thus, the total number of ready jobs of \(\tau_i\) in \(\Psi \cup \{J_{k,l}\}\) does not increase after \(t_d\). \(\square\)

There are two cases for \(t_d\): it is either a busy or a non-busy time instant. We will consider the non-busy case (Lemma 2) first and then the busy case (Lemmas 3-5).

Lemma 2. If \(t_d\) is a non-busy time instant, and the response time of each job of \(\tau_k\) released before \(J_{k,l}\) is at most \(x + T_k + C_k\),
then the response time of \( J_{k,l} \) is bounded by \( x + T_k + C_k \). (No conditions on \( x \) except \( x \geq 0 \) are implied in this lemma.)

**Proof.** By Lemma 1, the number of ready jobs in \( \Psi \cup \{ J_{k,l} \} \) does not increase after \( t_d \). Therefore, if \( t_d \) is not a busy time instant, then any later time instant is not busy, as jobs from \( \Psi \cup \{ J_{k,l} \} \) occupy fewer than \( m \) processors.

Thus, \( J_{k,l} \) is scheduled at the time instant it becomes ready, which might be after \( t_d \), as illustrated in Fig. 9. \( J_{k,l} \) becomes ready upon completion of \( J_{k,l-P_k} \), which was released by \( t_d - T_k - P_k - T_k \). By the lemma statement, \( J_{k,l-P_k} \) must complete by \( t_d - T_k - P_k - T_k + x + C_k + T_k = t_d - P_k \cdot T_k + x + C_k \). By (8), \( C_k \leq P_k \cdot T_k \), so \( J_{k,l} \) becomes ready by \( t_d + x \). As \( J_{k,l} \) is scheduled immediately upon becoming ready, it completes by \( t_d + x + C_k \), within \( x + T_k + C_k \) time from its release.

We now consider the case when \( t_d \) is busy.

**Def. 4.** Let \( t_0 \) denote the first busy instant such that \([t_0, t_d)\) is a busy interval. Let \( t_b \) denote the last time instant such that \([t_d, t_b)\) is a busy interval.

The following lemma limits the number of lower-priority jobs in \( \Psi \) that can affect the execution of higher-priority ones.

**Lemma 3.** A non-preemptive section of a job \( J_{i,j} \) in \( \Psi \) may block the execution of ready jobs in \( \Psi \cup \{ J_{k,l} \} \) within \([t_0, t_f)\) only if that section is scheduled at \( t_0 \). Moreover, such blocking may occur only within \([t_0, t_b)\).

**Proof.** Consider the interval \([t_0, t_f)\), depicted in Fig. 10 for the cases (a) \( t_b > t_f \), and (b) \( t_b \leq t_f \) (note that \( t_{av} \) is defined later in Lemma 5). We begin by showing, in both cases, that all time instants after \( t_0 \) are busy. By Def. 3, at \( t_b \), the at most \( m - 1 \) not-completed ready jobs in \( \Psi \cup \{ J_{k,l} \} \) are all scheduled. By Lemma 1, the number of ready jobs in \( \Psi \cup \{ J_{k,l} \} \) does not increase after \( t_d \). If a job \( J_{w,x} \in \Psi \cup \{ J_{k,l} \} \) becomes ready at some time \( t > t_d \), then \( J_{w,x-P_w} \) is completed at \( t \); the number of ready jobs in \( \Psi \cup \{ J_{k,l} \} \) does not increase at \( t \), and there is a processor available for \( J_{w,x} \) (namely, the processor where \( J_{w,x-P_w} \) completed). Additionally, as jobs in \( \Psi \cup \{ J_{k,l} \} \) have higher priority than those in \( \Psi \), they remain scheduled until they complete, so no time instant after \( t_0 \) is busy.

By Def. 3, if \( J_{i,j} \in \Psi \) blocks a job in \( \Psi \cup \{ J_{k,l} \} \) at \( t' \in [t_0, t_f) \), then \( t' \) is a busy instant. As no time instant after \( t_0 \) is busy, \( t' \in [t_0, t_0) \). \( J_{i,j} \) has lower priority than any job in \( \Psi \cup \{ J_{k,l} \} \), so it must therefore execute non-preemptively at every instant in \([t_0, t']\), or else it would be preempted. Thus, the non-preemptive section scheduled at \( t' \) must also be scheduled at \( t_0 \), and blocking by \( J_{i,j} \) occurs only within \([t_0, t_b)\).

