

COMP311: *COMPUTER ORGANIZATION!*

Lecture 8: Karnaugh Maps

tinyurl.com/comp311-fa25



KARNAUGH MAPS



Minimizing Logic

- Previously, we have discussed how to systematically convert logic specified as a truth table into sum-of-products (SOP).
- After that, we relied on cleverness, and luck to reduce these expressions via the properties and theorems of Boolean algebra.
- Today we will discuss a more systematic method for logic minimization, called a *Karnaugh map*, that is based on the *clustering* of terms.

Karnaugh Maps (K-Maps)

- Provide a graphical method to find simplified Boolean Equations
- K-maps make combinable terms (like the ones we just saw) easy to see by placing them next to each other on a grid!
- If you follow the procedure properly, you will end up with the most simplified equation!
- As an example consider the 2-variable Karnaugh map for the NAND function.

A	B	NAND
0	0	1
0	1	1
1	0	1
1	1	0

NAND truth table

A \ B	0	1
	0	1
0	1	1
1	1	0

NAND Karnaugh map

Because of this arrangement it is easier to see relationships between variables!



Karnaugh Maps (K-Maps)

A Karnaugh map is an *alternate truth-table layout* that places output terms such that shared variable states are adjacent. AS an example consider the 2-variable Karnaugh map for the NAND function.

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NAND truth
table

A \ B	0	1
0	1	1
1	1	0

NAND Karnaugh
map

The first row of 1s is where
 $Y = \sim A$ and the left column
is where
 $Y = \sim B$, which together gives

$$Y = \sim A + \sim B$$

*the De Morgan version of a
NAND gate*

Find the Equation

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

Find the Equation

	A	B	C	Y	
0	0	0	0	0	
1	0	0	1	1	$\bar{A}\bar{B}C$
2	0	1	0	0	
3	0	1	1	1	$\bar{A}BC$
4	1	0	0	0	
5	1	0	1	0	
6	1	1	0	0	
7	1	1	1	0	

$$Y = \bar{A}\bar{B}C + \bar{A}BC$$

$$Y = \bar{A}C$$

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC					
A		00	01	11	10
	0				
	1				

Each box in the K-Map corresponds to a single row in the truth table

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC					
A		00	01	11	10
	0	0			
	1				

Each box in the K-Map corresponds to a single row in the truth table

Row 0

$A = 0, B = 0, C = 0$

$Y = 0$

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC					
A		00	01	11	10
	0	0	1		
	1				

Each box in the K-Map corresponds to a single row in the truth table

Row 1

$A = 0, B = 0, C = 1$

$Y = 1$

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

		BC			
A		00	01	11	10
0	0	0 ₀	1 ₁		0 ₂
1	1				

Each box in the K-Map corresponds to a single row in the truth table

Row 2

$A = 0, B = 1, C = 0$

$Y = 0$

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC					
A		00	01	11	10
	0	0 ₀	1 ₁	1 ₃	0 ₂
	1				

Each box in the K-Map corresponds to a single row in the truth table

Row 3

$A = 0, B = 1, C = 1$

$Y = 1$

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

		BC				
A		00	01	11	10	
0	0	0 ₀	1 ₁	1 ₃	0 ₂	
1	1	0 ₄	0 ₅	0 ₇	0 ₆	

Each box in the K-Map corresponds to a single row in the truth table

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

Gray Code

BC

A

	00	01	11	10
0	0 ₀	1 ₁	1 ₃	0 ₂
1	0 ₄	0 ₅	0 ₇	0 ₆

Gray Code

- Successive values differ by only one bit

Gray Code

00

01

11

10

Binary Code

00

01

10

11



In binary code, two bits flip when we go from 1 to 2

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC					
A		00	01	11	10
	0	0 ₀	1 ₁	1 ₃	0 ₂
	1	0 ₄	0 ₅	0 ₇	0 ₆

Each square in a K-map differs from an adjacent square by a change in a single literal

