COMP311: *COMPUTER ORGANIZATION!*

Lecture 8: Karnaugh Maps

tinyurl.com/comp311-fa25

KARNAUGH MAPS

Minimizing Logic

- Previously, we have discussed how to systematically convert logic specified as a truth table into sum-of-products (SOP).
- After that, we relied on cleverness, and luck to reduce these expressions via the properties and theorems of Boolean algebra.
- Today we will discuss a more systematic method for logic minimization, called a *Karnaugh map*, that is based on the *clustering* of terms.

Karnaugh Maps (K-Maps)

- Provide a graphical method to find simplified Boolean Equations
- K-maps make combinable terms (like the ones we just saw) easy to see by placing them next to each other on a grid!
- If you follow the procedure properly, you will end up with the most simplified equation!
- As an example consider the 2-variable Karnaugh map for the NAND function.

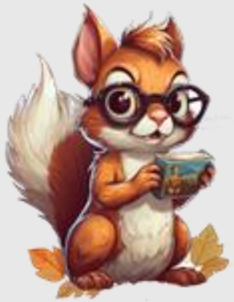
A	B	NAND
0	0	1
0	1	1
1	0	1
1	1	0

NAND truth table

A \ B	0	1
0	1	1
1	1	0

NAND Karnaugh map

Because of this arrangement it is easier to see relationships between variables!



Karnaugh Maps (K-Maps)

A Karnaugh map is an *alternate truth-table layout* that places output terms such that shared variable states are adjacent. AS an example consider the 2-variable Karnaugh map for the NAND function.

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NAND truth table

A \ B	0	1
0	1	1
1	1	0

NAND Karnaugh map

The first row of 1s is where Y = ~A and the left column is where Y = ~B, which together gives

Y = ~A + ~B

the De Morgan version of a NAND gate

Find the Equation

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

Find the Equation

	A	B	C	Y	
0	0	0	0	0	
1	0	0	1	1	$\bar{A}\bar{B}C$
2	0	1	0	0	
3	0	1	1	1	$\bar{A}BC$
4	1	0	0	0	
5	1	0	1	0	
6	1	1	0	0	
7	1	1	1	0	

$$Y = \bar{A}\bar{B}C + \bar{A}BC$$

$$Y = \bar{A}C$$

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0				
1				

Each box in the K-Map corresponds to a single row in the truth table

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	0 $_0$			
1				

Each box in the K-Map corresponds to a single row in the truth table

Row 0

A = 0, B = 0, C = 0

Y = 0

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	0 $_0$	1 $_1$		
1				

Each box in the K-Map corresponds to a single row in the truth table

Row 1

A = 0, B = 0, C = 1

Y = 1

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	0 $_{0}$	1 $_{1}$		0 $_{2}$
1				

Each box in the K-Map corresponds to a single row in the truth table

Row 2

A = 0, B = 1, C = 0

Y = 0

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	0 (0)	1 (1)	1 (3)	0 (2)
1				

Each box in the K-Map corresponds to a single row in the truth table

Row 3

A = 0, B = 1, C = 1

Y = 1

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	0 $_{0}$	1 $_{1}$	1 $_{3}$	0 $_{2}$
1	0 $_{4}$	0 $_{5}$	0 $_{7}$	0 $_{6}$

Each box in the K-Map corresponds to a single row in the truth table

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

Gray Code

A \ BC	00	01	11	10
0	0 $_0$	1 $_1$	1 $_3$	0 $_2$
1	0 $_4$	0 $_5$	0 $_7$	0 $_6$

Gray Code

- Successive values differ by only one bit

Gray Code	Binary Code
00	00
01	01
11	10
10	11

In binary code, two bits flip when we go from 1 to 2

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	0 $_0$	1 $_1$	1 $_3$	0 $_2$
1	0 $_4$	0 $_5$	0 $_7$	0 $_6$

Each square in a K-map differs from an adjacent square by a change in a single literal

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$\bar{A}\bar{B}C$

A \ BC	00	01	11	10
0	0 $_{0}$	$\boxed{1}$ $_{1}$	1 $_{3}$	0 $_{2}$
1	0 $_{4}$	0 $_{5}$	0 $_{7}$	0 $_{6}$

Let's examine the squares around box 1

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$\bar{A}\bar{B}C$

$\bar{A}\bar{B}\bar{C}$

A \ BC	00	01	11	10
0	0 $_{0}$	1 $_{1}$	1 $_{3}$	0 $_{2}$
1	0 $_{4}$	0 $_{5}$	0 $_{7}$	0 $_{6}$

Box 0 differs by the C literal

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$\bar{A}\bar{B}C$ $\bar{A}BC$

