1. Pseudo-code – describe algorithms
2. Asymptotic notation – discuss efficiency
3. Design techniques – design algorithms

1. Concentrate on the worst case
2. Ignore constant factors/ lower-order terms
3. Asymptotic analysis – for large values of n

A FAST algorithm is one for which the worst-case running time grows slowly with input size

Asymptotic notation

1. Pseudo-code – describe algorithms
2. Asymptotic notation – discuss efficiency
3. Design techniques – design algorithms

- Describe *growth* of functions.
- Focus on what's important by abstracting away low-order terms and constant factors.

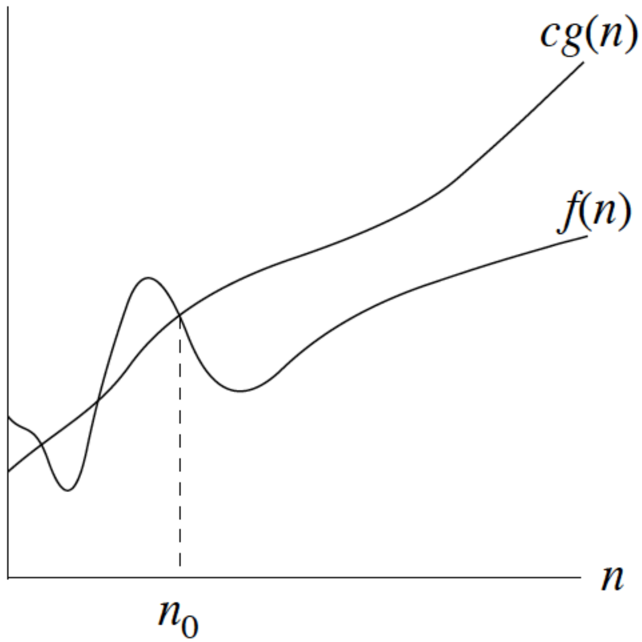
How we indicate running times of algorithms.

A way to compare "sizes" of functions:

$$
\begin{aligned}
O &\approx \le \\
\Omega &\approx \ge \\
\Theta &\approx = \\
o &\approx < \\
\omega &\approx >
\end{aligned}
$$

Asymptotic notation

$$O(g(n)) = \{f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \le f(n) \le cg(n) \text{ for all } n \ge n_0\} .$$

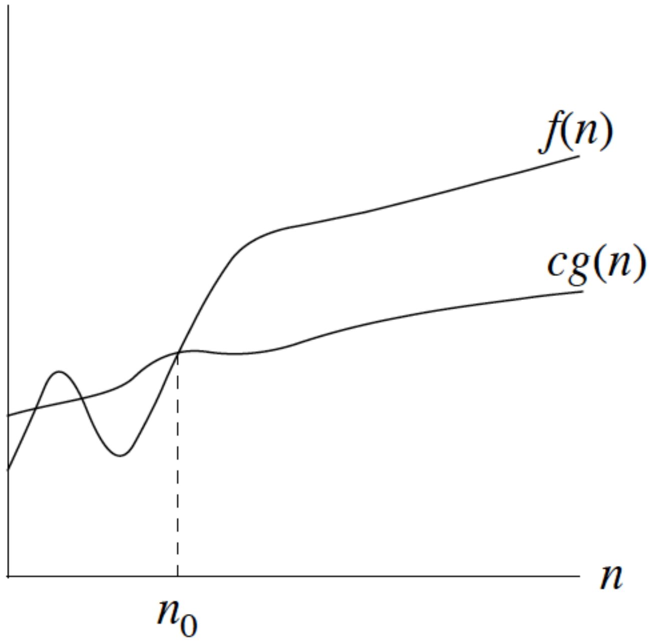


$g(n)$ is an ***asymptotic upper bound*** for $f(n)$.