Let \( W_d \) be \( C_k \) plus the total workload that can potentially prevent the execution of \( J_{k,l} \). By Lemma 3, \( W_d \) includes the workload of non-preemptive sections jobs of \( \Psi \) that are scheduled at \( t_0 \) and the workload of all jobs in \( \Psi \cup \{ J_{k,l} \} \).

By Lemma 4, given below, \( L(x) \), defined next, is an upper bound for \( W_d \).

\[
L(x) = (m-1)C_{max} + B_{max} + \max_{C \subseteq \mathcal{T} | \tau \leq T_{\mathcal{T}} \leq m-1} \left( \sum_{c \in \mathcal{C}} (u_c x + 2C_{c}) \right)
\]

**Lemma 4.** If \( t_d \) is a busy time instant, and the response time of each job \( J_{i,j} \in \Psi \) is at most \( x + T_i + C_i \), then \( W_d \leq L(x) \).

**Proof.** Let \( t_0 \) denote an arbitrarily small \( \varepsilon > 0 \) such that \([t_0, t_0) \) is a non-busy interval, illustrated in Fig. 11. Because \( \varepsilon \) is arbitrarily small, no scheduling events (jobs completions or releases) happen within \([t_0, t_0) \). To upper bound \( W_d \), we first bound the workload of jobs released before \( t_0 \) in Claims 1 and 2. Then we bound the workload of jobs released within \([t_0, t_d) \) in Claim 3. Finally, we bound how much of the workload is completed over \([t_0, t_d) \) in Claim 4. (For clarity, claim proofs end with ■ while other proofs end with □.)

Let \( a \) (resp., \( b \)) be the number of jobs in \( \Psi \cup \{ J_{k,l} \} \) (resp., \( \Psi \)) that are scheduled at \( t_0 \).
Claim 1. Consider the jobs that are scheduled at $t_0^-$. Their total non-completed workload at $t_0^-$ is at most $aC_{\text{max}} + bB_{\text{max}}$.

Proof. By Lemma 3, only non-preemptive sections of jobs in $\Psi$ can block the execution of jobs in $\Psi \cup \{J_{k,l}\}$. The maximal length of a non-preemptive section is $B_{\text{max}}$, and the number of such sections is $b$. The workload of a single job in $\Psi \cup \{J_{k,l}\}$ is bounded by $C_{\text{max}}$, and the number of such jobs is $a$. The total non-completed workload due to these jobs is upper bounded by $aC_{\text{max}} + bB_{\text{max}}$. ■

Let $\tau^*$ be the set of all tasks that have jobs in $\Psi \cup \{J_{k,l}\}$ that are released but not ready at $t_0^-$. 

Claim 2. The total non-completed workload at $t_0$ in $\tau^*$ is at most $\sum_{\tau_i \in \tau^*} (u_i x + 2C_i)$.

Proof. Denote the number of released jobs of a task $\tau_i \in \tau^*$ at $t_0^-$ as $s_i$. If $\tau_i$ has released but not ready jobs, then such jobs are not ready because certain preceding jobs of $\tau_i$ are not completed. By the definition of $P_i$ and job readiness, the first $P_i$ jobs of $\tau_i$ are ready, because $P_i$ jobs of $\tau_i$ can be scheduled in parallel. Thus, $s_i \geq P_i$. Note that the first $P_i$ of these jobs are scheduled at $t_0^- \ (t_0^-$ is a non-busy instant). Let $J_{i,j}$ be the earliest non-completed job of $\tau_i$ at $t_0^-$, and let $r$ be its release time. Because $J_{i,j} \in \Psi$, it cannot be $J_{k,l}$, i.e., $J_{i,j} \in \Psi$. Also, because $s_i$ and $\tau_i$ of $\tau_i$ are released at $t_0^-$, 

$$r \leq t_0^- - (s_i - 1)T_i.$$

Since $J_{i,j} \in \Psi$, it is completed at the latest by $r + x + T_i + C_i$. Because $J_{i,j}$ is not completed at $t_0^-$, $r + x + T_i + C_i \geq t_0^-$, or 

$$r \geq t_0^- - x - C_i - T_i.$$ 

Combining (3) and (4), $t_0^- - x - C_i - T_i \leq t_0^- - (s_i - 1)T_i$, which implies $s_i - 2 \leq (x + C_i)/T_i$, which in turn implies 

$$s_i \leq x/T_i + u_i + 2.$$ 

(5)