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$\bar{A}\bar{B}C$

BC

A

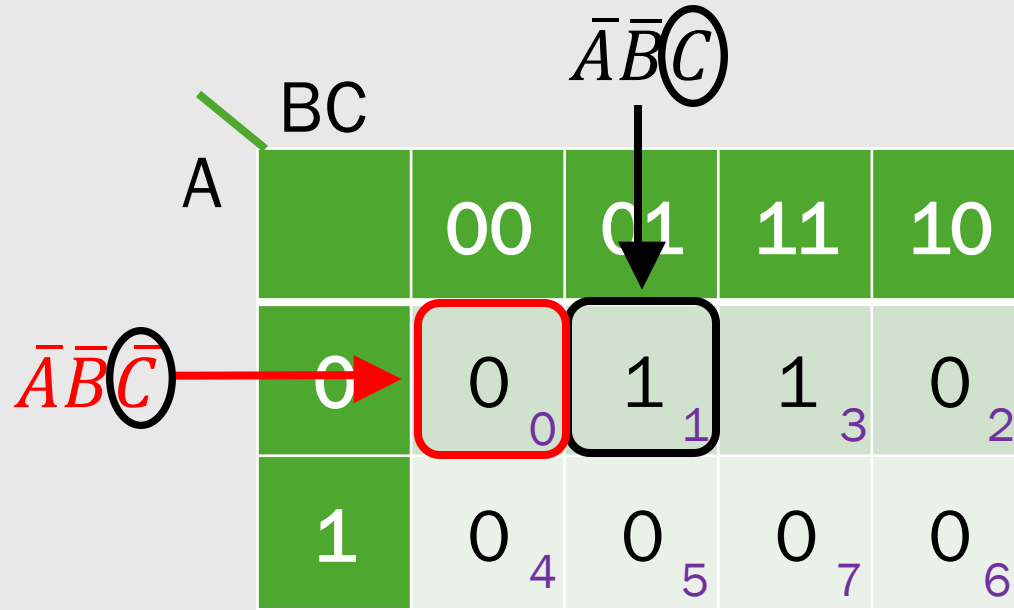
	00	01	11	10
0	0	1	1	0
1	0	0	0	0

Box 1 is highlighted around the value 1 in the cell corresponding to A=0, BC=01.

Let's examine the squares around box 1

3-input Function

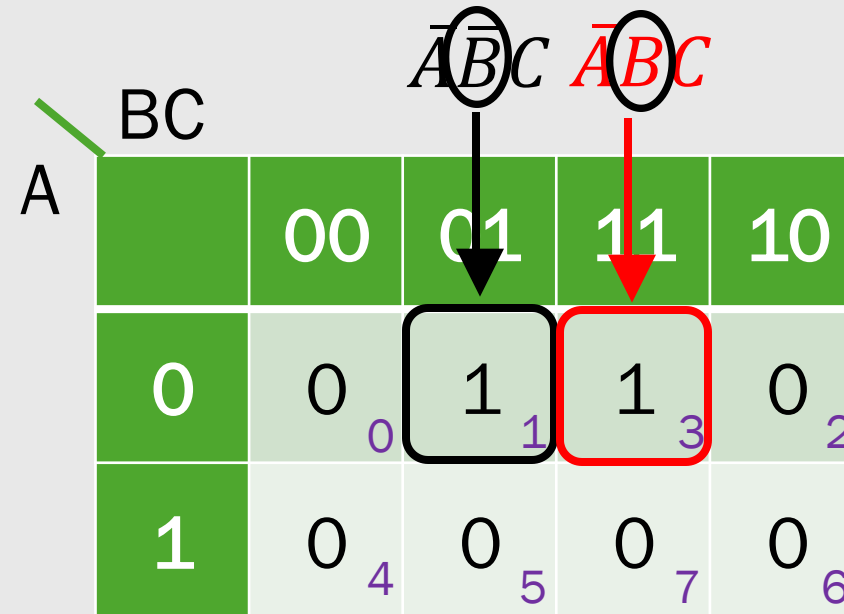
	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0



