Intro to control theory for robotics
PID & LQG

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Control Loops

- Use information from the output of an action to adjust the input of the action
- Process Variable
- Setpoint
- Manipulated variable

Motion

$$m\frac{d^2h}{dt^2} = -mg - b\frac{dh}{dt} - k(h - h_0)$$

- Ideal newtonian motion
- Easily solvable second order linear ODE

Noise

$$m\frac{d^2h}{dt^2} + b\frac{dh}{dt} + kh = -mg + kh_0 + \eta(t) + F(z(t))$$

- Real world system with unpredictable noise term η(t) causes errors z = h - h_0
- Measure error and compensate by applying force F(z(t))
- Can’t solve this because η(t) is unknown

PID Equation

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{d}{dt} e(t)$$

- Measure error, calculate how to fix the error, drive the signal back into the system in a way that pushes it towards equilibrium

- Goal is to manipulate the controlled variable such that the error is minimized

Proportional

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{d}{dt} e(t)$$

- present error
- provides stability against small disturbances

Integrated

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{d}{dt} e(t)$$

- Accumulated Error
  - provides stability against a steady disturbance
- Time dependence

Differentiated

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{d}{dt} e(t)$$

- predicted error
  - predicts system behavior
- improves control
- variable impact on system stability, often left off

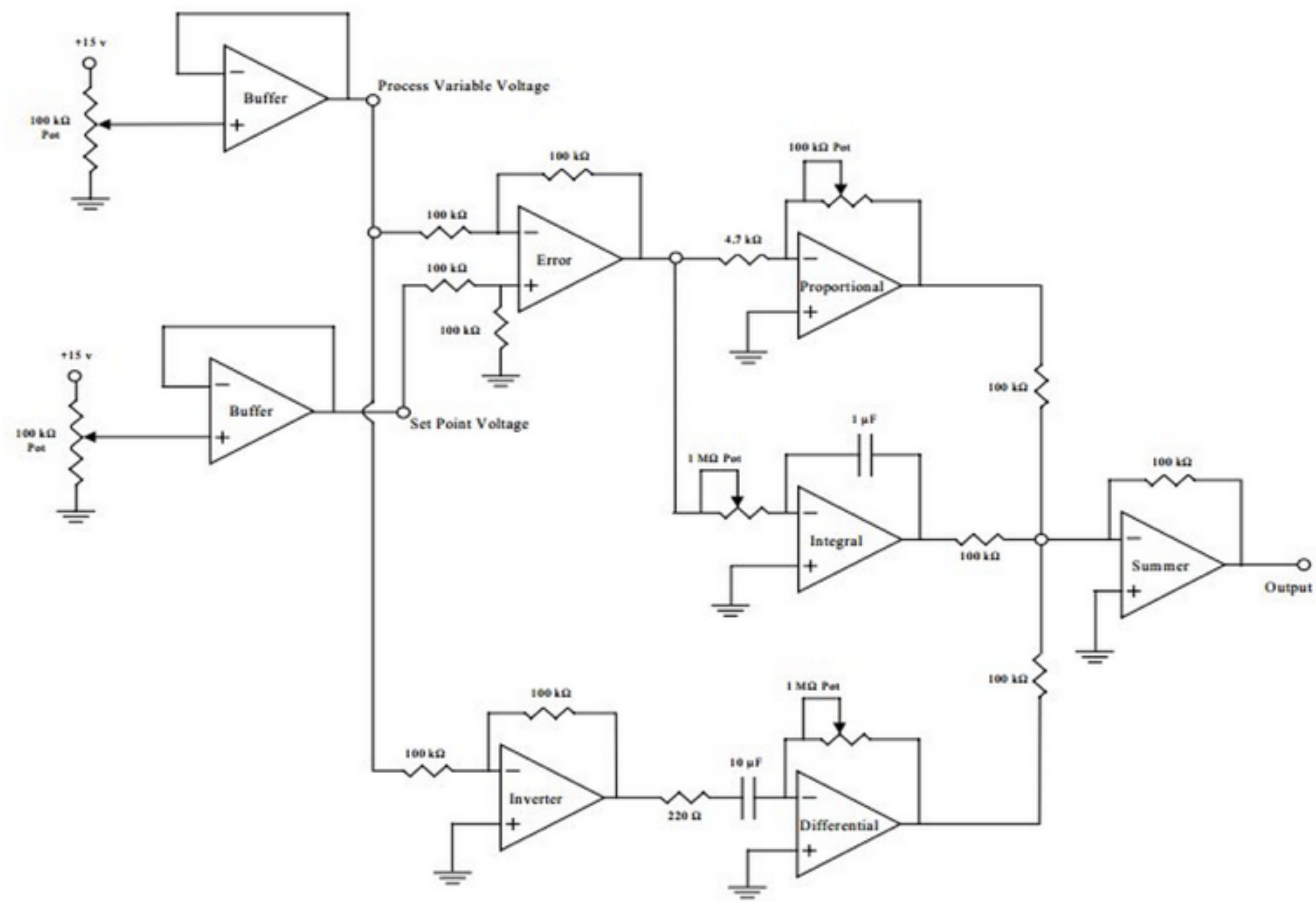
PID Controller

- responsiveness, overshoot, system oscillation.

