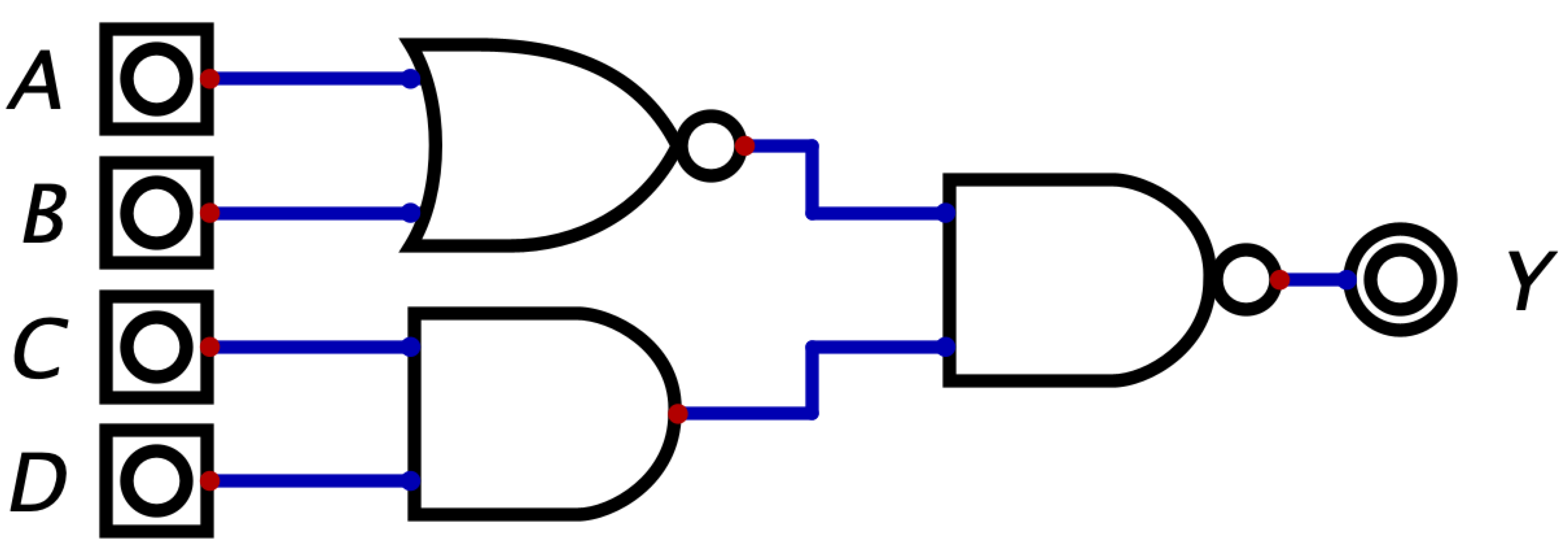
COMP311:
COMPUTER ORGANIZATION!

Lecture 6: Boolean Algebra, Digital Tutorial

tinyurl.com/comp311-fa25

FORMING LOGIC DIAGRAMS FROM EQUATIONS

Ex2: Forming Logic Diagrams from Equations $Y = \overline{\overline{(A+B)}CD}$



Ex3: Forming Logic Diagrams from Equations $Y = \overline{(\overline{A + B})(\overline{CD}) + E}$