Box 0 differs by the C literal

3-input Function

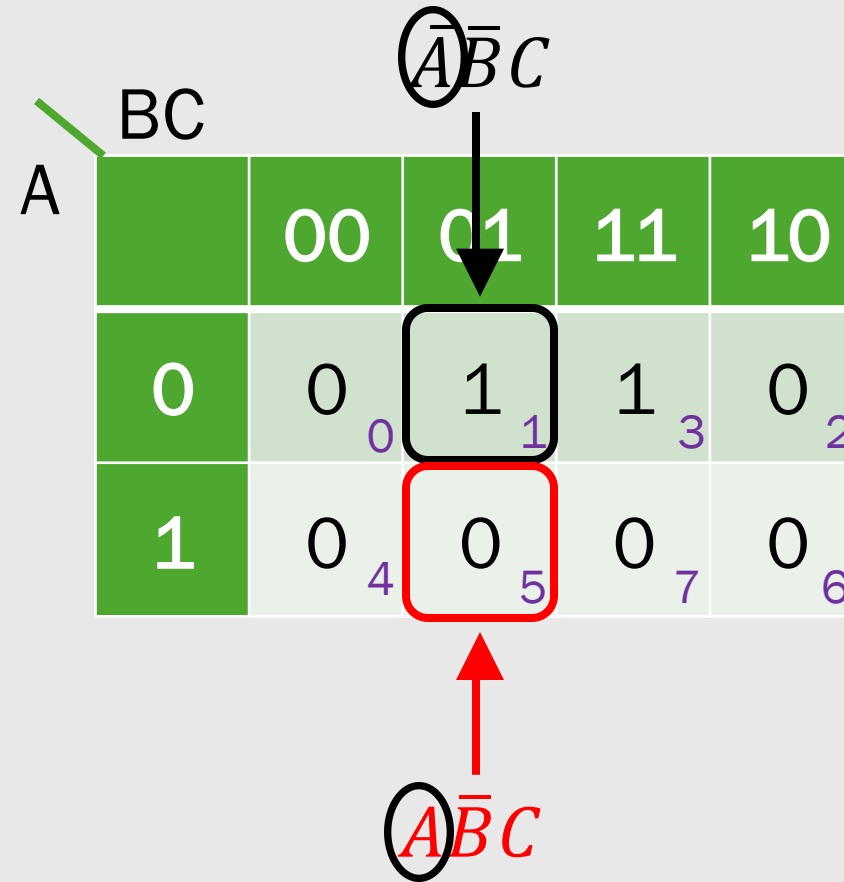
	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0



Box 3 differs by the B literal

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0



3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$\bar{A}\bar{B}C$

BC

A

	00	01	11	10
0	0	1	1	0
1	0	0	0	0

Diagram illustrating a 3-input function using a Karnaugh map. The map is a 3x4 grid with columns labeled BC (00, 01, 11, 10) and rows labeled A (0, 1). The entries are: Row 0: 00, 01, 11, 10; Row 1: 0, 0, 1, 0; Row 2: 1, 0, 0, 0. The entry 1 at (0, 1) is circled in black. The entries 0 at (1, 0) and (1, 2) are circled in red. The entry 1 at (0, 1) is labeled with a purple 1. The entries 0 at (1, 0) and (1, 2) are labeled with purple 4 and 7 respectively. The entry 0 at (0, 0) is labeled with a purple 0. The entry 0 at (0, 2) is labeled with a purple 3. The entry 0 at (0, 3) is labeled with a purple 2. The entry 0 at (1, 1) is labeled with a purple 5. The entry 0 at (1, 3) is labeled with a purple 6.

Diagonal entries are not considered adjacent

Simplification using the K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC		00	01	11	10
A	0	0	1	1	0
1	0	0	0	0	0

To graphically find the simplified equation, first group the terms together. Then, find the literals that the group has in common.

Simplification using the K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC		00	01	11	10
A	0	0	1 ₁	1 ₃	0 ₂
1	0 ₄	0 ₅	0 ₇	0 ₆	

Which literals do these entries share?

Simplification using the K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

		BC			
A		00	01	11	10
	0	0	1	1	0
	1	0	0	0	0

We can see that the entries inside of the group each contain $\bar{A}$ and C .

Simplification using the K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

		BC			
A					
		00	01	11	10
	0	0	1	1	0
	1	0	0	0	0

The literals that the group share form the simplified equation.

$$Y = \bar{A}C$$

Simplification using the K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

		BC			
A		00	01	10	11
0	0 ₀	1 ₁	0 ₂	1 ₃	
1	0 ₄	0 ₅	0 ₆	0 ₇	

If we had used binary code instead of gray code, this visual simplification would not have been possible!

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

		BC			
A		00	01	11	10
0		0	1	3	2
1		4	5	7	6

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC		00	01	11	10
A					
0		0	1	3	2
1		4	5	7	6

Step 1: Fill in the K-map

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

		BC			
A		00	01	11	10
0	1	1 ₀	1 ₁	1 ₃	1 ₂
1	0	0 ₄	0 ₅	0 ₇	0 ₆

Step 1: Fill in the K-map

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC					
A		00	01	11	10
	0	1 ₀	1 ₁	1 ₃	1 ₂
	1	0 ₄	0 ₅	0 ₇	0 ₆

Step 2: Group the terms together

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC					
A		00	01	11	10
	0	1 ₀	1 ₁	1 ₃	1 ₂
	1	0 ₄	0 ₅	0 ₇	0 ₆

Step 2: Group the terms together

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC					
A		00	01	11	10
	0	1 ₀	1 ₁	1 ₃	1 ₂
	1	0 ₄	0 ₅	0 ₇	0 ₆

Step 3: Find the literals that the group has in common

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC					
A		00	01	11	10
	0	1 ₀	1 ₁	1 ₃	1 ₂
	1	0 ₄	0 ₅	0 ₇	0 ₆

Step 3: Find the literals that the group has in common

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC					
A		00	01	11	10
0	0	1 ₀	1 ₁	1 ₃	1 ₂
1	1	0 ₄	0 ₅	0 ₇	0 ₆

Step 4: Derive the equation.