As the first $P_i$ released jobs of $\tau_i$ at $t_0^-$ are ready, the total workload of the remaining non-ready released jobs at $t_0$ is 

$$(s_i - P_i)C_i \leq \{\text{by (5)}\}$$ 

$$(x/T_i + u_i + 2 - P_i)C_i$$ 

$$= \{C_i/T_i = u_i\}$$ 

$$u_i x + 2C_i + (u_i - P_i)C_i$$ 

$$\leq \{r \text{ is feasible, so by (1), } u_i \leq P_i\}$$ 

$$u_i x + 2C_i.$$ 

Combining over all tasks in $\tau^*$, we have a total workload of at most $\sum_{\tau_i \in \tau^*} (u_i x + 2C_i)$, as claimed. ■

Claim 3. Consider the jobs in $\Psi \cup \{J_{k,l}\}$ that are not released at $t_0^-$. Their total generated workload over $[t_0, t_d)$ is at most $U(t_d - t_0)$.

Proof. By the definition of $\Psi$, all jobs in $\Psi$ have deadlines at or before $t_d$. The jobs of a task $\tau_i$ with releases and deadlines within $[t_0, t_d)$ generate a workload of at most $[(t_d - t_0)/T_i]C_i \leq u_i(t_d - t_0)$. Summing over all such jobs of all tasks of $\tau$ yields the claim. ■

Claim 4. The total workload completed in $[t_0, t_d)$ is $m(t_d - t_0)$.

Proof. If $t_0 \leq t_d$, then by the definition of $t_0$, $[t_0, t_d)$ is a busy interval, so the total completed workload is $m(t_d - t_0)$. ■

Now we can finally bound $W_d$:

$$W_d = \text{Workload at } t_0 \text{ of jobs scheduled at } t_0^-$$

+ \text{Workload at } t_0 \text{ of jobs released before, but not ready at } t_0^-$$

+ \text{Workload at } t_d \text{ of jobs released after } t_0^-$$

- \text{Workload completed within } [t_0, t_d)$$

$\leq \{\text{by Claims 1-4}\}$

$$aC_{\text{max}} + bB_{\text{max}} + \sum_{\tau_i \in \tau^*} (u_i x + 2C_i)$$

$$+ U(t_d - t_0) - m(t_d - t_0)$$

$$\leq \{r \text{ is feasible, so } U \leq m\}$$

$$aC_{\text{max}} + bB_{\text{max}} + \sum_{\tau_i \in \tau^*} (u_i x + 2C_i)$$

(6)

Note that, by the definition of $t_0^-$, at least one processor is not occupied with a job from $\Psi \cup \{J_{k,l}\}$ at $t_0^-$, so $a \leq (m - 1)$. Additionally, the total number of scheduled jobs at $t_0^-$ cannot exceed $m$. Thus, because $B_{\text{max}} \leq C_{\text{max}}$, we have 

$$aC_{\text{max}} + bB_{\text{max}} \leq (m - 1)C_{\text{max}} + B_{\text{max}}.$$ 

(7)

Also, any task $\tau_i \in \tau^*$ has exactly $P_i$ ready jobs scheduled at $t_0^-$, while their total number is at most $(m - 1)$. Thus, 

$$\sum_{\tau_i \in \tau^*} P_i \leq m - 1.$$ 

(8)

Combining (6), (7) and (8), and recalling (2), we get $W_d \leq (m - 1)C_{\text{max}} + B_{\text{max}} + \sum_{\tau_i \in \tau^*} (u_i x + 2C_i) \leq L(x)$.

Lemma 5. If $t_d$ is a busy time instant, and the response time of each job $J_{i,j} \in \Psi$ is at most $x + T_i + C_i$, where 

$$mx \geq L(x),$$ 

then the response time of $J_{k,l}$ is bounded by $x + T_k + C_k$. 