Tuning

- adjustment of gain to the optimum values to obtain a desired control response
- Various offline and online methods to tune PID
- Ziegler-Nichols method:
    - Set Ki and Kd to zero, increase Kp until the output begins to oscillate periodically (Ku, Pu). Use this information to determine Kp, Ki, Kd formulaically.

Analog Diagram



Digital PID

- Discretize general equation
- Easier to tune
- Communicate directly with computer
- Sometimes less stable than analog controller

Limitations

- Linear and symmetric - performance in non-linear systems is inconsistent
- does't guarantee optimal control of the system or system stability.

Linear-quadratic regulator

- System dynamics are described by set of linear ODE
- Cost is described by a quadratic function
- Feedback controller, like PID
- Settings to the controller that governs a process are found using an algorithm to minimize cost function with supplied constraints

LQR Algorithm

- Automated way of finding an appropriate state-feedback controller
- Different LQR Algorithms for
  - Finite-horizon, continuous time
  - Infinite-horizon, continuous time
  - Finite-horizon, discrete time
  - Infinite-horizon, discrete time
- Solve the representative Riccati equation in one of several ways.

Linear-quadratic Estimator

- Kalman filter
- uses series of measurements over time containing inaccuracies
- produces estimates of unknown variables

LQE Algorithm

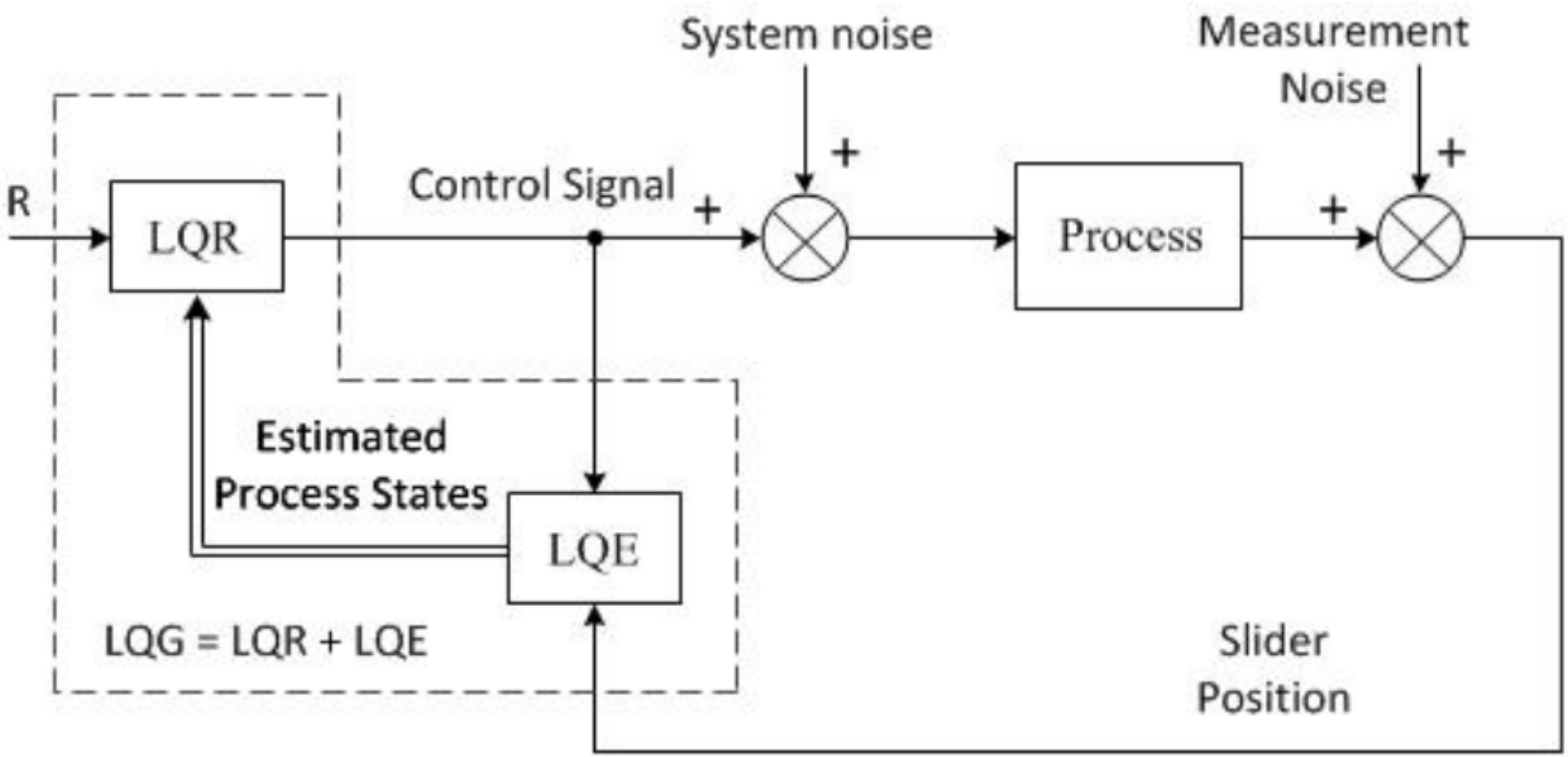
- Prediction step: produces estimates of current state variables and uncertainties
- Observation step: observes next measurement
- Adjustment step: estimates are updated via weighted average