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC					
A		00	01	11	10
	0	1 ₀	1 ₁	1 ₃	1 ₂
	1	0 ₄	0 ₅	0 ₇	0 ₆

Step 4: Derive the equation.

$$Y = \bar{A}$$



K-MAP GROUPS



Ex1: Reading A K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

BC

A

	00	01	11	10
0	0	0	0	0
1	0	0	1	1

The K-map shows the output Y for input combinations of A, B, and C. The cells are labeled with their decimal indices (0-7) in the bottom right corner. The cells containing 1 are at indices 6 and 7, which are highlighted with a red box.

Ex1: Reading A K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

BC					
A		00	01	11	10
	0	0 ₀	0 ₁	0 ₃	0 ₂
	1	0 ₄	0 ₅	1 ₇	1 ₆

$$Y = AB$$

Ex2: Reading A K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC		00	01	11	10
A	0	0 ₀	1 ₁	0 ₃	0 ₂
1	0 ₄	0 ₅	0 ₇	0 ₆	

Ex2: Reading A K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A	BC				
		00	01	11	10
	0	0 ₀	1 ₁	0 ₃	0 ₂
1	0 ₄	0 ₅	0 ₇	0 ₆	

$$Y = \bar{A}\bar{B}C$$

Ex3: Reading a K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

BC		00	01	11	10
A	0	0 ₀	0 ₁	0 ₃	0 ₂
	1	1 ₄	1 ₅	1 ₇	0 ₆

Ex3: Reading a K-Map

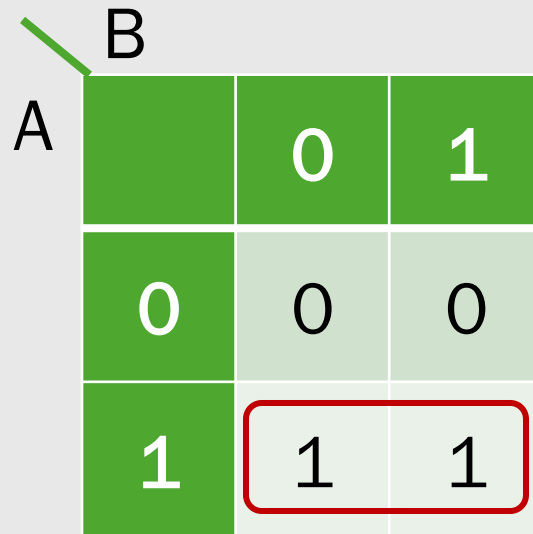
	A	B	C	Y
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

BC		00	01	11	10
A	0	0 ₀	0 ₁	0 ₃	0 ₂
1	1 ₄	1 ₅	1 ₇	0 ₆	

$$Y = A\bar{B} + AC$$

K-Map Grouping Rules

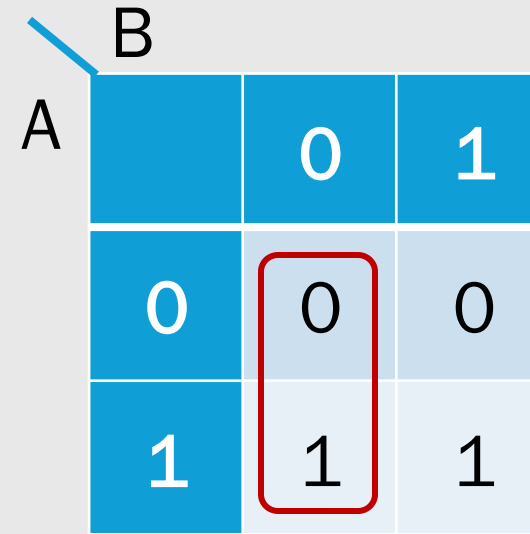
All squares in each circle must contain 1's



A 3x3 K-map for variables A and B. The map is color-coded: green for 1s and light green for 0s. A red rectangle highlights a group of two 1s in the bottom row, columns 2 and 3. The group is labeled 'Right'.

