Statistical Control of Nonlinear Diffusion:
An Integrated Approach in Image Processing

Segmentation and Classification of Multi-Echo
MR Images: an Integrated Approach

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A dissertation proposal in partial fulfillment of the degree requirements for the Ph.D. in Computer Science

Abstract

I propose to explore diffusion as a filtering mechanism for multivalued medical images. My intended approach is to modify conductance parameters of the diffusion equation adaptively based upon local statistical properties of the image. This form of nonlinear or variable conductance diffusion (VCD) will be implemented for filtering not only images with multiple values, but also the measured higher order statistical moments of the images. As this research develops, effort will be directed toward employing these techniques to filtering multivalued images in the presence of nonlinear intensity and contrast distortions typically present in images acquired using medical magnetic resonance instruments.

1. Introduction

The first task in the computer aided visualization of medical images is object definition, the isolation of organs and other solid masses (such as tumors) within medical images. In Magnetic Resonance Imaging (MR) it is possible to modify the acquisition process to emphasize the distribution of water and fatty tissue throughout the represented anatomy. Images with multiple values per pixel (each value generated using different acquisition parameters) may be generated to improve the recognition of disease processes, improving diagnosis and treatment. Trained clinicians are able to view multiple 2D representations of the separate image values and mentally synthesize them into single coherent representation.

In MR images even simple approaches to object definition are often difficult to automate. MR images contain distortions many of which exhibit large scale and are often intensity dependent. By virtue of their ability to adapt to local image statistics, humans are capable of perceiving a great wealth of information in MR data. Traditional computer mechanisms for classification are too sensitive to absolute intensity and do not incorporate either the geometry of objects within the image or the scale at which information is represented. On the other hand, MR images have the advantage of providing multiple intensities (multivalued intensity) at each location.

I intend to construct a family of mechanisms to filter multivalued MR images for improved tissue classification. Filtering systems based on variable conductance diffusion (VCD) incorporate geometry and scale with spatially adaptive nonlinear filters. I wish to extend the VCD metaphor to incorporate multiple values and recognize local statistical properties within MR data. The research my be extended to address clinical questions using diffusion based classifiers on MR images.

Thesis:

Statistical control of variable conductance diffusion reduces error in the segmentation of multivalued magnetic resonance images in the presence of noise and non-stationary distortions of signal intensity.

1.1. Background

Given an intensity function $I(\bar{x},t)$, (where $\bar{x}$ is spatial location and t represents time), consider the isotropic diffusion equation

$$\frac{\partial I(\bar{x},t)}{\partial t} = \nabla \bullet c\nabla I(\bar{x},t) \tag{1}$$

In nonlinear or variable conductance diffusion (VCD) the equation is modified to incorporate both scale and geometry. The constant conductance value c becomes a function, g(), of both geometry (*e.g.*, gradient magnitude at a particular pixel location, $\bar{x}$) and scale (where scale is the size of the aperture at which the gradient is measured). The modified gradient magnitude VCD equation is

$$\frac{\partial I(\bar{x},t)}{\partial t} = \nabla \bullet \left(g\left(\nabla I(\bar{x},t) \otimes G(\bar{x},s(t))\right)\nabla I(\bar{x},t)\right) \tag{2}$$

$$g\left(\nabla I(\bar{x},t) \otimes G(\bar{x},s(t))\right) = e^{-\frac{\left\|\nabla I(\bar{x},t)\otimes G(\bar{x},s(t))\right\|^2}{k^2}} \tag{3}$$

The symbol ⊗ is the convolution operation. The control parameter k is the conductance parameter and it remains constant. It controls the rate of the variable conductance given a perceived intensity gradient at aperture s(t). The aperture $G(\bar{x},s(t))$ is a Gaussian convolution kernel whose scale or aperture, s(t) is monotonically decreasing with time t. The resulting equation describes an adaptive filter for reducing noise within images while preserving boundary information.