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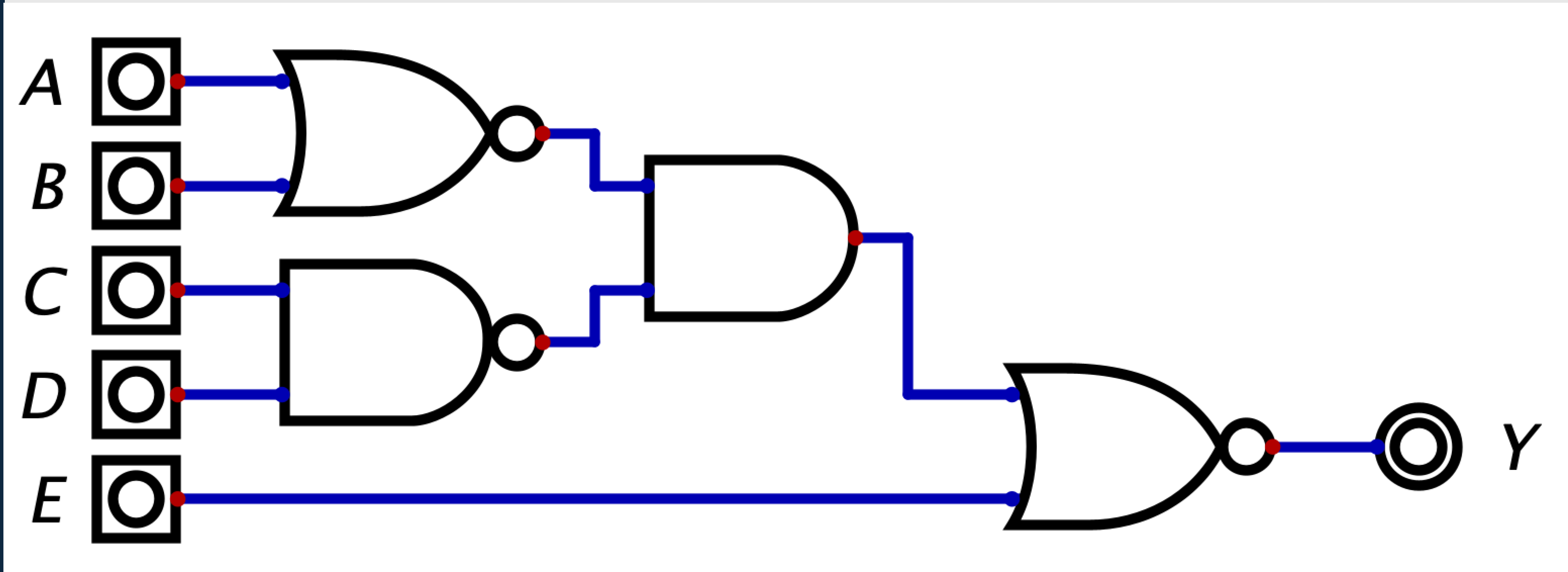


FORMING TRUTH TABLES FROM EQUATIONS

Ex2: Forming Truth Tables from Equations

$$Y = AB + \bar{C}$$

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Ex3: Forming Truth Tables from Equations

$$Y = A + \bar{A}B\bar{C}$$

A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Ex3:Forming Truth Tables from Equations

$$Y = A + \bar{A}B\bar{C}$$

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Ex4: Forming Truth Tables from Equations

$$Y = \bar{A}\bar{B} + \overline{AB}$$

A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Ex4: Forming Truth Tables from Equations

$$Y = \bar{A}\bar{B} + \overline{AB}$$

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

FORMING EQUATIONS FROM TRUTH TABLES

Ex1: Forming Equations from Truth Tables

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Ex1: Forming Equations from Truth Tables

A	B	C	Y	
0	0	0	1	$\bar{A}\bar{B}\bar{C}$
0	0	1	0	
0	1	0	1	$\bar{A}B\bar{C}$
0	1	1	0	
1	0	0	0	
1	0	1	1	$A\bar{B}C$
1	1	0	0	
1	1	1	0	

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}C$$

Ex2: Forming Equations from Truth Tables

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Ex2: Forming Equations from Truth Tables

A	B	C	Y	
0	0	0	0	
0	0	1	1	$\bar{A}\bar{B}C$
0	1	0	0	
0	1	1	0	
1	0	0	1	$A\bar{B}\bar{C}$
1	0	1	0	
1	1	0	1	$AB\bar{C}$
1	1	1	1	ABC