Proof. Note that under G-EDF, $J_{k,l}$ cannot be preempted after its deadline $t_d$ (which is $T_t$ time units after $J_{k,l}$’s release). Thus, it is enough to prove that $J_{k,l}$ is scheduled at some point within $[t_d, t_d + x]$. Let $t_{av}$ ("av" means a processor is available—see Fig. 10(b)) denote the first time instant after $t_d$ such that some processor exists that is not executing a job in $\Psi$ or any non-preemptive section of a job in $\Psi$ that is scheduled at time $t_0$ (and hence executes continually in $[t_0, t_{av}]$). Note that $t_b \leq t_{av}$. We consider three cases, depending on how much processor allocation $J_{k,l}$ receives within $[t_0, t_{av}]$.

Case 1. $J_{k,l}$ is completed before $t_{av}$.

In this case, the response time of $J_{k,l}$ is bounded by $T_k + t_{av} - t_d$. Note that $t_{av} \leq t_d + W_d/m$ (because $W_d$ is the workload that keeps all processors busy), so

$$t_{av} - t_d \leq W_d/m \leq \{\text{by Lemma 4}\} L(x)/m \leq \{\text{by (9)}\} mx/m = x.$$ 

This ensures a response-time bound of $x + T_k + C_k$ for $J_{k,l}$.

Case 2. $J_{k,l}$ is ready, but not completed at $t_{av}$.

Let $\delta$ denote the amount of execution completed for $J_{k,l}$ before $t_d$. Because, by Lemma 4, the total remaining workload at $t_d$ is at most $L(x)$, and at least $C_k - \delta$ units of $J_{k,l}$ remain to be completed, it follows that $t_{av} - t_d \leq (L(x) - (C_k - \delta))/m$. By Lemma 3, $J_{k,l}$ cannot be blocked by jobs or non-preemptive sections that do not contribute to $W_d$. Thus, the completion time of $J_{k,l}$ is bounded by

$$t_{av} + C_k - \delta - t_d \leq (L(x) - (C_k - \delta))/m + C_k - \delta = L(x)/m + (C_k - \delta)(1 - 1/m) \leq \{\text{by (9)}\} mx/m + (C_k - \delta)(1 - 1/m) \leq x + C_k.$$ 

Case 3. $J_{k,l}$ is not ready at $t_{av}$.

In this case, $J_{k,l} - P_k$ (which is in $\Psi \cup \{J_{k,l}\}$) is not finished by $t_{av}$. This predecessor is released at the latest by time $t_d - (P_k + 1) - T_k$. By the lemma statement, $J_{k,l} - P_k$ should complete at the latest by $t_d - (P_k + 1) - T_k + x + T_k + C_k = t_d + x - P_k - T_k + C_k$. By (1), $C_k \leq P_k - T_k$, so $J_{k,l}$ will be ready at the latest by $t_d + x$. By Lemma 3, $J_{k,l}$ is not blocked by any job, because $t_b \leq t_{av}$. That ensures the response-time bound.

We now can conclude both the busy and the non-busy $t_d$ cases in the following theorem.

Theorem 1. Every job $J_{i,j}$ of every task $\tau_i \in \tau$ completes within $x + T_i + C_i$ time units after its release for any $x > 0$ such that $x$ satisfies (9).

Proof. Follows by induction over $\prec$, applying Lemma 2 or Lemma 5.

We now introduce some terminology that is used in obtaining a closed-form expression for $x$ that it is of relevance in the context of the processing graphs that motivate this work.

Def. 5. Call a task $\tau_i$ $p$-restricted (parallelism-restricted) if $P_i < m$, and non-$p$-restricted if $P_i \geq m$. Also, let

$$U_{res}^b = \sum_{i} u_i \quad \text{and} \quad C_{res}^b = \sum_{i} C_i,$$

$b$ largest values $\tau_i$ is $p$-restricted

$$U_{res}^n = \sum_{i} \{\text{u}_i \times (2C_i)\} \quad \text{and} \quad C_{res}^n = \sum_{i} C_i,$$

$b$ largest values $\tau_i$ is $p$-restricted

and let $U_{res} = U_{res}^n$ and $C_{res} = C_{res}^n$.

