Lower Degree Predicates for the Additively Weighted Voronoi Diagram

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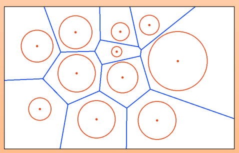
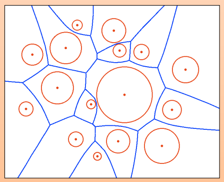
Abstract

This work considers the problem of incrementally constructing additively weighted Voronoi diagrams in $\mathbb{R}^2$. Incremental constructions assume a diagram of *k* sites and considers the insertion of a new site *s*. In general, this type of construction consist of two steps: first, identify the the *conflict region*, the set of diagram components which *s* destroys; second, repair the conflict region as to return the diagram to a valid state. In the paper *Dynamic Additively Weighted Voronoi Diagrams in 2d*, Karavelas and Yvinec describe such a procedure, determining the conflict region using predicates of algebraic degree 14. We propose a different set of predicates for determining this region, which achieves the same result, but has an algebraic degree of only 6. In addition, this method handles degeneracies in a manner which results in a diagram insensitive to insertion order. Finally, we show that implementing these lower degree predicates result in 39 to 66 percent less running time.

Power diagram

$$\delta(s_i,x) = \|p_i - x\|^2 - w_i^2$$

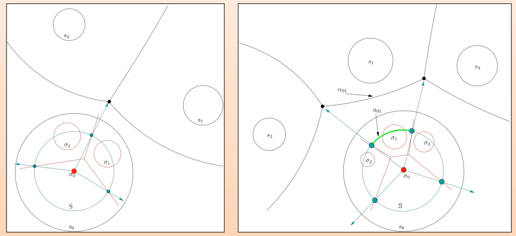




AW-Voronoi diagram

$$\delta(s_i,x) = \|p_i - x\| - w_i$$

Correspondence



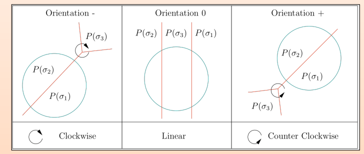
(Boissonnat and Karavelas): The projection of a cell of the AW-Voronoi diagram onto a sphere, $\mathbb{S}$, coincides with the intersection of the Power diagram and $\mathbb{S}$.

Specifically, given a set of *n* sites $\{s_1, s_2, \ldots, s_n\}$ in $\mathbb{R}^d$, the projection of the partial AW-Voronoi cell of $s_i=(p_i, w_i)$ onto $\mathbb{S}$, a unit hypersphere center at p_i corresponds to the intersection between the Power diagram of $\Sigma=\{\sigma_1, \sigma_2, \ldots, \sigma_n\}$ and $\mathbb{S}$ where,

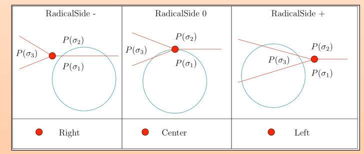
$$\sigma_j = (c_j, r_j, \alpha_j), \qquad c_j = \frac{q_j}{\alpha_j}, \qquad r_j = \frac{\omega_j^*}{\alpha_j}$$

$$q_j = p_j - p_i, \qquad \omega_j^* = w_j - w_i, \qquad \alpha_j = \begin{cases} q_i^2 - \omega_j^{*2}, & i = j \\ 1, & i \neq j \end{cases}$$

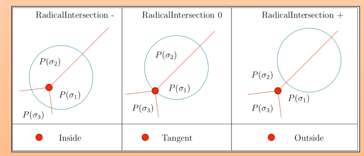
Predicates



$$sign\left(\begin{vmatrix} \alpha_1 & x_1 & y_1 \\ \alpha_2 & x_2 & y_2 \\ \alpha_3 & x_3 & y_3 \end{vmatrix}\right)$$

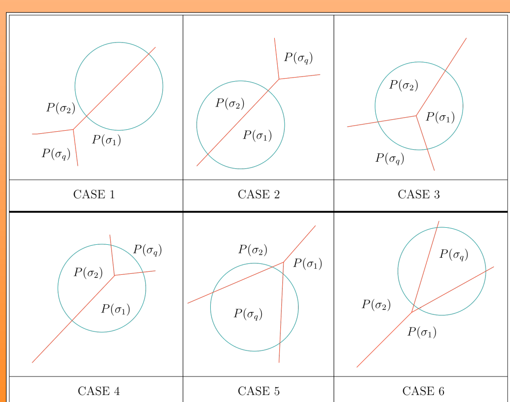


$$sign\left(-\begin{vmatrix} \alpha_1 & x_1 \\ \alpha_2 & x_2 \end{vmatrix} * \begin{vmatrix} \alpha_1 & r_1 & x_1 \\ \alpha_2 & r_2 & x_2 \\ \alpha_3 & r_3 & x_3 \end{vmatrix} - \begin{vmatrix} \alpha_1 & y_1 \\ \alpha_2 & y_2 \end{vmatrix} * \begin{vmatrix} \alpha_1 & r_1 & y_1 \\ \alpha_2 & r_2 & y_2 \\ \alpha_3 & r_3 & y_3 \end{vmatrix}\right)$$

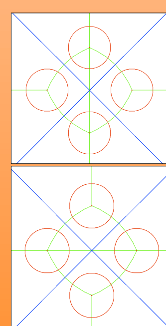


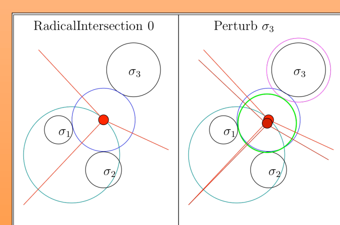
$$sign\left(\begin{vmatrix} \alpha_1 & r_1 & y_1 \\ \alpha_2 & r_2 & y_2 \\ \alpha_3 & r_3 & y_3 \end{vmatrix}^2 + \begin{vmatrix} \alpha_1 & x_1 & r_1 \\ \alpha_2 & x_2 & r_2 \\ \alpha_3 & x_3 & r_3 \end{vmatrix}^2 - \begin{vmatrix} \alpha_1 & x_1 & y_1 \\ \alpha_2 & x_2 & y_2 \\ \alpha_3 & x_3 & y_3 \end{vmatrix}^2\right)$$

An Example: Vertex Conflict



Degeneracies





Results

- 39-66% speed up for evaluating predicates
- 10-20% reduction in exact arithmetic in nearly degenerate inputs
- Vertex and Edge conflict predicates reduced from degree 14 to degree 6
- Method for handling degeracies insensitive to insertion order

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