Degree-driven algorithm design
for computing the Voronoi diagram

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Summary
In the paper Robust Proximity Queries: an Illustration of Degree-Driven Algorithm Design, Liotta, Preparata, and Tamassia derive a structure from the Voronoi diagram whose computation requires five times the precision of the input, but supports proximity queries in \( O(\log n) \) time, with only two times the input precision. This work considers how this structure can be computed directly, using at most triple precision in \( O(n \log n + \log g) \) time where \( g \) is the bisector length.

Implicit Voronoi Diagram and the Cell Graph
Given a set of \( n \) sites \( S = \{s_1, s_2, \ldots, s_n\} \) whose coordinates are \( b \)-bit integers, the implicit Voronoi diagram \( V^*(S) \) [LPT97] contains two parts

- **Topological**: The Planar embedding of the Voronoi diagram of \( S \).
- **Geometric**: For each vertex \((v_x, v_y)\) of the Voronoi diagram of \( S \), the implicit Voronoi diagram, \( V^*(S) \) stores the half integers

\[
v_x^* = \begin{cases} 
    v_x & \text{if } 0 \leq v_x \leq 2^b - 1 \text{ and } v_x \in \mathbb{Z}, \\
    \lfloor v_x \rfloor + \frac{1}{2} & \text{if } 0 \leq v_x \leq 2^b - 1 \text{ and } v_x \notin \mathbb{Z}, \\
    0 & v_x < 0, \\
    2^b - \frac{1}{2} & v_x > 2^b - 1.
\end{cases}
\]

Predicates and Operations

- **BisectorInCell**
- **Stabbing Ordering**

Incremental Construction
Using these predicates and operations we update the cell graph in a manner similar to Sugihara and Iri [SI92] and carve out the region of the newly inserted site.

References