In his dissertation research Whitaker suggested that nonlinear diffusion implicitly warps distances within the image. Eberly extends this metaphor in his dissertation to include the concept of scale space and states that nonlinear diffusion may be consider to be *linear* diffusion in a non-Euclidean space. The modification of Euclidean distances to some other distance metric define the parameters of the non-linear diffusion equation. That is, given a distance metric for a non-Euclidean space one may generate a nonlinear diffusion equation that operates in Euclidean space and vice versa. The scale space distance metric corresponding to the linear equation (1) is

$$dl^2 = \left(\frac{d\bar{x} \bullet d\bar{x}}{c^2}\right)^2 + \left(\frac{dt}{c}\right)^2 \tag{4}$$

where arc-length l is expressed in terms of Euclidean distances in $\bar{x}$, time t, and is normalized by the conductance parameter c. In equation 2 and other versions of the nonlinear diffusion equation conductance becomes a function of gradient magnitude or other measures of dissimilarity. Consequently, it is equivalent to discuss the structure of the conductance function and the distance metric associated with the corresponding scale space. I shall use the term *diffusion metric* to describe the combined effects of the sampling operation and the modified conductance function.

There are several unanswered questions regarding variable conductance diffusion. Through my dissertation I wish to explore (a) different forms of the scale selection function s(t), (b) different diffusion metrics based upon sampling operators other than gradient magnitude, and (c) the geometry of multivalued images.

2. Approach

I am exploring the use of operators other than Gaussian convolution with gradient values as the basis for VCD. Multivalued MR images do not represent a well behaved vector space. No implicit bases are available to aid in the determination of multivalued intensity gradients. This incommensurable nature of the multiple values of multiecho MR imaging leads me to explore statistical measures of the image intensity values.

MR is particularly sensitive to distortions of intensity. Anomalies in the induced magnetic field often result in image regions with depressed intensity gain combined with a decrease in contrast in those regions. This property encourages my research toward exploring statistical operators of the higher order central moments of local image intensity as measures that are insensitive to shifts in absolute intensity. Normalizing these moments by the variance yields statistics that are insensitive to fluctuations of both intensity and contrast.

The successful completion of this dissertation will require me to address the following issues.

- Identify a local statistical operator (or family of operators) that is invariant with respect to spatial translation, spatial rotation, and variations of intensity gain and contrast.
- Explore the use of higher order moments of these operators, in particular second, third and fourth order moments
- Extend these measures to multivalued images.
- Develop a multiscale analysis method based upon these operators.
- Employ this invariant operator in a VCD equation and demonstrate its utility in filtering magnetic resonance images so as to achieve segmentation and improved tissue classification.

2.1. Previous Work / Completed Tasks

Some work is already in progress to address the issues listed above. I have begun analysis and implementation of a family of spatially adaptive statistical operators. I perform a statistical analysis on the neighborhood surrounding a pixel by measuring probability distributions within a Gaussian envelope centered about that pixel.

Consider the measurement made using a Gaussian sampling kernel to be the average or expected value of I over the neighborhood surrounding location $\bar{x}$ defined by the aperture value s. This implies that the local mean over this neighborhood can be defined by

Local Mean
$$\mu_I(\bar{x}|s) = \langle I(\bar{x})|s\rangle = \sum_{\text{neighborhood}(s)} P(I(\bar{x}))I(\bar{x}) = G(\bar{x},s) \otimes I(\bar{x}) \qquad (5)$$

where $\langle I(\bar{x})|s\rangle$ is the local expected value of $I(\bar{x})$ measured with aperture s and $P(I(\bar{x}))$ is the probability of observing the value $I(\bar{x})$.

Beyond the calculation of a local mean, it is straightforward to calculate a value for the local variance and the related standard deviation over the neighborhood specified by scale s.

Local Variance

$$\begin{aligned} \mu_I^{(2)}(\bar{x}|s) &= \left\langle \left(I(\bar{x}) - \mu_I(\bar{x}|s)\right)^2 \middle| s \right\rangle \\ &= \sum_{\text{neighborhood}(s)} P(I(\bar{x}))\left(I(\bar{x}) - \mu_I(\bar{x}|s)\right)^2 \\ &= G(\bar{x},s) \otimes (I(\bar{x}))^2 - \left(\mu_I(\bar{x}|s)\right)^2 \end{aligned} \tag{6}$$

Local Standard Deviation

$$\sigma_I(\bar{x}|s) = \sqrt{\mu_I^{(2)}(\bar{x}|s)} \tag{7}$$

Generally, like gradient magnitude, local variance increases and then falls back as you pass across an object edge. Observe that the third and fourth local central moments are easily calculated in a similar fashion.