Corollary 1. The response time of any task $\tau_i \in \tau$ is bounded by $x + T_i + C_i$, where

$$x = \frac{(m - 1)C_{max} + B_{max} + 2C_{res}}{m - U_{res}}.$$ 

Furthermore, if there exists $P_{min} \geq 1$ such that for every $p$-restricted task $\tau_i$, $P_i \geq P_{min}$, then $U_{res}$ and $C_{res}$ in (10) can be replaced with $U_{res}^b$ and $C_{res}^b$, where $b = \lceil(m - 1)/P_{min}\rceil$.

Proof. Note that the task subset $\tau^* \in (2)$ consists of only $p$-restricted tasks, because $\sum_{i} P_i \leq m - 1$ (see (8)), while $P_i \geq m$ for any non-$p$-restricted task. Thus,

$$\max_{\tau^* \subset \tau, P_i \leq m - 1} \left(\sum_{\tau_i \in \tau^*}(u_i x + 2C_i)\right) = \max_{\tau_i \subset \tau^* \text{consists of } p\text{-restricted tasks}} \left(\sum_{\tau_i \in \tau^*}(u_i x + 2C_i)\right) \leq \sum_{\tau_i \subset \tau^*}(u_i x + 2C_i) = U_{res} x + 2C_{res}.$$ 

Hence, by (2), $L(x) \leq (m - 1)C_{max} + B_{max} + U_{res} x + 2C_{res}$. Because, by (10), $mx = (m - 1)C_{max} + B_{max} + U_{res} x + U_{res} x + 2C_{res} \geq L(x)$, $x$ satisfies (9). Therefore, by Theorem 1, $x + T_i + C_i$ is a response-time bound for any task $\tau_i$.

If for every $p$-restricted task $\tau_i$, $P_i \geq P_{min}$, then $|\tau^*| \leq \lfloor(m - 1)/P_{min}\rfloor$, as $\sum P_i \leq m - 1$. In this case, only the $\lfloor(m - 1)/P_{min}\rfloor$ $p$-restricted tasks with the highest corresponding values have to be considered in $U_{res}$ and $C_{res}$. 

Recall that we are interested in $rp$-sporadic tasks obtained via our graph-transformation process. Tasks corresponding to supernodes will generally be $p$-restricted, while other tasks will not. Hence, the corollary above is useful in our context.

The results of this section provide clear tradeoffs. For example, if an OpenVX graph has a cycle with utilization exceeding 1.0 that must execute sequentially, then bounded response times for that graph cannot be ensured. Our analysis shows that, by allowing parallelism within such a cycle, this result can be reversed. Furthermore, Corollary 1 shows that response-time bounds can be lowered by increasing $P_i$ values, i.e., by sacrificing some accuracy.
B. Improved Bounds

The basic bound just derived can be improved via several techniques that we omitted above due to space constraints. We briefly mention those techniques here.

Improved definition of a busy time instant. Let \( m^+ = \lceil U \rceil \).

Then, Def. 3 can be modified in the following way: a time instant is busy if at least \( m^+ \) jobs in \( \Psi \cup \{ J_{k,1} \} \) are scheduled, or there is a non-scheduled ready job in \( \Psi \cup \{ J_{k,1} \} \). With this change, the \((m - 1)C_{\max} + B_{\max}\) sub-expression in the definition of \( L(x) \) becomes \((m^+ - 1)C_{\max} + (m - m^+ + 1)B_{\max}\), and \( \sum_{\tau \in \tau^*} P_i \leq m - 1 \) becomes \( \sum_{\tau \in \tau^*} P_i \leq m^+ - 1 \). This change yields a significant improvement for low-utilization task sets.

Accurate accounting of ready jobs. In Claim 1 of Lemma 4, we bounded the maximal workload of any ready job at \( t_0^* \), and this bound is improved for \( \Psi \cup \{ J_{k,1} \} \) such that \( \Psi \cup \{ J_{k,1} \} \) is scheduled, or there is a non-scheduled ready job in \( \Psi \cup \{ J_{k,1} \} \). With this change, the \((m - 1)C_{\max} + B_{\max}\) sub-expression in the definition of \( L(x) \) becomes \((m^+ - 1)C_{\max} + (m - m^+ + 1)B_{\max}\), and \( \sum_{\tau \in \tau^*} P_i \leq m - 1 \) becomes \( \sum_{\tau \in \tau^*} P_i \leq m^+ - 1 \). This change yields a significant improvement for low-utilization task sets.