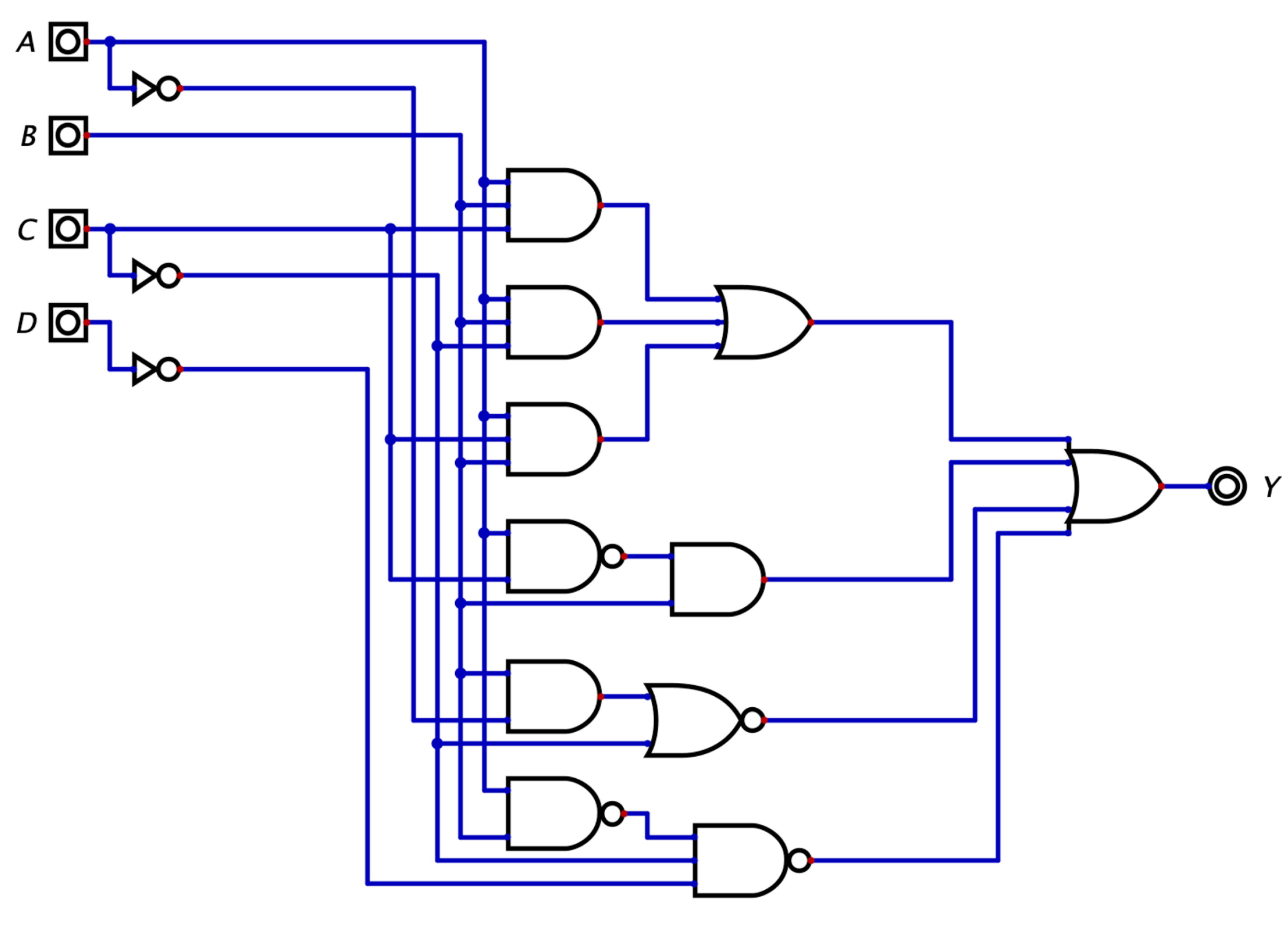
$$Y = \bar{A}\bar{B}C + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