Other Central Moments:

$$\mu_I^{(3)}(\bar{x}|s) = G(\bar{x},s) \otimes (I(\bar{x}))^3 - 3\mu_I(\bar{x}|s)\mu_I^{(2)}(\bar{x}|s) - \left(\mu_I(\bar{x}|s)\right)^3 \tag{8}$$

$$\mu_I^{(4)}(\bar{x}|s) = G(x,s) \otimes (I(\bar{x}))^4 - 4\mu_I(\bar{x}|s)\mu_I^{(3)}(\bar{x}|s) - 6\left(\mu_I(\bar{x}|s)\right)^2 \mu_I^{(2)}(\bar{x}|s) - \left(\mu_I(\bar{x}|s)\right)^4 \tag{9}$$

The use of the Gaussian distribution as the neighborhood function is a result of constraining the sampling filter to be invariant with respect to particular transformations of $\bar{x}$. An additional constraint placed upon the operators is that the sampled measurements of the array of $I(\bar{x})$ be invariant with respect to linear transformations of I. Such measures may be obtained by normalizing the central moments with multiples/exponents of the standard deviation. The resulting measures are described as *skewness* and *kurtosis*. Their local manifestations, given a sampling aperture s, may be defined as

Local Skewness

$$\gamma_I^{(3)}(\bar{x}|s) = \frac{\mu_I^{(3)}(\bar{x}|s)}{\left(\sigma_I(\bar{x}|s)\right)^3} \tag{10}$$

Local Kurtosis

$$\gamma_I^{(4)}(\bar{x}|s) = \frac{\mu_I^{(4)}(\bar{x}|s)}{\left(\sigma_I(\bar{x}|s)\right)^4} \tag{11}$$

The Skewness operator, like the Laplacian operator, generally begins polarizing in the proximity of an object boundary and changes polarity across the border. Kurtosis reflects the peakedness of the local probability distribution. Its behavior is similar to the gradient magnitude of the variance operation, increasing as samples are taken near an object boundary and then decreasing again, generating a local minimum at the object border.

Figure 1 shows an example image with each of the local statistical measurements applied at a fixed sampling aperture. The image on the far left shows the original figure with a raster resolution of 128x128 pixels and zero mean Gaussian distributed white noise. SNR of this image is 4 to 1.

The images from left to right are the local mean, the local variance, and the third and fourth local central moments. The sampling aperture has a spatial standard deviation of three pixels.

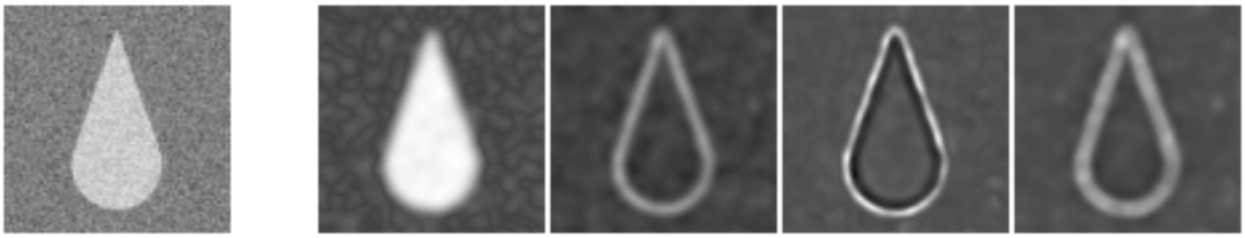

Figure 1. A test image and local statistics of that image.

Pilot studies surrounding a diffusion equation based upon these operators have been completed. A 0th order VCD implementation from the statistically based diffusion equation was tested and performed well on noisy computer generated phantom images. The scale selection parameter was derived in these equations from a rudimentary multiscale analysis of the images to be filtered. “Critical” scales were estimated by finding the roots or zero crossings of $\partial\mu_I^{(2)}(\bar{x}|s)/\partial s$, the first derivative of local variance with respect to increasing aperture s.