Compliant-vector analysis. We considered every task to have the same value \( x \). We could have instead applied compliant-vector analysis [16] and [15], which assigns a distinct \( x_i \) to each task \( \tau_i \). This approach yields lower response-time bounds.

GEL schedulers. The provided analysis easily extends to any GEL (G-EDF-like) scheduler (even with the above improvements applied). Such a scheduler prioritizes each job by a priority point, a point in time a constant distance from its release. We focused on G-EDF merely for simplicity. However, considering GEL schedulers more broadly enables DAG response times to be lowered by determining priority points via linear optimization [43]. Also, as FIFO is a GEL scheduler, the same analysis can be applied for FIFO-scheduled GPUs.

C. GPUs as Schedulable Entities

The final comment above suggests the possibility of considering GPUs as schedulable entities instead of synchronization objects as we have done. However, the former creates some surprising analysis difficulties, as illustrated next.

Ex. 5. Consider the cycle depicted in Fig. 12 in a system with one CPU and one GPU. Observe that the total utilization of this cycle is \( 1.0 \). However, both the CPU and the GPU are not fully utilized. Thus, there could exist other GPU work on the same platform that causes some amount of blocking for the GPU task in the figure. When considering this cycle from a CPU point of view, where time spent accessing the GPU (including both execution and blocking) is viewed as suspension time away from the CPU, the GPU blocking results in an overloaded system and unbounded response times.

As this example suggests, it turns out that, with GPUs considered as schedulable entities, we must consider a given cycle from both a CPU perspective—in which case time accessing a GPU is suspension time away from CPU execution—and from a GPU perspective—in which case time executing on a CPU is suspension time away from GPU execution. Determining such suspension times requires determining GPU and CPU response times, respectively. Thus, we have a circularity: in order to determine CPU and GPU response times, we need to know CPU and GPU response times! Note that this circularity is unique to nodes within cycles—other nodes are not so affected.

While this circularity may seem rather devastating, we have actually devised several workarounds to it, but we lack sufficient space to explain them. In any event, we mention this issue here to provide some indication as to why we opted for the simpler synchronization-based approach in this first work on dealing with arbitrary cycles in OpenVX graphs.

V. Case Study Evaluation

We evaluated our approach via a case study of a CV pedestrian tracking application for which the graph contains a cycle. In this section, we describe our pedestrian tracking experimental setup, and then present the results of varying the minimum history requirement for the cycle induced by tracking, and discuss the effects on analytical and observed response times and on the tracking accuracy.

A. Experimental Setup

We chose for our case study a pedestrian tracking application using the Histogram of Oriented Gradients (HOG) method for detecting the pedestrians from camera image frames. This type of application would be important in an ADAS, as it enables the car to take action depending on the trajectories of pedestrians or other dynamic obstacles.

Pedestrian tracking via HOG. HOG computes gradients within the image at a range of different scales, and performs classification to detect objects within each scale. The computation cost increases with the number of image scales, but each scale enables detection of a pedestrian at a different distance from the camera. HOG was evaluated in [44], and we used their work as a starting point, as it is available online. As in
TABLE I: Analytical and observed end-to-end response times. A bound of N/A indicates a violated feasibility condition.

<table>
<thead>
<tr>
<th></th>
<th>Sequential</th>
<th>$p = 1$</th>
<th>$p = 2$</th>
<th>$p = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical Bound (ms)</td>
<td>N/A</td>
<td>N/A</td>
<td>927.27</td>
<td>928.37</td>
</tr>
<tr>
<td>Observed Maximum Response Time (ms)</td>
<td>25250.67</td>
<td>572.81</td>
<td>713.53</td>
<td>537.60</td>
</tr>
<tr>
<td>Observed Average Response Time (ms)</td>
<td>11765.23</td>
<td>293.63</td>
<td>280.86</td>
<td>293.07</td>
</tr>
</tbody>
</table>

[Fig. 13: CDF of observed response times for varying $p$.]