	B		
A		0	1
	0	0	0
	1	1	1

Right



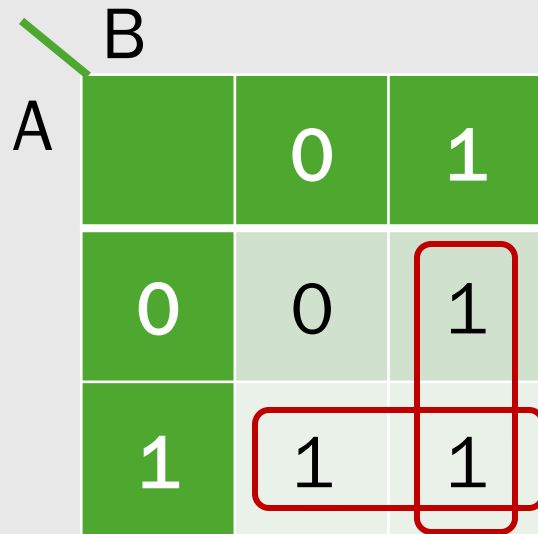
A 3x3 K-map for variables A and B. The map is color-coded: blue for 1s and light blue for 0s. A red rectangle highlights a group of two 1s in the bottom row, columns 2 and 3. The group is labeled 'Wrong'.

	B		
A		0	1
	0	0	0
	1	1	1

Wrong

K-Map Grouping Rules

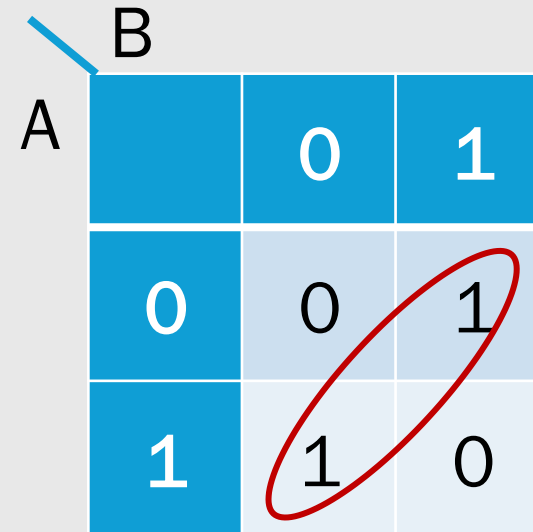
Groups may be horizontal or vertical, but not diagonal



A 3x3 Karnaugh map for variables A and B. The map is colored green for 1s and light green for 0s. The top row is labeled 'B' and the left column is labeled 'A'. The map contains 1s at (A=0, B=1), (A=1, B=0), (A=1, B=1), and (A=1, B=2). The map is divided into four 2x2 quadrants. The top-left quadrant (A=0, B=0) is green. The top-right quadrant (A=1, B=0) is light green. The bottom-left quadrant (A=0, B=1) is light green. The bottom-right quadrant (A=1, B=1) is light green. Two red rectangles highlight valid groupings: one horizontal group of 1s in the bottom row (A=1, B=0 and A=1, B=1) and one vertical group of 1s in the right column (A=1, B=0 and A=1, B=1).

	B=0	B=1	B=2
A=0	0	1	0
A=1	0	1	1

Right



A 3x3 Karnaugh map for variables A and B. The map is colored blue for 1s and light blue for 0s. The top row is labeled 'B' and the left column is labeled 'A'. The map contains 1s at (A=0, B=1), (A=1, B=0), (A=1, B=1), and (A=1, B=2). The map is divided into four 2x2 quadrants. The top-left quadrant (A=0, B=0) is blue. The top-right quadrant (A=1, B=0) is light blue. The bottom-left quadrant (A=0, B=1) is light blue. The bottom-right quadrant (A=1, B=1) is light blue. A red oval highlights an invalid diagonal grouping of 1s at (A=1, B=0) and (A=0, B=1).

	B=0	B=1	B=2
A=0	0	1	0
A=1	0	1	1

Wrong

K-Map Grouping Rules

Groups must contain 2^n cells where $n = 0, 1, 2$, etc.