THE LAWS OF BOOLEAN ALGEBRA

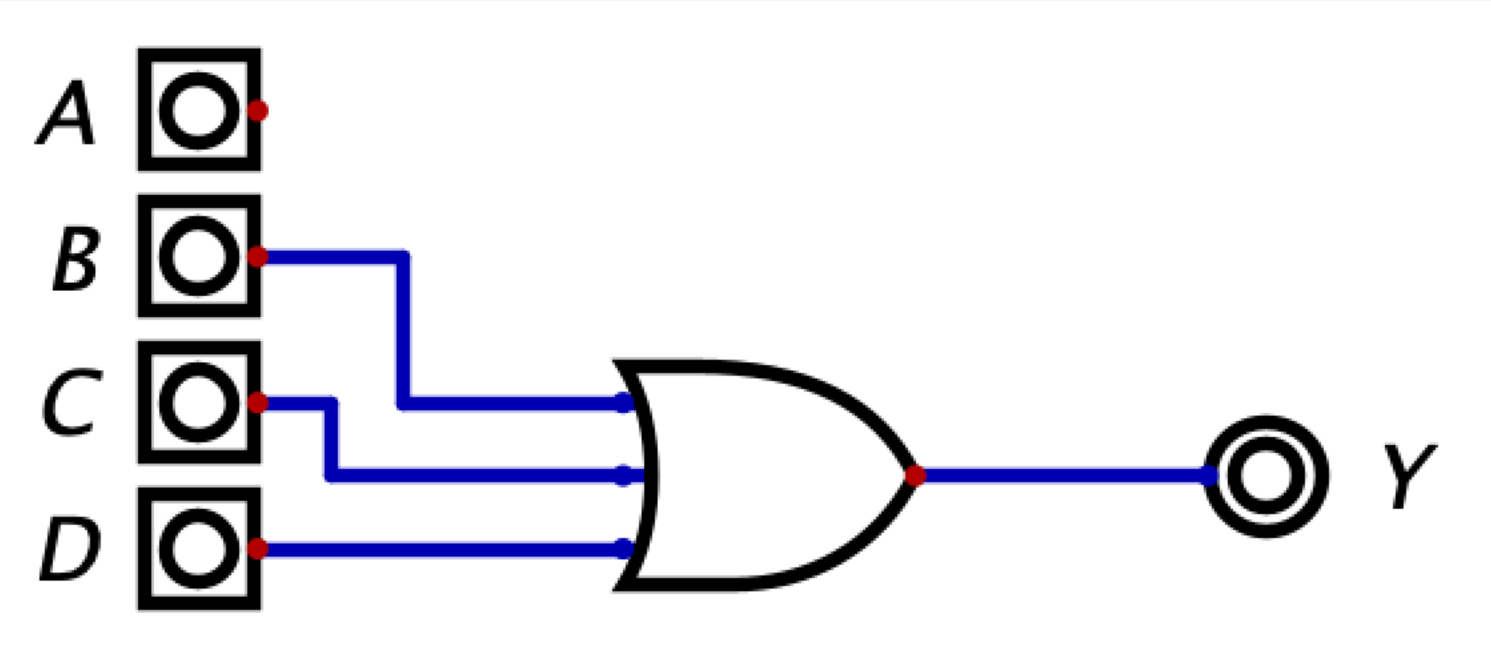
Optimizing Our Circuits

- Fewer transistors means
  - *Smaller Area*
  - *Lower power consumption*
  - *Shorter delay from input to output*
  - *Lower cost*

Optimizing Our Gates



Optimizing Our Gates



Boolean Algebra Laws

Name	OR Form	AND Form
Identity	$A + 0 = A$	$A \cdot 1 = A$
Null	$A + 1 = 1$	$A \cdot 0 = 0$
Idempotent	$A + A = A$	$A \cdot A = A$
Complement	$A + \overline{A} = 1$	$A \cdot \overline{A} = 0$
Commutative	$A + B = B + A$	$A \cdot B = B \cdot A$
Associative	$(A + B) + C = A + (B + C)$	$(A \cdot B) \cdot C = A \cdot (B \cdot C)$
Distributive	$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$	$A + (B \cdot C) = (A + B) \cdot (A + C)$
De Morgan's	$\overline{A + B} = \overline{A} \cdot \overline{B}$	$\overline{A \cdot B} = \overline{A} + \overline{B}$

Your Turn!