The features computed by HOG are provided to a classifier such as a support vector machine (SVM), which determines whether a potential detection is a pedestrian. The output is a series of rectangles of varying sizes and positions. Over time (i.e., frames of the video), detections of a given pedestrian can be matched to form a track of positions. This process requires matching a current-frame detection from a track based on the prior frame (or older, if $p > 1$), resulting in a cycle.

We achieved intra-task parallelism by replicating each task; we replicated HOG $P_i = m$ times (non-$p$-restricted node) and the tracking node $P_i = p$ times ($p$-restricted, as a supernode). In our experiments we varied the value of $p$. Each frame of the video was passed to only one of the HOG replicas, in round-robin order. Similarly, only one of the $p$ tracking supernodes processed the resulting detections for a given frame.

**Test platform.** We performed our experiments on a platform with two eight-core Intel CPUs and 32 GB of DRAM. The CPU cores each have a 32-KB L1 data and and 32-KB instruction caches and a 1-MB L2 cache. All eight cores on a socket share an 11-MB L3 cache. The platform additionally has an NVIDIA 1070 GPU, and was configured to run Ubuntu 16.04 with the 2017.1 LITMUSRT kernel [30].

**Competing workloads.** We chose as competing workloads a set of GPU-using tasks that increase the blocking suffered by the tracking supernode. To measure the full effect of this contention, we ran HOG on the CPU, and configured tracking to perform computations on both the CPU and the GPU.

**B. Results**

Our goal was to measure the impact, in terms of response times and accuracy, of varying $p$ for a given graph in the presence of resource contention that results in a higher utilization for that graph’s supernode. In our experiments, we ran the single instance of pedestrian tracking alongside two instances of the GPU-using competing workload task with period 50 ms.

**Impact of $p$ on response times.** We used FeatherTrace [6] to measure the worst-case execution times of each task, and took the 99th percentile value over 10,000 samples. The HOG and tracking tasks had a period of 25 ms, corresponding to the camera frames being processed at 40 frames per second (FPS) (CV applications typically target between 30 and 60 FPS). Each competing task had a period of 50 ms and accessed the GPU for 2 ms, so $B_{\text{max}} = 32$ ms. The number of competing tasks was chosen to be the maximum such that $U < m$.

We computed the response-time bound of each task using Corollary 1 in Sec. IV-A. The utilization constraints are violated for sequential scheduling and for $p = 1$, so no bounds could be computed. The resulting end-to-end response time bounds are listed in Table I for varying $p$, along with the observed worst- and average-case end-to-end response times. The response-time distributions are plotted in Fig. 13.

**Obs. 1.** The system is unschedulable if the supernode is not replicated ($p = 1$) or if graph is scheduled sequentially.

Under sequential scheduling, both HOG and tracking effectively have $p = 1$. HOG in particular had high worst-case execution time, so the end-to-end response time of the graph far exceeded its period, and in fact grew without bound. This is evident in the observed response time. When $p = 1$, the inflation due to potential GPU blocking causes the tracking node to have a utilization higher than 1, which violates the feasibility condition (1) in Sec. III.

**Obs. 2.** The analytical response-time bounds for $p \geq 2$ are almost identical.

This is expected behavior; due to space constraints, the bound we presented in Corollary 1 is somewhat conservative, and remains the same if $p$ increases but the number of $p$-restricted tasks remains significantly smaller than $m$ (this case study includes a single $p$-restricted task).

**Obs. 3.** The analytical response-time bounds upper-bounded the observed response times for $p \geq 2$.

Although the analytical bounds presented are quite high, they are conservative upper bounds for the observed response times given in Table I.

**Obs. 4.** Observed maximum response times decrease with increasing $p$, but observed average response times increase.

This trend is shown in Table I. Although intra-task parallelism allows for shorter response times in the worst case, the number of jobs competing with the job of interest at a given time increases, resulting in worse average-case behavior.

**Impact of $p$ on accuracy.** Bounded response times for (previously unschedulable) cycles come at a price: accuracy drops as $p$ increases. To fully assess the impact on accuracy, a study of multiple CV workloads with varying $p$ values would...
be required. In this paper, we instead seek to demonstrate that allowing a small amount of restricted parallelism does not necessarily translate to a large drop in accuracy.