BC

A

	0	1
0	0	0
1	1	0

Right

BC

A

	00	01	11	10
0	0	0	0	0
1	1	1	1	0

Right

BC

A

	00	01	11	10
0	0	0	0	0
1	1	1	1	0

Wrong

BC

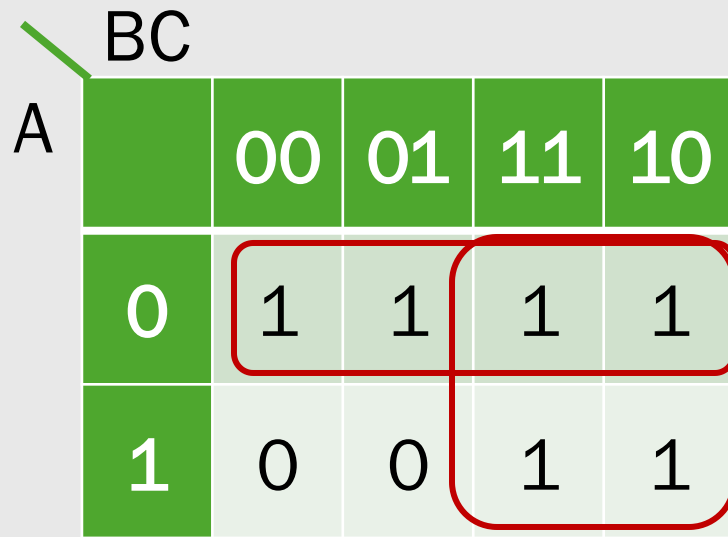
A

	00	01	11	10
0	1	1	0	0
1	1	1	1	0

Right

K-Map Grouping Rules

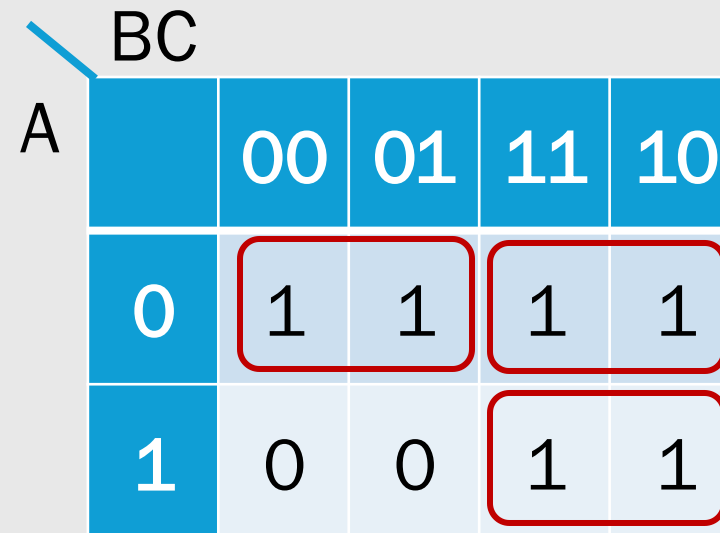
Each group must be as large as possible



A Karnaugh map for variables A and BC. The map is a 2x5 grid. The columns are labeled with BC values: 00, 01, 11, 10. The rows are labeled with A values: 0, 1. The cells are colored green. The values in the cells are: (A=0, BC=00) is empty, (A=0, BC=01) is 1, (A=0, BC=11) is 1, (A=0, BC=10) is 1, (A=1, BC=00) is 0, (A=1, BC=01) is 0, (A=1, BC=11) is 1, (A=1, BC=10) is 1. Two red boxes highlight the groups: one box covers the four 1s in the A=0 row, and another box covers the four 1s in the BC=11 and BC=10 columns.

	BC				
A		00	01	11	10
0		1	1	1	1
1	0	0	0	1	1

Right



A Karnaugh map for variables A and BC. The map is a 2x5 grid. The columns are labeled with BC values: 00, 01, 11, 10. The rows are labeled with A values: 0, 1. The cells are colored blue. The values in the cells are: (A=0, BC=00) is empty, (A=0, BC=01) is 1, (A=0, BC=11) is 1, (A=0, BC=10) is 1, (A=1, BC=00) is 0, (A=1, BC=01) is 0, (A=1, BC=11) is 1, (A=1, BC=10) is 1. Two red boxes highlight the groups: one box covers the four 1s in the A=0 row, and another box covers the two 1s in the A=1 row, BC=11 and BC=10 columns. This grouping is incorrect because it does not cover all 1s in the largest possible groups.

