A QUICK INTRODUCTION TO PROBABILITY

(Appendix C: you don't need to know all of Appendix C, but whenever confused look there.)

- A ***random process***: One *outcome* from some set of outcomes (called the ***sample space*** for the random process) is sure to occur, but it is impossible to predict with certainty which outcome that will be.
- An ***event*** is a subset of the sample space
- The ***Kolmogorov Axioms of Probability***: Let S be a sample space. A ***probability function*** P from the set of all events in S to the real numbers satisfies the following three axioms. For all events A and B in S

  1) $0 \leq P(A) \leq 1$
  2) $P(\{\}) = 0$ and $P(S) = 1$
  3) If $A \cap B = \{\}$ then $P(A \cup B) = P(A) + P(B)$

  A useful consequence (not an axiom – can be derived):

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- A ***random variable (RV)*** is a function from a sample space to the real numbers

  Suppose the possible values of a random variable are $a_1, a_2, \ldots, a_n$, which occur with probabilities $p_1, p_2, \ldots, p_n$. The ***expected value*** (or *expectation*) of the random variable is

$$\sum_{i=1}^{n} a_i p_i$$

  (Illustration: three coin tosses. RV is number of heads.)

- ***Indicator variables***: a convenient method for converting between probabilities and expectations. For an event A

$$I(A) = \begin{cases} 1 & \text{if the event } A \text{ occurs} \\ 0 & \text{otherwise} \end{cases}$$

- **Lemma 5.1** (p 118):

  Given a sample space S and an event A in S. Let $X_A = I(A)$ Then $E(X_A) = P(A)$.

- ***Linearity of expectation*** (stated in Appendix C – page 1198):

$$E(A + B) = E(A) + E(B)$$

  A "proof by diagram" – columns are all the outcomes in the sample space, labeled by their probability of occurrence. One row each for events A and B. LHS sums down the columns, and then across. RHS sums across the rows, and then down the columns.

  Linearity of expectation is a very simple but extremely useful concept.