2.2. Joint Moments of Multivalued Images

Extending the previous metaphor for local statistics to multiple values, we arrive at the following definitions. Consider a multivalued image $\bar{I}$ where an individual pixel value $\bar{I}(\bar{x})$ may be expressed as an ordered tuple of values $[I_1(\bar{x}), I_2(\bar{x}), \ldots, I_m(\bar{x})]$ where m is the number of values per pixel. The average or expected value of $\bar{I}$ over the neighborhood surrounding location $\bar{x}$ specified by the aperture value s is

<u>Local Multivalued Mean</u>

$$\mu_{\bar{I}}(\bar{x}|s) = \langle \bar{I}(\bar{x})|s \rangle = \sum_{\text{neighborhood}(s)} P(\bar{I}(\bar{x}))\bar{I}(\bar{x}) = \bar{G}(\bar{x},s) \otimes \bar{I}(\bar{x}) = \begin{bmatrix} G(\bar{x},s) \otimes I_1(\bar{x}) \\ \vdots \\ G(\bar{x},s) \otimes I_m(\bar{x}) \end{bmatrix} \tag{12}$$

Einstein summation notation (ESN) abbreviates this multivalued mean to $\mu_{I_i}(\bar{x}|s) = G(\bar{x},s) \otimes I_i(\bar{x})$ where the subscript value i is considered to range over all possible values (1...m).

Continuing the generalization to multiple values, it is possible to calculate a value for the local covariance over the multiple values in the neighborhood specified by the scale s.

<u>Local Covariance</u> (13)

$$\mu_{\bar{I}}^{(2)}(\bar{x}|s) =$$

$$\begin{bmatrix} G(\bar{x},s) \otimes (I_1(\bar{x}))^2 - (\mu_{I_1}(\bar{x}|s))^2 & \cdots & G(\bar{x},s) \otimes (I_1(\bar{x}))(I_m(\bar{x})) - (\mu_{I_1}(\bar{x}|s))(\mu_{I_m}(\bar{x}|s)) \\ \vdots & \ddots & \vdots \\ G(\bar{x},s) \otimes (I_m(\bar{x}))(I_1(\bar{x})) - (\mu_{I_m}(\bar{x}|s))(\mu_{I_1}(\bar{x}|s)) & \cdots & G(\bar{x},s) \otimes (I_m(\bar{x}))^2 - (\mu_{I_m}(\bar{x}|s))^2 \end{bmatrix}$$

Higher order joint moments can be generated. These moments are descriptive of the shape of the joint distribution of the multiple values in the neighborhood about $\bar{x}$.

2.2.1. Moments of Images with Two Values

For the remainder of this discussion, let m = 2. That is, $\bar{I}(\bar{x}) = [I_1(\bar{x}), I_2(\bar{x})]$. The mean and covariance matrix of $\bar{I}$ over the neighborhood surrounding location $\bar{x}$ specified by s is then

Local Bivariate Mean
$$\mu_{\bar{I}}(\bar{x}|s) = \begin{bmatrix} \mu_{I_1}(\bar{x}|s) \\ \mu_{I_2}(\bar{x}|s) \end{bmatrix} = \begin{bmatrix} G(\bar{x},s) \otimes I_1(\bar{x}) \\ G(\bar{x},s) \otimes I_2(\bar{x}) \end{bmatrix} \tag{14}$$

Local Covariance Matrix

$$\mu_{\bar{I}}^{(2)}(\bar{x}|s) = \begin{bmatrix} \mu_{\bar{I}}^{(2,0)}(\bar{x}|s) & \mu_{\bar{I}}^{(1,1)}(\bar{x}|s) \\ \mu_{\bar{I}}^{(1,1)}(\bar{x}|s) & \mu_{\bar{I}}^{(0,2)}(\bar{x}|s) \end{bmatrix}$$
$$= \begin{bmatrix} G(\bar{x},s) \otimes \left(I_1(\bar{x})\right)^2 - \left(\mu_{I_1}(\bar{x}|s)\right)^2 & G(\bar{x},s) \otimes (I_1(\bar{x}))(I_2(\bar{x})) - \left(\mu_{I_1}(\bar{x}|s)\right)\left(\mu_{I_2}(\bar{x}|s)\right) \\ G(\bar{x},s) \otimes (I_1(\bar{x}))(I_2(\bar{x})) - \left(\mu_{I_1}(\bar{x}|s)\right)\left(\mu_{I_2}(\bar{x}|s)\right) & G(\bar{x},s) \otimes \left(I_2(\bar{x})\right)^2 - \left(\mu_{I_2}(\bar{x}|s)\right)^2 \end{bmatrix} \tag{15}$$