	BC				
A		00	01	11	10
0		1	1	1	1
1	0	0	0	1	1

Wrong

K-Map Grouping Rules

Each cell containing a 1 must be in at least one group

BC

A

	00	01	11	10
0	0	1	0	1
1	0	0	0	1

Right

BC

A

	00	01	11	10
0	0	1	0	1
1	1	0	0	1

Wrong

K-Map Grouping Rules

Groups may wrap around the table

		BC			
A		00	01	11	10
	0	1	0	0	1
	1	1	0	0	1

K-Map Grouping Rules

Every circle should contain at least one unique 1

BC

A

	00	01	11	10
0	1	1	1	1
1	0	0	1	1

Right

BC

A

	00	01	11	10
0	1	1	1	1
1	0	0	1	1

Wrong

Ex: Reading a K-Map

		BC			
A		00	01	11	10
	0	0 ₀	0 ₁	0 ₃	1 ₂
	1	0 ₄	1 ₅	1 ₇	1 ₆

Ex: Reading a K-Map

	BC				
A		00	01	11	10
0	0	0 ₀	0 ₁	0 ₃	1 ₂
1	0	0 ₄	1 ₅	1 ₇	1 ₆

$$Y = AC + B\bar{C}$$

Ex: Reading a K-Map

		BC			
A		00	01	11	10
	0	0 ₀	0 ₁	0 ₃	1 ₂
	1	0 ₄	1 ₅	1 ₇	1 ₆

Wrong – the blue group does not contain a unique 1. This grouping would produce a logically equivalent equation, but it would not be the most simplified.

		BC			
A		00	01	11	10
	0	0 ₀	0 ₁	0 ₃	1 ₂
	1	0 ₄	1 ₅	1 ₇	1 ₆

Wrong – the blue group is not a power of 2. We cannot derive a term for this group.

Use a K-Map to find the simplified SOP Equation

	A	B	C	Y
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

BC					
A		00	01	11	10
	0				
	1				
		0	1	3	2
		4	5	7	6

Use a K-Map to find the simplified SOP Equation

	A	B	C	Y
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

BC					
A		00	01	11	10
	0	1 ₀	0 ₁	1 ₃	1 ₂
	1	1 ₄	0 ₅	1 ₇	1 ₆

$$Y = B + \bar{C}$$

4-variable K-Map

		CD			
AB		00	01	11	10
	00				
	01				
	11				
	10				

Use Gray code order so that adjacent cells only differ by one literal.

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

4-variable K-Map

		CD			
AB		00	01	11	10
	00	0			
	01				
	11				
	10				

This cell is 0000, so it corresponds to row 0

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

4-variable K-Map

		CD			
AB		00	01	11	10
	00				
	01				
	11				
	10				

This cell is 0001, so it corresponds to row 1

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

4-variable K-Map

		CD			
AB		00	01	11	10
	00				
	01				
	11				
	10				

This cell is 0110, so it corresponds to row 6

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

4-variable K-Map

		CD			
AB		00	01	11	10
	00				
	01				
	11				
	10				

This cell is 1111, so it corresponds to row 15

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

4-variable K-Map

		CD			
AB		00	01	11	10
	00				
	01				
	11				
	10				
		0	1	3	2
		4	5	7	6
		12	13	15	14
		8	9	11	10

Use Gray code order so that adjacent cells only differ by one literal.

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

Ex: 4-variable K-Map

		CD			
AB		00	01	11	10
	00	0	1	3	2
	01	4	5	7	6
	11	12	13	15	14
	10	8	9	11	10

	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

Ex: 4-variable K-Map

		CD			
AB		00	01	11	10
	00	0 ₀	0 ₁	1 ₃	1 ₂
	01	1 ₄	1 ₅	0 ₇	0 ₆
	11	1 ₁₂	1 ₁₃	0 ₁₅	0 ₁₄
	10	0 ₈	0 ₉	0 ₁₁	1 ₁₀

	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

Ex: 4-variable K-Map

		CD			
AB		00	01	11	10
	00	0 ₀	0 ₁	1 ₃	1 ₂
	01	1 ₄	1 ₅	0 ₇	0 ₆
	11	1 ₁₂	1 ₁₃	0 ₁₅	0 ₁₄
	10	0 ₈	0 ₉	0 ₁₁	1 ₁₀

$$Y = B\bar{C} + \bar{A}\bar{B}C + \bar{B}C\bar{D}$$

	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

Q2.1: Reading a K-Map

CD

AB

	00	01	11	10
00	1	0	0	0
01	1	1	1	0
11	1	1	1	1
10	1	0	0	0

WS Q2.1: Reading a K-Map

CD

AB

	00	01	11	10
00	1	0	0	0
01	1	1	1	0
11	1	1	1	1
10	1	0	0	0

$$Y = \bar{C}\bar{D} + BD + AB$$

WS 2.2: Reading a K-Map

CD

AB

	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	1	1

Q2.2: Reading a K-Map

AB \ CD					
	00	01	11	10	
	00	0	0	1	1
	01	1	1	1	1
	11	1	1	0	0
	10	0	0	1	1

$$Y = B\bar{C} + \bar{B}C + \bar{A}C$$

AB \ CD					
	00	01	11	10	
	00	0	0	1	1
	01	1	1	1	1
	11	1	1	0	0
	10	0	0	1	1

$$Y = B\bar{C} + \bar{B}C + \bar{A}B$$

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

AB \ CD	CD			
	00	01	11	10
00				
01				
11				
10				

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

AB \ CD	CD			
	00	01	11	10
00	1			
01	1			
11	1			
10	1			

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + \textcolor{red}{ABC}\bar{D} + ACD + \bar{A}\bar{C}\bar{D} + A\bar{B}C\bar{D}$$