Using the laws that you have just learned, simplify the following Boolean expression:

$$A \cdot (A + B)$$

Name	OR Form	AND Form
Identity	$A + 0 = A$	$A \cdot 1 = A$
Null	$A + 1 = 1$	$A \cdot 0 = 0$
Idempotent	$A + A = A$	$A \cdot A = A$
Complement	$A + \overline{A} = 1$	$A \cdot \overline{A} = 0$
Commutative	$A + B = B + A$	$A \cdot B = B \cdot A$
Associative	$(A + B) + C = A + (B + C)$	$(A \cdot B) \cdot C = A \cdot (B \cdot C)$
Distributive	$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$	$A + (B \cdot C) = (A + B) \cdot (A + C)$
De Morgan's	$\overline{A + B} = \overline{A} \cdot \overline{B}$	$\overline{A \cdot B} = \overline{A} + \overline{B}$

Your Turn!

$$\mathbf{A \cdot (A + B)}$$

$$AA + AB$$
$$A + AB$$
$$A(1 + B)$$
$$A(1)$$
$$A$$

Name	OR Form	AND Form
Identity	$A + 0 = A$	$A \cdot 1 = A$
Null	$A + 1 = 1$	$A \cdot 0 = 0$
Idempotent	$A + A = A$	$A \cdot A = A$
Complement	$A + \overline{A} = 1$	$A \cdot \overline{A} = 0$
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Associative	$(A + B) + C = A + (B + C)$	$(A \cdot B) \cdot C = A \cdot (B \cdot C)$
Distributive	$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$	$A + (B \cdot C) = (A + B) \cdot (A + C)$
De Morgan’s	$\overline{A + B} = \overline{A} \cdot \overline{B}$	$\overline{A \cdot B} = \overline{A} + \overline{B}$

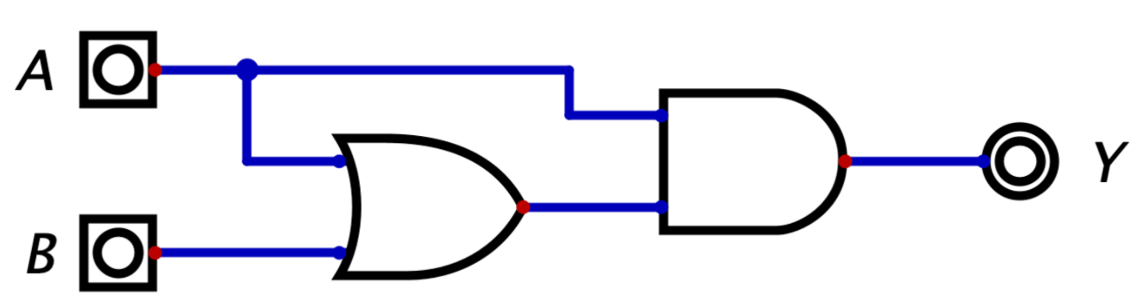
Absorption Law: $A \cdot (A + B) = A$

Recall: Transistor Count

Gate	Number of Transistors
NOT	2
AND	6
OR	6
NAND	4
NOR	4
XOR	12
XNOR	12

Comparing Number of Transistors Used

Original Circuit



$$Y = A \cdot (A + B)$$

Number of Transistors = 12

Simplified Circuit

$$Y = A$$

Number of Transistors = 0

Your Turn!