2.2.2. Higher Order Joint Moments

The matrix element $\mu_{\bar{I}}^{(1,1)}(\bar{x}|s) = G(\bar{x},s) \otimes (I_1(\bar{x}))(I_2(\bar{x})) - \left(\mu_{I_1}(\bar{x}|s)\right)\left(\mu_{I_2}(\bar{x}|s)\right)$ is the *covariance* of $\bar{I}(\bar{x})$ and is also the first joint moment of $I_1(\bar{x})$ and $I_2(\bar{x})$. The entire range of joint moments may be generated from the following equation. Note that as with the moments of scalar functions, joint moments may be expressed as polynomial functions of the elements of $\bar{I}(\bar{x})$ and $\mu_{\bar{I}}(\bar{x}|s)$.

Local Joint Moments

$$\begin{aligned} \mu_{\bar{I}}^{(k,r)}(\bar{x}|s) &= \left\langle \left(I_1(\bar{x}) - \mu_{I_1}(\bar{x}|s)\right)^k \left(I_2(\bar{x}) - \mu_{I_2}(\bar{x}|s)\right)^r \right\rangle \\ &= \int \left(I_1(\bar{x}) - \mu_{I_1}(\bar{x}|s)\right)^k \left(I_2(\bar{x}) - \mu_{I_2}(\bar{x}|s)\right)^r G(\bar{x},s)\ d\bar{x} \end{aligned} \tag{16}$$

2.2.3. Critical Scale

I define critical scale s_n to be those points in scale space where

Multivalued Critical scale (s_n)
$$\frac{\partial\left(\det\left(\mu_{\bar{I}}^{(2)}(\bar{x}|s_n)\right)\right)}{\partial s} \approx 0 \tag{17}$$

The determinant of the covariance matrix $\det\left(\mu_{\bar{I}}^{(2)}(\bar{x}|s_n)\right)$ is a scalar measure of the overall spread of the distribution about the mean in terms of area. In homogenous regions within objects this value is relatively constant with changing aperture s. At object boundary locations the nonlinear boundary discontinuity causes a dramatic increase in the variance functions as the aperture widens to include non-object pixels. By constraining the determinant of the covariance to be minimal (its derivative with respect to s to be exactly or near 0), I hypothesize that the largest sample size may be acquired and thus produce the optimal estimate of the distribution parameters about that pixel location.

2.3. Characteristic Functions

If u is a random variable and f(u) describes its density function, the *characteristic function* of u is defined to be $\Phi(\omega)$ the Fourier transform of f(u). Similarly the inverse Fourier transform of $\Phi(\omega)$ yields the original probability density function.

$$\Phi(\omega) = \int_{-\infty}^{\infty} f(u) e^{i\omega u} du \qquad\qquad f(u) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi(\omega) e^{-i\omega u} d\omega \tag{18a,b}$$

In the bivariate case the relationship between the two random variables is described by the *joint characteristic function*. Given two random variables u_1 and u_2 such that $\bar{u} = [u_1, u_2]$ and their joint distribution function $f(u_1,u_2)$, the joint characteristic function $\Phi(\omega_1,\omega_2)$ and its inverse are the integrals

$$\Phi(\omega_1,\omega_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u_1,u_2) e^{i(\omega_1 u_1 + \omega_2 u_2)} du_1 \, du_2 \tag{19a}$$

$$f(u_1,u_2) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Phi(\omega_1,\omega_2) e^{-i(\omega_1 u_1 + \omega_2 u_2)} d\omega_1 \, d\omega_2 \tag{19b}$$

From the moment theorem it is possible to obtain $f(u_1,u_2)$ from its joint moments. Given a series of central moments that describe $f(u_1,u_2)$, it is possible to generate the probability distribution using (19b) and the following equation.

$$\Phi(\omega_1,\omega_2) = \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{k=0}^{\infty} \binom{n}{k} \mu_{\bar{u}}^{(k,n-k)} (i\omega_1)^k (i\omega_2)^{n-k} \tag{20}$$

3. Proposed Research Direction

I propose to generate a new diffusion metric based upon the local statistical derivations outlined above. This metric will be applied to filter multivalued MR images and the results measured by analyzing the improved confidence and accuracy of subsequent tissue classification.