A \ BC	00	01	11	10
0	0 $_{0}$	1 $_{1}$	1 $_{3}$	0 $_{2}$
1	0 $_{4}$	0 $_{5}$	0 $_{7}$	0 $_{6}$

Box 3 differs by the B literal

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$\bar{A}\bar{B}C$

A \ BC	00	01	11	10
0	0 (0)	$\boxed{1}$ (1)	1 (3)	0 (2)
1	0 (4)	$\boxed{0}$ (5)	0 (7)	0 (6)

$A\bar{B}C$

Box 5 differs by the A literal

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$\bar{A}\bar{B}C$

A \ BC	00	01	11	10
0	0_0	$\boxed{1}_1$	1_3	0_2
1	$\boxed{0}_4$	0_5	$\boxed{0}_7$	0_6

Diagonal entries are not considered adjacent

Simplification using the K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	0_{0}	1_{1}	1_{3}	0_{2}
1	0_{4}	0_{5}	0_{7}	0_{6}

To graphically find the simplified equation, first group the terms together. Then, find the literals that the group has in common.

Simplification using the K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	0 $_{0}$	1 $_{1}$	1 $_{3}$	0 $_{2}$
1	0 $_{4}$	0 $_{5}$	0 $_{7}$	0 $_{6}$

Which literals do these entries share?

Simplification using the K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	0_{0}	1_{1}	1_{3}	0_{2}
1	0_{4}	0_{5}	0_{7}	0_{6}

We can see that the entries inside of the group each contain $\bar{A}$ and C.

Simplification using the K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	0 (0)	1 (1)	1 (3)	0 (2)
1	0 (4)	0 (5)	0 (7)	0 (6)

The literals that the group share form the simplified equation.

$$Y = \bar{A}C$$

Simplification using the K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	10	11
0	0 $_0$	1 $_1$	0 $_2$	1 $_3$
1	0 $_4$	0 $_5$	0 $_6$	0 $_7$

If we had used binary code instead of gray code, this visual simplification would not have been possible!

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

Step 1: Fill in the K-map

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	1_0	1_1	1_3	1_2
1	0_4	0_5	0_7	0_6

Step 1: Fill in the K-map

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	1_0	1_1	1_3	1_2
1	0_4	0_5	0_7	0_6

Step 2: Group the terms together

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	1 (0)	1 (1)	1 (3)	1 (2)
1	0 (4)	0 (5)	0 (7)	0 (6)

Step 2: Group the terms together

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	1_{0}	1_{1}	1_{3}	1_{2}
1	0_{4}	0_{5}	0_{7}	0_{6}

Step 3: Find the literals that the group has in common

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	1_{0}	1_{1}	1_{3}	1_{2}
1	0_{4}	0_{5}	0_{7}	0_{6}

Step 3: Find the literals that the group has in common

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	1 (0)	1 (1)	1 (3)	1 (2)
1	0 (4)	0 (5)	0 (7)	0 (6)

Step 4: Derive the equation.

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	1 (0)	1 (1)	1 (3)	1 (2)
1	0 (4)	0 (5)	0 (7)	0 (6)

Step 4: Derive the equation.

$$Y = \bar{A}$$

K-MAP GROUPS

Ex1: Reading A K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

A \ BC	00	01	11	10
0	0 $_0$	0 $_1$	0 $_3$	0 $_2$
1	0 $_4$	0 $_5$	1 $_7$	1 $_6$

Ex1: Reading A K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

A \ BC	00	01	11	10
0	0 $_0$	0 $_1$	0 $_3$	0 $_2$
1	0 $_4$	0 $_5$	1 $_7$	1 $_6$

$$Y = AB$$

Ex2: Reading A K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	0 (0)	1 (1)	0 (3)	0 (2)
1	0 (4)	0 (5)	0 (7)	0 (6)

Ex2: Reading A K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A \ BC	00	01	11	10
0	0 (0)	1 (1)	0 (3)	0 (2)
1	0 (4)	0 (5)	0 (7)	0 (6)

$$Y = \bar{A}\bar{B}C$$

Ex3: Reading a K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

A \ BC	00	01	11	10
0	0_{0}	0_{1}	0_{3}	0_{2}
1	1_{4}	1_{5}	1_{7}	0_{6}

Ex3: Reading a K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

A \ BC	00	01	11	10
0	0_0	0_1	0_3	0_2
1	1_4	1_5	1_7	0_6

$$Y = A\bar{B} + AC$$

K-Map Grouping Rules

All squares in each circle must contain 1’s

A \ B	0	1
0	0	0
1	1	1

Right

A \ B	0	1
0	0	0
1	1	1

Wrong

K-Map Grouping Rules

Groups may be horizontal or vertical, but not diagonal

A \ B	0	1
0	0	1
1	1	1

Right

A \ B	0	1
0	0	1
1	1	0

Wrong

K-Map Grouping Rules

Groups must contain 2^n cells where n = 0, 1, 2, etc.