Simplify the following Boolean expression: $\boldsymbol{B + \bar{B}C}$

Name	OR Form	AND Form
Identity	$A + 0 = A$	$A \cdot 1 = A$
Null	$A + 1 = 1$	$A \cdot 0 = 0$
Idempotent	$A + A = \underline{A}$	$A \cdot A = \underline{A}$
Complement	$A + \bar{A} = 1$	$A \cdot \bar{A} = 0$
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Your Turn!

$$\boldsymbol{B + \bar{B}C}$$

$$(B+\bar{B})(B+C)$$
$$(1)(B+C)$$
$$B+C$$

Name	OR Form	AND Form
Identity	$A + 0 = A$	$A \cdot 1 = A$
Null	$A + 1 = 1$	$A \cdot 0 = 0$
Idempotent	$A + A = A$	$A \cdot A = A$
Complement	$A + \bar{A} = 1$	$A \cdot \bar{A} = 0$
Commutative	$A + B = B + A$	$A \cdot B = B \cdot A$
Associative	$(A + B) + C = A + (B + C)$	$(A \cdot B) \cdot C = A \cdot (B \cdot C)$
Distributive	$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$	$A + (B \cdot C) = (A + B) \cdot (A + C)$
De Morgan's	$\overline{A + B} = \bar{A} \cdot \bar{B}$	$\overline{A \cdot B} = \bar{A} + \bar{B}$

Recall: De Morgan's Theorem

$$\overline{A + B} = \bar{A} \cdot \bar{B}$$

$$\overline{AB} = \bar{A} + \bar{B}$$

DeMorgan’s

$$\overline{AB + CD}$$

- How would you apply DeMorgan’s to this expression?

DeMorgan's

$$\overline{AB + CD}$$

- How would you apply DeMorgan's to this expression?
- We group our terms based on the lowest operator precedence under the bar
- The order of operations for Boolean Algebra is **NOT, then AND, then OR.** Expressions inside brackets are always evaluated first.
- In this case, the operator with the lowest precedence is OR

$$(\overline{AB})(\overline{CD})$$

$$(\bar{A} + \bar{B})(\bar{C} + \bar{D})$$

DeMorgan's

$$\overline{\bar{A}B + \bar{C}}$$

- How would you apply DeMorgan's to this expression?

DeMorgan's

$$\overline{\bar{A}B + \bar{C}}$$

- How would you apply DeMorgan's to this expression?

$$\overline{\bar{A}B}C$$

$$(A + \bar{B})C$$

DeMorgan's

- How would you apply DeMorgan's to this expression?

$$\overline{(\bar{A} + B)(C + D)\overline{(E + F)} + G}$$

DeMorgan’s

- How would you apply DeMorgan’s to this expression?

$$\overline{(\bar{A} + B)(C + D)\overline{(E + F)} + G}$$

$$\overline{(\bar{A} + B)(C + D)\overline{(E + F)}} \cdot \bar{G}$$

$$(\overline{\bar{A} + B} + \overline{C + D} + E + F) \cdot \bar{G}$$

$$(A\bar{B} + \bar{C}\bar{D} + E + F) \cdot \bar{G}$$

Your Turn!

Simplify the following Boolean expression:

$$ABC + AB\overline{C} + \overline{AC}B + ACB + \overline{\overline{AB}\,\overline{C}\,\overline{D}} + \overline{\overline{A}B + \overline{C}}$$

Name	OR Form	AND Form
Identity	$A + 0 = A$	$A \cdot 1 = A$
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Your Turn!

$$ABC + AB\bar{C} + \overline{AC}B + ACB + \overline{\overline{AB}\,\overline{C}\,\overline{D}} + \overline{\bar{A}B + \bar{C}}$$

Your Turn!