Using the first order statistical operators, multiscale analysis and a pilot VCD system have been applied to MR images. However, a complete analysis and derivation of the diffusion equation remains to be completed. In particular a multivalued derivation has not been implemented. Statistical central moments above the second order are also not employed in the current system. This section outlines a proposed diffusion system as well as broadly describes some of the remaining research tasks.

3.1. Proposed Diffusion Metric

Heretofore, all diffusion has been based upon gradient differences of some invariant measure or some zeroth order property such as intensity. Gradient operations normalized by some diffusion parameter have formed the bulk of the similarity metrics under study. Using the multiscale statistical analysis described above, it is now possible to compare the *probability distributions* of two voxels to measure their similarity.

First, equations 20 can be modified to reflect local statistics of bivariate images measured with sampling aperture s. Moreover, the infinite series may be abbreviated to only include the first four moment terms to reduce the complexity of the operation.

$$\Phi(\omega_1,\omega_2) = \sum_{n=0}^{4} \frac{1}{n!} \sum_{k=0}^{4} \binom{n}{k} \left(\mu_{\bar{I}}^{(k,n-k)}(\bar{x}|s)\right)(i\omega_1)^k (i\omega_2)^{n-k} \tag{21}$$

Pixel to pixel similarity may now be evaluated on the correspondence of their local characteristic functions or probability distributions. One similarity metric upon which to generate conductance is the Euclidean norm of the multivalued gradient vector of the first four moments. That is, $\left\| \nabla \left[\mu_I(\bar{x}|s_n), \mu_I^{(2)}(\bar{x}|s_n), \mu_I^{(3)}(\bar{x}|s_n), \mu_I^{(4)}(\bar{x}|s_n) \right] \right\|$. This basic metric, while crude, measures the similarity of mean intensity and local probability density among neighboring pixels giving equal weight to each of the moment values. Explorations into this and other metrics that give separate weighting to the local moments are currently underway.

3.2. Proposed Milestones / Intermediate Goals

In addition to the development and analysis of a multivalued nonlinear diffusion system, several issues remain unexplored. The following list indicates work proposed to complete this dissertation.

- Statistical Geometry

  One assertion that I am making regarding the skewness property of image structure is that it should generate geometry similar to the Laplacian operation, but invariant to linear transformations of intensity.

  As part of an exploration of the properties of the higher order operators I intend to show that, like the Canny edge operator, skewness zero crossings generate closed curves within the image. At critical scale, these contours should show reduced segmentation error (oversegmentation/undersegmentation) when compared to the Canny operator.

- Multiscale Local Statistics

  The multiscale analysis reported in the previous work was based upon measurements of local variance through varying aperture s. A commensurate derivation of multiscale skewness and kurtosis will be required as part of this research. The identification of critical scale s with regard to each of the higher order moments is required.

  Blom showed in his research that while measurements of higher order derivative values are very sensitive to noise at small scale, they are less sensitive than lower order derivatives when measured using a large sampling aperture. I expect that local statistical moments will exhibit a similar behavior as scale increases. That is, as the sampling aperture increases, I expect the calculated local higher order moments will become less sensitive to noise.

- Diffusion Equations

  I envision a family of diffusion equations to match the family of statistical operators, just as Whitaker developed a family to match the derivative operators of various orders. In the pilot work, a simple diffusion equation was developed using the derivative of the local variance with respect to scale to find critical scale, and local variance was employed as a diffusion parameter.

Each moment operator has its own diffusion equation, with the geometry being diffused (gradient, intensity, curvature, noise, etc.) corresponding to the order of the moment. I intend to develop an entire family of diffusion equations matching each of the moments that I described in section 2. I also expect to derive the generating function for these equations, generalizing to even higher order moments.

I expect that the resulting family of statistical sampling operations will provide a better model for the probability distribution of the noise process than a strict assumption of Gaussian distributed, zero mean additive white noise. Moreover, these operators will recognize and evaluate the changes in these distributions across object boundaries. Consequently, the diffusion metrics based upon these sampling operators will enable the filtering and subsequent segmentation of images where a variety of noise elements are present, including fuzziness of boundaries and possibly bifurcations of branching structures.