A \ B	0	1
0	0	0
1	1	0

Right

A \ BC	00	01	11	10
0	0	0	0	0
1	1	1	1	0

Right

A \ BC	00	01	11	10
0	0	0	0	0
1	1	1	1	0

Wrong

A \ BC	00	01	11	10
0	1	1	0	0
1	1	1	1	0

Right

K-Map Grouping Rules

Each group must be as large as possible

A \ BC	00	01	11	10
0	1	1	1	1
1	0	0	1	1

Right

A \ BC	00	01	11	10
0	1	1	1	1
1	0	0	1	1

Wrong

K-Map Grouping Rules

Each cell containing a 1 must be in at least one group

A \ BC	00	01	11	10
0	0	1	0	1
1	0	0	0	1

Right

A \ BC	00	01	11	10
0	0	1	0	1
1	1	0	0	1

Wrong

K-Map Grouping Rules

Groups may wrap around the table

A \ BC	00	01	11	10
0	1	0	0	1
1	1	0	0	1

K-Map Grouping Rules

Every circle should contain at least one unique 1

A \ BC	00	01	11	10
0	1	1	1	1
1	0	0	1	1

Right

A \ BC	00	01	11	10
0	1	1	1	1
1	0	0	1	1

Wrong

Ex: Reading a K-Map

A \ BC	00	01	11	10
0	0 $_{0}$	0 $_{1}$	0 $_{3}$	1 $_{2}$
1	0 $_{4}$	1 $_{5}$	1 $_{7}$	1 $_{6}$

Ex: Reading a K-Map

A \ BC	00	01	11	10
0	0 (0)	0 (1)	0 (3)	1 (2)
1	0 (4)	1 (5)	1 (7)	1 (6)

$$Y = AC + B\bar{C}$$

Ex: Reading a K-Map

A \ BC	00	01	11	10
0	0_{0}	0_{1}	0_{3}	1_{2}
1	0_{4}	1_{5}	1_{7}	1_{6}

Wrong – the blue group does not contain a unique 1. This grouping would produce a logically equivalent equation, but it would not be the most simplified.

A \ BC	00	01	11	10
0	0_{0}	0_{1}	0_{3}	1_{2}
1	0_{4}	1_{5}	1_{7}	1_{6}

Wrong – the blue group is not a power of 2. We cannot derive a term for this group.

Use a K-Map to find the simplified SOP Equation

	A	B	C	Y
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

Use a K-Map to find the simplified SOP Equation

	A	B	C	Y
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

A \ BC	00	01	11	10
0	1 (0)	0 (1)	1 (3)	1 (2)
1	1 (4)	0 (5)	1 (7)	1 (6)

$$Y = B + \bar{C}$$

4-variable K-Map

AB \ CD	00	01	11	10
00				
01				
11				
10				

Use Gray code order so that adjacent cells only differ by one literal.

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

4-variable K-Map

AB \ CD	00	01	11	10
00	0			
01				
11				
10				

This cell is 0000, so it corresponds to row 0

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

4-variable K-Map

AB \ CD	00	01	11	10
00	0	1		
01				
11				
10				

This cell is 0001, so it corresponds to row 1

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

4-variable K-Map

AB \ CD	00	01	11	10
00	0	1		
01				6
11				
10				

This cell is 0110, so it corresponds to row 6

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

4-variable K-Map

AB \ CD	00	01	11	10
00	0	1		
01				6
11			$\boxed{15}$	
10				

This cell is 1111, so it corresponds to row 15

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

4-variable K-Map

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Use Gray code order so that adjacent cells only differ by one literal.