As $p$ increases, the difference between the “current” position of a pedestrian and their last-tracked position increases. As a result, the track might be lost, to be started anew at a later frame. Therefore, we expect that the total number of tracks maintained throughout the video (including tracks abandoned when pedestrians are “lost”) will increase with increasing $p$.

In practice, $p$ might represent the maximum age of historical results available in a given cycle, i.e., newer results could be used, if available. In our experiments, however, we assume that $p$ corresponds to the actual age of the historical results available in order to demonstrate the worst-case accuracy. This worst-case behavior effectively partitions the frames into distinct sets. For example, if $p = 2$, then data produced by frames 0, 2, 4, 6, ... will never be available to frames 1, 3, 5, 7, ... and vice versa; in this case, a given pedestrian corresponds to two separate tracks, one for each set of frames.

We chose as a metric for accuracy the total number of tracks maintained throughout the video, including tracks abandoned when pedestrians are lost. Given the divisions of frames based on $p$, we consider this total on a per-frame-set basis. Figs. 14 and 15 depict the total tracks for 100 frames of the video. The solid line indicates the total track count for $p = 1$. Fig. 14 depicts the total track counts for the two frame sets for $p = 2$, and Fig. 15 depicts the three frame sets for $p = 3$.

**Obs. 5.** Accuracy is comparable for $p = 1$ and $p = 2$.

This is supported by Fig. 14. For $p = 2$, the two sets of even and odd frames effectively result in two different video sequences, each with half the frame rate of the original. The even frame sequence for $p = 2$ maintains the same number of tracks as the “ground truth” of $p = 1$, and after the first few frames, the odd frame sequence tracks only one fewer pedestrian. Additionally, the two sequences for $p = 2$ only differ by at most one tracked pedestrian.

**Obs. 6.** Accuracy significantly decreases for $p = 3$.

This can be seen in comparing Figs. 14 and 15. For $p = 3$, pedestrians effectively move three times as far as $p = 1$ between “consecutive” frames of a given sequence. As a result, pedestrians are more frequently lost, as evidenced by the higher total track count for one of the $p = 3$ sets in Fig. 15. Furthermore, the three sequences corresponding to $p = 3$ in Fig. 15 differ greatly from each other, indicating that the results are much less stable as $p$ increases.

As mentioned above, the results presented here assume that $p$ represents the exact age of historical results available in a given cycle. If $p$ instead represented the maximum age of results, we expect that higher values of $p$ could be used without significant impact on the accuracy. We plan to explore such implications in future work.

**VI. CONCLUSION**

We have presented the first ever work on guaranteeing response-time bounds for OpenVX graphs that have arbitrary cycles. Such graphs are crucial to consider in real-time certification processes applicable to autonomous vehicles due to the prevalence of uses cases where historical information must be tracked. Our results reveal interesting tradeoffs pertaining to graph cycles that hinge on response times, allowed parallelism, and CV accuracy. We discussed an approach to enable such tradeoffs to be explored that involves transforming an OpenVX graph to an “equivalent” sporadic task set for which allowed intra-task parallelism is a settable per-task parameter. We introduced the rp-sporadic task model to enable the formal study of such task sets, and derived response-time bounds that are applicable to any feasible task set under this model. Additionally, our work can be applied to any graph that contains a cycle, including those from motion planning and machine-learning applications.

This paper opens up many avenues for future work. First, as discussed in Secs. IV-B and IV-C, we made certain simplifying assumptions in our analysis due to space constraints; we intend to fully explore all of the options mentioned there for easing these assumptions. Second, like in prior work, our approach does not allow specifying desired response-time bounds (doing so would introduce utilization constraints). We will explore system design choices and their impacts on resulting bounds. Third, we intend to extend our experimental efforts to consider higher-level notions of accuracy in autonomous driving, such as missed obstacles when engaged in actual driving scenarios, and to perform a large-scale study of the tradeoff between response times and accuracy for a broad set of autonomous driving applications. Finally, we intend to develop a tool that will enable CV programmers to graphically specify OpenVX programs that are then automatically transformed to fine-grained implementations with response-time analysis.
VII. CONCLUSION

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