Degree-driven algorithm design for computing the Voronoi diagram

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Summary

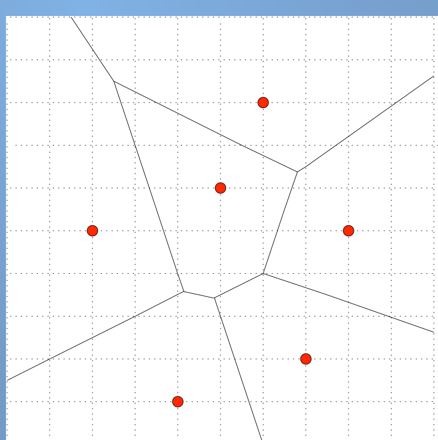
In the paper *Robust Proximity Queries: an Illustration of Degree-Driven Algorithm Design*, Liotta, Preparata, and Tamassia derive a structure from the Voronoi diagram whose computation requires five times the precision of the input, but supports proximity queries in O(log *n*) time, with only two times the input precision. This work considers how this structure can be computed directly, using at most triple precision in O(*n* (log *n* + log *g*)) time where *g* is the bisector length.

Implicit Voronoi Diagram and the Cell Graph

Given a set of *n* sites $S = \{s_1, s_2, \ldots, s_n\}$ whose coordinates are *b*-bit integers, the *implicit Voronoi diagram* $V^*(S)$ [LPT97] contains two parts

- Topological: The Planar embedding of the Voronoi diagram of *S.*
- Geometric: For each vertex (v_x, v_y) of the Voronoi diagram of *S*, the implicit Voronoi diagram, $V^*(S)$ stores the half integers

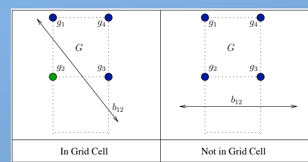
$$v_x^* = \begin{cases} v_x & \text{if } 0 \le v_x \le 2^b - 1 \text{ and } v_x \in \mathbb{Z}, \\ \lfloor v_x \rfloor + \frac{1}{2} & \text{if } 0 \le v_x \le 2^b - 1 \text{ and } v_x \notin \mathbb{Z}, \\ 0 & v_x < 0, \\ 2^b - \frac{1}{2} & v_x > 2^b - 1. \end{cases}$$



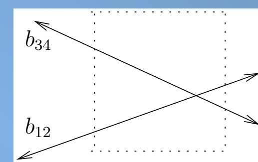
Lower Layer: Voronoi diagram
Middle Layer: Implicit Voronoi diagram
Top Layer: Cell Graph

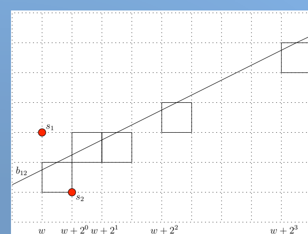
Predicates and Operations

BisectorInCell

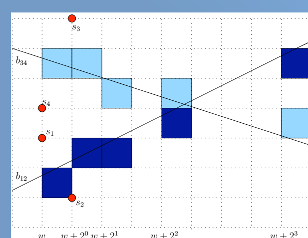


Stabbing Ordering





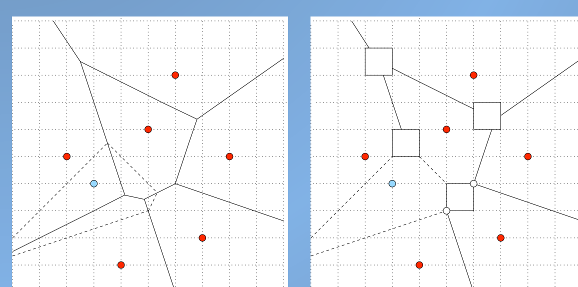
Bisector Walk



Bisector Intersection

Incremental Construction

Using these predicates and operations we update the cell graph in a manner similar to Sugihara and Iri [SI92] and carve out the region of the newly inserted site.



References

G. Liotta, F. P. Preparata, and R. Tamassia. Robust proximity queries: an illustration of degree-driven algorithm design. In SCG ’97, pages 156–165, New York, NY, USA, 1997. ACM.

K. Sugihara and M. Iri. Construction of the Voronoi diagram for ‘one million’ generators in single-precision arithmetic. Proceedings of the IEEE, 80(9):1471–1484, 1992.