		CD			
AB		00	01	11	10
	00	1			
	01	1			
	11	1			1
	10	1			

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + \textcolor{red}{ACD} + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

		CD			
AB		00	01	11	10
	00	1			
	01	1			
	11	1		1	1
	10	1		1	

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

AB \ CD	CD				
	00	01	11	10	
00	1			1	
01	1			1	
11	1		1	1	
10	1		1		

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + \textcolor{red}{A}\bar{B}C\bar{D}$$

AB \ CD	CD				
	00	01	11	10	
00	1			1	
01	1			1	
11	1		1	1	
10	1		1	1	

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

		CD				
AB			00	01	11	10
	00	1	0	0	1	
	01	1	0	0	1	
	11	1	0	1	1	
	10	1	0	1	1	

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

A 5x5 Karnaugh Map for variables A, B, C, and D. The vertical axis is labeled 'AB' and the horizontal axis is labeled 'CD'. The 'AB' labels are 00, 01, 11, 10. The 'CD' labels are 00, 01, 11, 10. The map contains 1s in the following cells: (00,00), (00,10), (01,00), (01,10), (11,00), (11,10), (11,11), (11,10), (10,00), (10,10), (10,11), (10,10). Red lines group the 1s into two groups: a group of four 1s in the first two columns (C=0) and a group of four 1s in the last two columns (C=1).

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	1	1
10	1	0	1	1

$$Y = AC + \bar{D}$$

Recall: Boolean Algebra simplification

$$ABC + AB\bar{C} + \overline{AC}B + ACB + \overline{\overline{AB}\bar{C}\bar{D}} + \overline{\overline{AB} + \bar{C}}$$

$$AB(C + \bar{C}) + B(\overline{AC} + AC) + AB + C + D + (\overline{\overline{AB}})C$$

$$AB(1) + B(1) + AB + C + D + (A + \bar{B})C$$

$$AB + B + AB + C + D + AC + \bar{B}C$$

$$AB + B + C + D + AC + \bar{B}C$$

$$B(A + 1) + C(1 + A) + \bar{B}C + D$$

$$B(1) + C(1) + \bar{B}C + D$$

$$B + C + \bar{B}C + D$$

$$B + C(1 + \bar{B}) + D$$

$$B + C(1) + D$$

$$B + C + D$$

Let's use a K-Map Instead

1. Put the equation in SOP form.
2. Fill out the K-Map.
3. Find the simplified equation.

Reminder: Sum of Products / Product of Sums

Sum of Products (SOP)

- When two or more products (AND) are summed (OR) together

$$Y = AB + A\bar{C}$$

Product of Sums (POS)

- When two or more sums (OR) are composed (AND) together

$$Y = (A + B)(C + A)$$

Using a K-Map to Simplify a Boolean Equation

1. Put the equation in SOP form
2. Fill out the K-Map
3. Find the simplified equation

$$ABC + ABC\bar{C} + \overline{AC}B + ACB + \overline{\overline{AB}\overline{C}\overline{D}} + \overline{\overline{AB} + \overline{C}}$$

AB \ CD	CD			
	00	01	11	10
00				
01				
11				
10				

Using a K-Map to Simplify a Boolean Equation

$$ABC + AB\bar{C} + \overline{AC}B + ACB + \overline{\overline{AB}\overline{C}\overline{D}} + \overline{\overline{AB} + \bar{C}}$$

1. Put the equation in SOP form

$$ABC + AB\bar{C} + \bar{A}B + \bar{A}C + ACB + AB + C + D + AC + \bar{B}C$$

Using a K-Map to Simplify a Boolean Equation

1. Put the equation in SOP form
2. Fill out the K-Map

AB \ CD	CD			
	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	1	1	1	1
10	0	1	1	1

$$ABC + ABC\bar{C} + \bar{A}B + \bar{A}C + ACB + AB + C + D + AC + \bar{B}C$$

Using a K-Map to Simplify a Boolean Equation

1. Put the equation in SOP form
2. Fill out the K-Map
3. Find the simplified equation

$$B + C + D$$

	CD				
AB	00	01	11	10	
00	0	1	1	1	
01	1	1	1	1	
11	1	1	1	1	
10	0	1	1	1	