- Comparison Studies

  The validity of this research must be evaluated relative to other segmentation mechanisms currently available. There are existing studies comparing new segmentation techniques to hand-segmented images. One such study has been made available to me for the purpose of comparing this research to other methods.

  I intend to add new elements to existing comparison studies as well as conduct some of my own. In particular, I wish to repeat the experiments performed by the Brigham and Women's hospital in accuracy testing and speed on their data and compare the results to hand segmentation, semi-automated results, and the results they achieved using gain-field equalization system. This study was performed on data that does not have a significant intensity distortion. The extension that I am proposing will show that the research performed in this dissertation performs as well as existing mechanisms on relatively well behaved data.

  In addition, I will perform other studies comparing the results between semi-automated systems in use today with the proposed research on data where intensity distortions are a significant component. Since existing mechanisms do not have a means of incorporating the distortion fields into their structure, significant improvements are expected.

4. Validation Issues

Targeting Clinical Applications

Filtering techniques generated as part of this dissertation research are valid only if they are capable of performing well on clinical data. Clinical data, and especially those images generated using MR, are not always free of artifact, nor are they selected for their mathematical purity. More often they are selected on the basis of patient need, and it is this need that I am addressing.

The techniques developed here should be robust enough to perform well on realistically distorted images. They should be capable of processing large volumes of data, and they should not require significant “tinkering” or parameter tweaking in order to produce optimal image results. Indeed, much of the motivation for deriving a parameter control mechanism based on statistical image features is to free the user from parameter selection.

This pursuit of clinical applications constrains the systems developed to be suitable for production environments. The user interfaces and the hardware necessary to support the processing of these images should be compatible with clinical environments. It is my intention to supply clinicians with these tools as a means of testing the results and proving the robustness of the approaches.

Real Data / Real Truth

In order to generalize this technique for MRI dual echo images, it will be necessary to perform this method on true MRI images. Some of that work is in progress now, but no absolute quantitative measurements are possible, only quantitative comparisons with other methods.

In addition to comparison studies to other techniques, comparisons to absolute truth may be performed in the course of this dissertation. Images with known boundaries and known feature strength will be required. Dr. Benjamin Tsui of the BMME department at UNC-Chapel Hill has designed a phantom for the study of MR images. He has made that device available for my use. In addition, Dr. David Beard has generated MR images of food items for the study of volume measurement. Phantom studies and other available data will be used for comparing the new analysis techniques to "truth" as measured in the laboratory.

5. Completion Criteria

The goal for the code development for this dissertation is to generate a working system for the segmentation and classification of MR images. In particular, the system will be viewed as successful if it is capable of classifying MR images acquired using surface coils. Such images are fairly common in the clinic and represent a class of images that inherently contain non-stationary intensity distortions.

The research goal for this dissertation is to demonstrate the efficacy of a combined statistical and geometric approach to the classification of medical images. In essence, the addition of scale and geometry to statistical pattern recognition introduces a family of structuring elements of image, allowing for faster and more accurate object definition. The research will be considered complete if demonstrable and repeatable improvements in image classification accuracy and speed are achieved.

6. Contribution

The research program presented here represents a possible significant advance in the segmentation and classification of multivalued medical data. The validity of the research program will be reflected in its ability to satisfy the driving application in clinical situations. Moreover, the intended formalization of the relationship between current variable conductance diffusion techniques and their corresponding expressions in a statistical feature space represents a foundation not only for this work, but also for the continued exploration of relaxation filtering in other applications.

7. The scope of this dissertation

It is my intention to apply this work to segmentation and classification of real medical MR data. A goal for this research is to perform accurate diffusion, respecting boundary information, even in the presence of nonhomogeneous signal strength. Indeed, it is precisely this task that drives this research. The eventual goal is to generalize the techniques presented here to 3D multivalued data, though completion of the intended body of the research is not considered contingent upon the successful extension to 3D.

Although there may be an abundance of information to be applied in several different weighted images of the same anatomy, it is important to avoid excessive dimensionality in this treatment of VCD. Each additional value extends the dimensions of the space to be analyzed and explored. For simplicity and focus, only two valued images of two spatial dimensions will be considered.