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

Ex: 4-variable K-Map

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

Ex: 4-variable K-Map

AB \ CD	00	01	11	10
00	0_{0}	0_{1}	1_{3}	1_{2}
01	1_{4}	1_{5}	0_{7}	0_{6}
11	1_{12}	1_{13}	0_{15}	0_{14}
10	0_{8}	0_{9}	0_{11}	1_{10}

	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

Ex: 4-variable K-Map

AB \ CD	00	01	11	10
00	0 (0)	0 (1)	1 (3)	1 (2)
01	1 (4)	1 (5)	0 (7)	0 (6)
11	1 (12)	1 (13)	0 (15)	0 (14)
10	0 (8)	0 (9)	0 (11)	1 (10)

$$Y = B\bar{C} + \bar{A}\bar{B}C + \bar{B}C\bar{D}$$

	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

Q2.1: Reading a K-Map

AB \ CD	00	01	11	10
00	1	0	0	0
01	1	1	1	0
11	1	1	1	1
10	1	0	0	0

WS Q2.1: Reading a K-Map

AB \ CD	00	01	11	10
00	1	0	0	0
01	1	1	1	0
11	1	1	1	1
10	1	0	0	0

$$Y = \bar{C}\bar{D} + BD + AB$$

WS 2.2: Reading a K-Map

AB \ CD	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	1	1

Q2.2: Reading a K-Map

AB \ CD	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	1	1

$$Y = B\bar{C} + \bar{B}C + \bar{A}C$$

AB \ CD	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	1	1

$$Y = B\bar{C} + \bar{B}C + \bar{A}B$$

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

CD

AB

	00	01	11	10
00				
01				
11				
10				

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

AB \ CD	00	01	11	10
00	1			
01	1			
11	1			
10	1			

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + {\color{red}ABC\bar{D}} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

AB \ CD	00	01	11	10
00	1			
01	1			
11	1			1
10	1			

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

AB \ CD	00	01	11	10
00	1			
01	1			
11	1		1	1
10	1		1	

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

AB \ CD	00	01	11	10
00	1			1
01	1			1
11	1		1	1
10	1		1	

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

AB \ CD	00	01	11	10
00	1			1
01	1			1
11	1		1	1
10	1		1	1

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	1	1
10	1	0	1	1

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	1	1
10	1	0	1	1

$$Y = AC + \bar{D}$$

Recall: Boolean Algebra simplification

$$ABC + AB\bar{C} + \overline{AC}B + ACB + \overline{\overline{AB}\,\bar{C}\,\bar{D}} + \overline{\bar{A}B + \bar{C}}$$

$$AB(C + \bar{C}) + B(\overline{AC} + AC) + AB + C + D + (\overline{\bar{A}B})C$$

$$AB(1) + B(1) + AB + C + D + (A + \bar{B})C$$

$$AB + B + AB + C + D + AC + \bar{B}C$$

$$AB + B + C + D + AC + \bar{B}C$$

$$B(A + 1) + C(1 + A) + \bar{B}C + D$$

$$B(1) + C(1) + \bar{B}C + D$$

$$B + C + \bar{B}C + D$$

$$B + C(1 + \bar{B}) + D$$

$$B + C(1) + D$$

$$B + C + D$$

Let’s use a K-Map Instead

1. Put the equation in SOP form.
2. Fill out the K-Map.
3. Find the simplified equation.

Reminder: Sum of Products / Product of Sums

Sum of Products (SOP)

- When two or more products (AND) are summed (OR) together

$$Y = AB + A\bar{C}$$

Product of Sums (POS)

- When two or more sums (OR) are composed (AND) together

$$Y = (A + B)(C + A)$$

Using a K-Map to Simplify a Boolean Equation

1. Put the equation in SOP form
2. Fill out the K-Map
3. Find the simplified equation

$$ABC + AB\bar{C} + \overline{AC}B + ACB + \overline{\overline{AB}\,\overline{C}\,\overline{D}} + \overline{\bar{A}B + \bar{C}}$$

AB \ CD	00	01	11	10
00				
01				
11				
10				

Using a K-Map to Simplify a Boolean Equation

$$ABC + AB\bar{C} + \overline{AC}B + ACB + \overline{\overline{AB}\,\bar{C}\,\bar{D}} + \overline{\bar{A}B + \bar{C}}$$

1. Put the equation in SOP form

$$ABC + AB\bar{C} + \bar{A}B + \bar{A}C + ACB + AB + C + D + AC + \bar{B}C$$

Using a K-Map to Simplify a Boolean Equation

1. Put the equation in SOP form
2. Fill out the K-Map

AB \ CD	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	1	1	1	1
10	0	1	1	1

$$ABC + AB\bar{C} + \bar{A}B + \bar{A}C + ACB + AB + C + D + AC + \bar{B}C$$

Using a K-Map to Simplify a Boolean Equation

1. Put the equation in SOP form
2. Fill out the K-Map
3. Find the simplified equation

AB \ CD	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	1	1	1	1
10	0	1	1	1

$$\boldsymbol{B + C + D}$$