If $f(n) \in O(g(n))$, we write $f(n) = O(g(n))$

Asymptotic notation

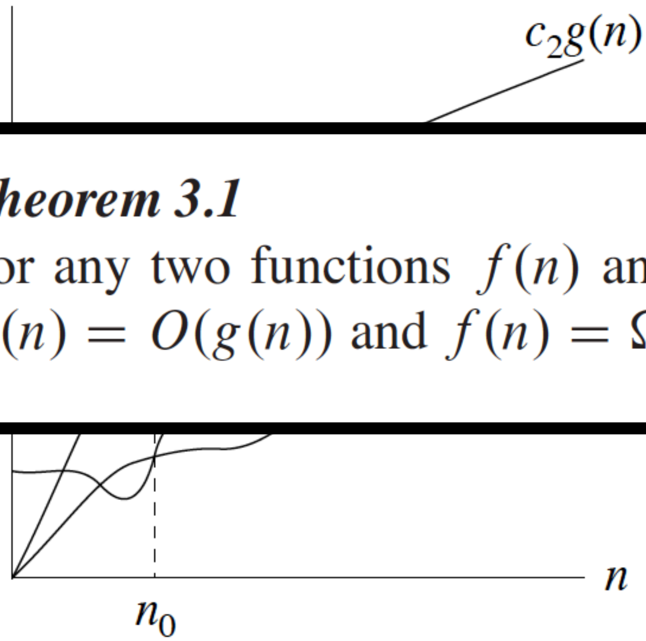
$$\Omega(g(n)) = \{f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \le cg(n) \le f(n) \text{ for all } n \ge n_0\} .$$



$g(n)$ is an ***asymptotic lower bound*** for $f(n)$.

Asymptotic notation

$\Theta(g(n)) = \{ f(n) :$ there exist positive constants c_1, c_2, and n_0 such that $0 \le c_1 g(n) \le f(n) \le c_2 g(n)$ for all $n \ge n_0 \}$.



Theorem 3.1
For any two functions $f(n)$ and $g(n)$, we have $f(n) = \Theta(g(n))$ if and only if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$. ■

$g(n)$ is an ***asymptotically tight bound*** for $f(n)$.

Asymptotic notation

$$o(g(n)) = \{f(n) : \text{for all constants } c > 0, \text{ there exists a constant } n_0 > 0 \text{ such that } 0 \le f(n) < cg(n) \text{ for all } n \ge n_0\}$$

$$: \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0.$$

$$\omega(g(n)) = \{f(n) : \text{for all constants } c > 0, \text{ there exists a constant } n_0 > 0 \text{ such that } 0 \le cg(n) < f(n) \text{ for all } n \ge n_0\}$$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$$

Asymptotic notation in equations

When on right-hand side

$O(n^2)$ stands for some anonymous function in the set $O(n^2)$.

$2n^2 + 3n + 1 = 2n^2 + \Theta(n)$ means $2n^2 + 3n + 1 = 2n^2 + f(n)$ *for some* $f(n) \in \Theta(n)$. In particular, $f(n) = 3n + 1$.

When on left-hand side

No matter how the anonymous functions are chosen on the left-hand side, there is a way to choose the anonymous functions on the right-hand side to make the equation valid.

Interpret $2n^2 + \Theta(n) = \Theta(n^2)$ as meaning *for all* functions $f(n) \in \Theta(n)$, there exists a function $g(n) \in \Theta(n^2)$ such that $2n^2 + f(n) = g(n)$.

Can chain together:

$$\begin{aligned} 2n^2 + 3n + 1 &= 2n^2 + \Theta(n) \\ &= \Theta(n^2) . \end{aligned}$$

ASYMPTOTIC NOTATION: PROPERTIES

Transitivity:

$f(n) = \Theta(g(n))$ and $g(n) = \Theta(h(n)) \Rightarrow f(n) = \Theta(h(n))$.

Same for O, Ω, o, and ω.

Reflexivity:

$f(n) = \Theta(f(n))$.

Same for O and Ω.

Symmetry:

$f(n) = \Theta(g(n))$ if and only if $g(n) = \Theta(f(n))$.

Transpose symmetry:

$f(n) = O(g(n))$ if and only if $g(n) = \Omega(f(n))$.

$f(n) = o(g(n))$ if and only if $g(n) = \omega(f(n))$.

COMPARISON OF FUNCTIONS

$f(n)$ is ***asymptotically smaller*** than $g(n)$ if $f(n) = o(g(n))$.
$f(n)$ is ***asymptotically larger*** than $g(n)$ if $f(n) = \omega(g(n))$.

No trichotomy. Although intuitively, we can liken O to $\leq$, Ω to $\geq$, etc., unlike real numbers, where $a < b$, $a = b$, or $a > b$, we might not be able to compare functions.

Example: $n^{1+\sin n}$ and n, since $1 + \sin n$ oscillates between 0 and 2.

Some problems from the text: 3.1-3, 3.1-4, 3-2

Let f(n) and g(n) denote non-negative functions of n. Prove that max(f(n), g(n)) = Θ(f(n) + g(n))