- This process is linear as the previous calculated state and uncertainty matrix are stored with each step.

Linear-quadratic-gaussian control problem

- Introduce Gaussian noise to system, consider Gaussian measurement uncertainty
- Composed of LQR and Linear-quadratic estimator
- Separation principle: the problem of designing an optimal feedback controller for a stochastic system can be solved by designing an optimal observer and an optimal deterministic controller

LQG



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LQG Problem

$$\frac{d\mathbf{x}(t)}{dt} = A(t)\mathbf{x}(t) + B(t)\mathbf{u}(t) + \mathbf{v}(t)$$
$$\mathbf{y}(t) = C(t)\mathbf{x}(t) + \mathbf{w}(t)$$

- **x** as vector of state variables
- **u** as vector of control inputs
- **y** as vector of measured outputs
- Gaussian system noise **v**, Gaussian measurement noise **w**

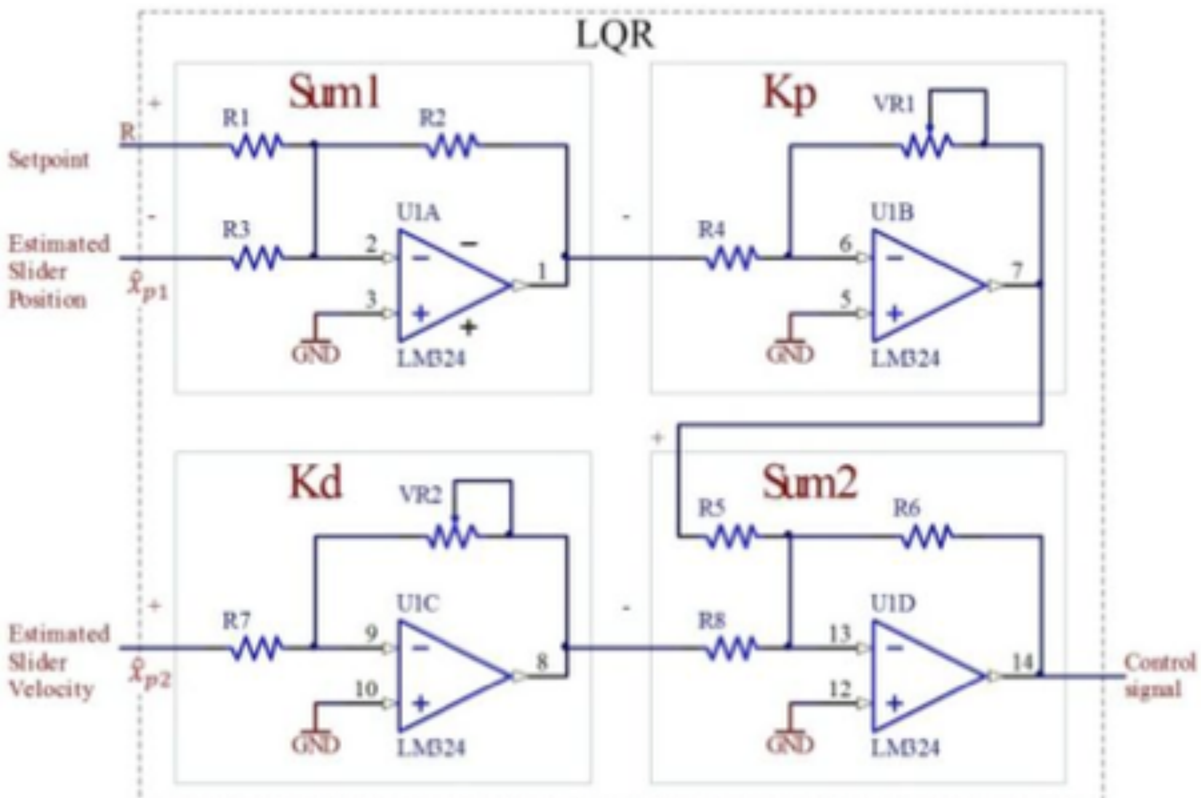
LQG Controller

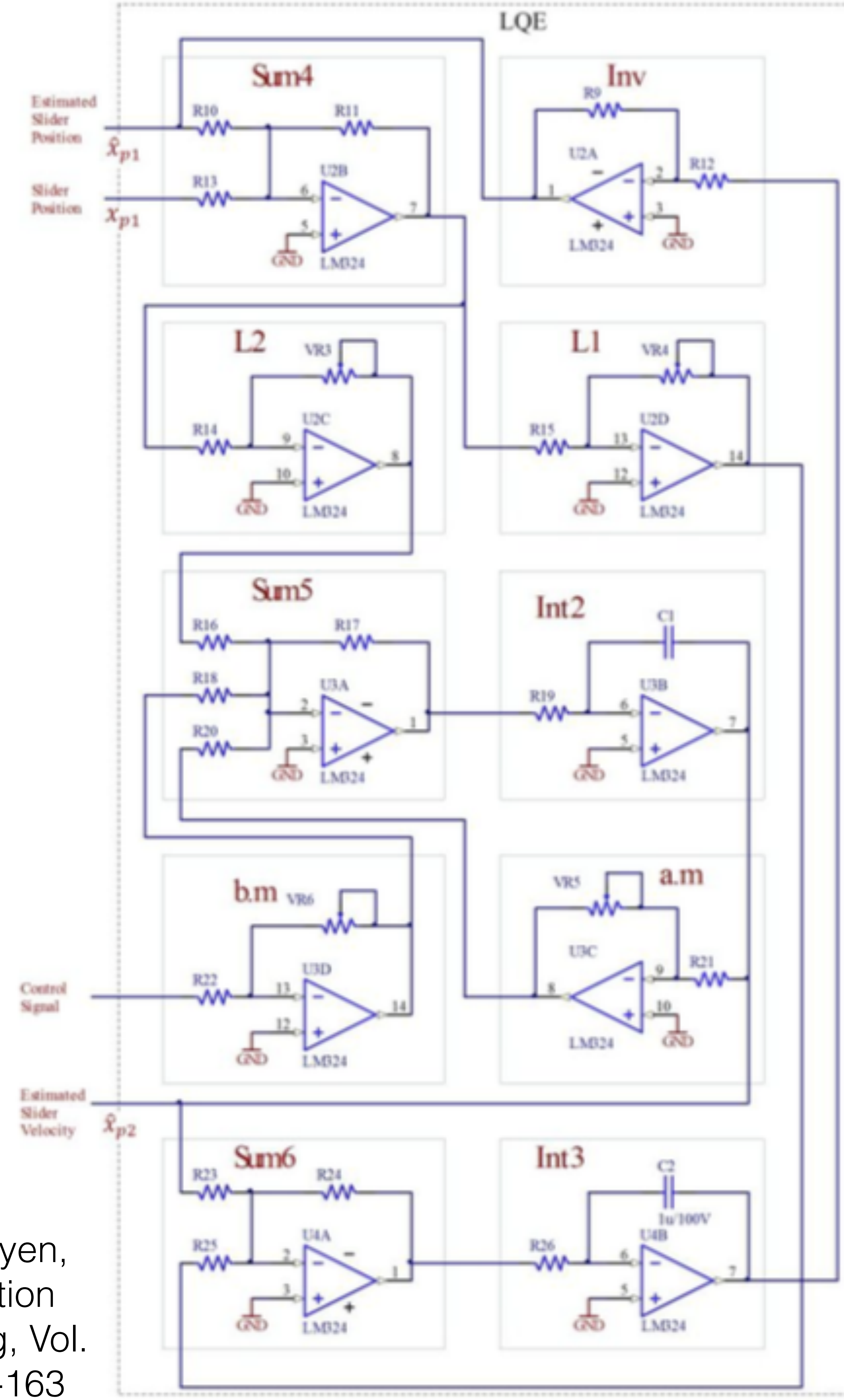
$$\frac{d\hat{\mathbf{x}}(t)}{dt} = A(t)\hat{\mathbf{x}}(t) + B(t)\mathbf{u}(t) + K(t)(\mathbf{y}(t) - C(t)\hat{\mathbf{x}}(t))$$

$$\mathbf{u}(t) = -L(t)\hat{\mathbf{x}}(t)$$

- LQE estimate $\hat{\mathbf{x}}$(t)
- LQE gain K(t)
- Feedback gain L(t)
- K(t), L(t) determined by solving matrix Riccati

Analog LQG





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