MRI is subject not only to distortions in intensity, but also to deformations of the geometry in the image. This work targets object definition; the distortions of geometry are not expected to adversely affect the proposed filtering techniques. This dissertation does not extend to correcting the geometric warping of MR images.

8. Tentative Schedule

December 92 Pilot system for multiple cluster feature space problem. Draft paper for submission to IPMI '93.

January 93 Ph.D. Oral Examination **(29 January and 1 February 93)**

June 93 IPMI 93 - Present preliminary work **(17 June 93)**
Present paper - "Using SPR Techniques to Control Variable Conductance Diffusion"

August 93 SIGGRAPH 93
Chair Tutorial: *3D Visualization in Medicine* **(1 August 93)**

January 94 Begin teaching requirement (Instructor for COMP 140)

Research on scale control

February 94 Continue teaching COMP 140

Draft note on scale control

March 94 Continue teaching COMP 140
Attend AAAI Symposium **(21-23 March 94)**
Applications Of Computer Vision In Medical Image Processing
Present paper - "Scale and Statistics in Variable Conductance Diffusion"

Research on multivalued statistical VCD

April 94 Complete teaching COMP 140

Continue research on multivalued statistical VCD

May 94 Attend ECCV '94 **(2-6 May 94)**
Present research at the meeting of Nonlinear Diffusion Consortium

Continue research on multivalued statistical VCD

June 94 Attend S/CAR '94 **(12-15 June 94)**
Present paper - "Interactive 3D Medical Visualization: A Parallel Approach to Surface Rendering 3D Medical Data"

Continue research on multivalued statistical VCD
Begin implementation of multivalued statistical VCD system

July 94 SIGGRAPH 94 **(24-29 July 94)**
Lecturer Tutorial: *3D Visualization of Medical Data*
Draft Dissertation Chapter 1 (Introduction/Goals)

Continue implementation of multivalued statistical VCD system
Research on higher order moments of statistical vector spaces

August 94 Reading and Research
Draft Dissertation Chapter 2 (Background/VCD)

Continue research on higher order moments of statistical vector spaces
Begin collecting clinical data and alternate results for comparison

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September 94 Status meeting with Dissertation Committee
Draft Dissertation Chapter 3 (Background/MRI)

Continue research on higher order moments of statistical vector spaces
Continue collecting clinical data and alternate results for comparison

October 94	Attend VBC ‘94 **(4-7 October 94)**	
	Draft Dissertation Chapter 4 (Probability and Scale)	
	Continue research on higher order moments of statistical vector spaces Begin implementation of multivalued statistical VCD system based on higher order moments Continue collecting clinical data and alternate results for comparison	
November 94	Draft publication of results from work in progress for IPMI ‘95	
	Draft Dissertation Chapter 5 (Geometry of Parameterized Feature Spaces)	
	Begin analysis of clinical data and comparison studies with VCD results Continue implementation of multivalued statistical VCD system based on higher order moments	
December 94	Tuning and exploring multivalued VCD of higher order moments... indicate future directions	
	Draft Dissertation Chapter 6 (Adaptive VCD - Statistical Pattern Anisotropic Diffusion [SPAD?]) Draft Dissertation Chapter 9 (Future Directions)	
	Continue analysis of clinical data and comparison studies of other methods with VCD results Wrap-up implementation of multivalued statistical VCD system based on higher order moments	
January 95	Wrap up exploration... begin drafting notes on research phase	
	Draft Dissertation Chapter 7 (Diffusing Higher Order Geometry and Statistical Moments)	
	Wrap-up analysis of clinical data and comparison studies of other methods with VCD results	
February 95	Completion, proofreading, feedback, and despair	
	Draft Dissertation Chapter 8 (Results from the Clinic) Rewrite and revise dissertation draft	
March 95	Defense **(27 March 95)**	
	Rewrite and revise dissertation draft Revisit Chapter 1 (Introduction/Goals) revise to improve cohesiveness of the dissertation	
	Prepare for defense Defend.	
Apr.-May 95	Repair and revise dissertation text Submit to Graduate School Interview for positions, get on with life.	

Preliminary Reading List

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