$$ABC + AB\bar{C} + \overline{AC}B + ACB + \overline{\overline{AB}\,\overline{C}\,\overline{D}} + \overline{\bar{A}B + \bar{C}}$$

$$AB(C + \bar{C}) + B(\overline{AC} + AC) + AB + C + D + (\overline{\bar{A}B})C$$

$$AB(1) + B(1) + AB + C + D + (A + \bar{B})C$$

$$AB + B + AB + C + D + AC + \bar{B}C$$

$$AB + B + C + D + AC + \bar{B}C$$

$$B(A + 1) + C(1 + A) + \bar{B}C + D$$

$$B(1) + C(1) + \bar{B}C + D$$

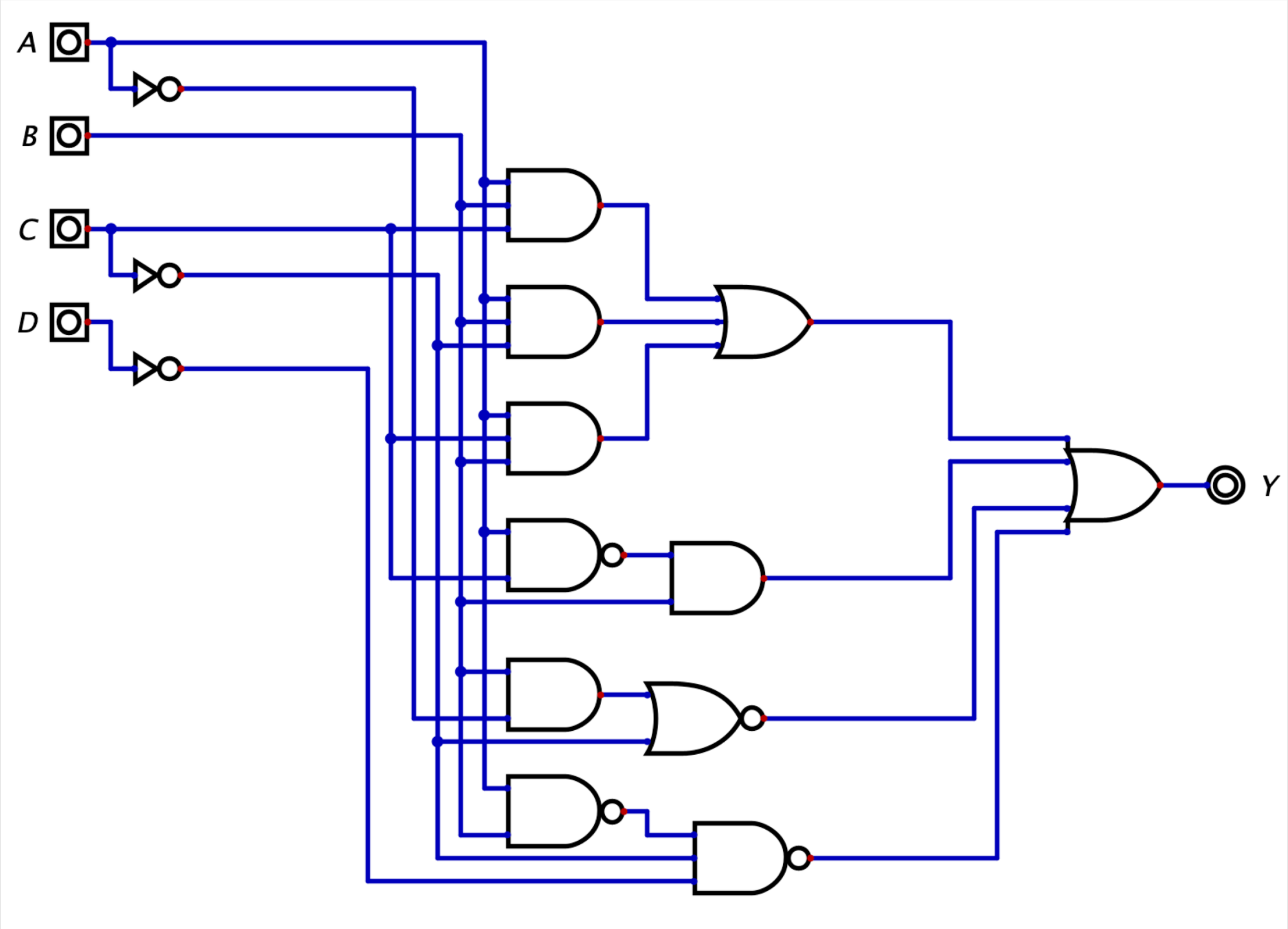
$$B + C + \bar{B}C + D$$

$$B + C(1 + \bar{B}) + D$$

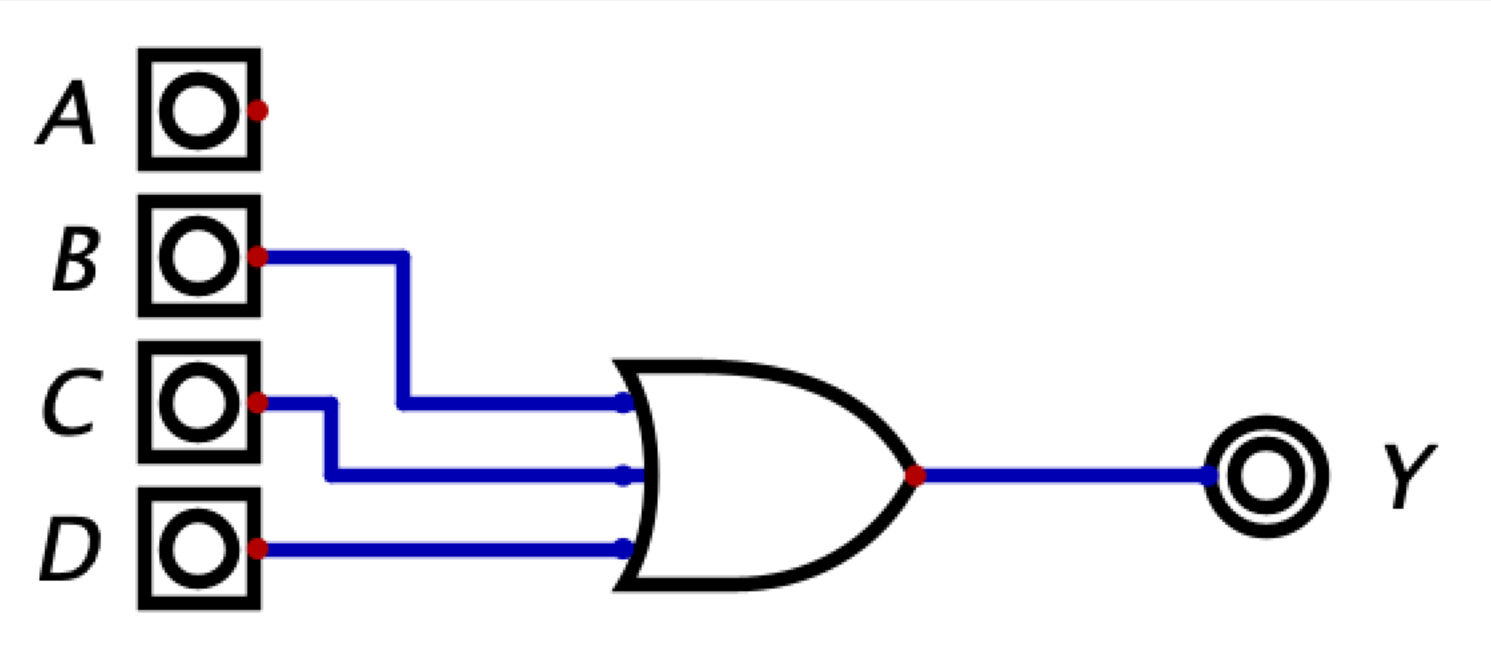
$$B + C(1) + D$$

$$B + C + D$$

Optimizing Our Gates



Optimizing Our Gates



Yay! We did it!

DIGITAL TUTORIAL