Definitions

Definitions

- Literal
  - A single variable. May be complemented.
  - Ex1: A
  - Ex2: $\bar{A}$
- Product Term
  - AND of literals
  - Ex: $A\bar{B}C$
  - Not a product term: $A\overline{BC}$

Minterm

Product term in which all variables appear once

Minterm	Not a Minterm
$\bar{A}\bar{B}\bar{C}$	A
$\bar{A}BC$	$\bar{A}C$
$\bar{A}B\bar{C}$	BC
ABC	

Example assumes that our only variables are A, B, and C

Minterm

A	B	C	Minterm	Minterm name
0	0	0	$\bar{A}\bar{B}\bar{C}$	m_0
0	0	1	$\bar{A}\bar{B}C$	m_1
0	1	0	$\bar{A}B\bar{C}$	m_2
0	1	1	$\bar{A}BC$	m_3
1	0	0	$A\bar{B}\bar{C}$	m_4
1	0	1	$A\bar{B}C$	m_5
1	1	0	$AB\bar{C}$	m_6
1	1	1	ABC	m_7

Deriving Equation Using Minterms

- Take the sum of the minterms of the rows whose output is 1

A	B	C	F	Minterm	Minterm Name
0	0	0	1	$\bar{A}\bar{B}\bar{C}$	m_0
0	0	1	0	$\bar{A}\bar{B}C$	m_1
0	1	0	1	$\bar{A}B\bar{C}$	m_2
0	1	1	0	$\bar{A}BC$	m_3
1	0	0	0	$A\bar{B}\bar{C}$	m_4
1	0	1	1	$A\bar{B}C$	m_5
1	1	0	0	$AB\bar{C}$	m_6
1	1	1	1	ABC	m_7

Our equation, F, can be represented in the following ways:

$$F = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}C + ABC$$

$$F(A, B, C) = \sum(m_0, m_2, m_5, m_7)$$

Sum of Products

- When two or more products (AND) are summed (OR) together

Sum of Products

$$Y = AB + A\bar{C}$$

Not Sum of Products

$$Y = (A + B)(C + A)$$

Canonical Sum of Products

- A sum of products form in which each product contains all literals

Canonical Sum of Products Form $F = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \mathrm{A}\bar{B}C + ABC$

Simplified Sum of Products Form $F = \bar{A}\bar{C} + \mathrm{AC}$

When simplifying Boolean expressions, we usually want to find the simplified sum of products form!

Maxterm

A	B	C	Maxterm	Maxterm Name
0	0	0	$A + B + C$	M_0
0	0	1	$A + B + \bar{C}$	M_1
0	1	0	$A + \bar{B} + C$	M_2
0	1	1	$A + \bar{B} + \bar{C}$	M_3
1	0	0	$\bar{A} + B + C$	M_4
1	0	1	$\bar{A} + B + \bar{C}$	M_5
1	1	0	$\bar{A} + \bar{B} + C$	M_6
1	1	1	$\bar{A} + \bar{B} + \bar{C}$	M_7

Deriving Equation Using Maxterms

- Take the product of the maxterms of the rows whose output is 0

A	B	C	F	Maxterm	Maxterm Name
0	0	0	1	$A + B + C$	M_0
0	0	1	0	$A + B + \bar{C}$	M_1
0	1	0	1	$A + \bar{B} + C$	M_2
0	1	1	0	$A + \bar{B} + \bar{C}$	M_3
1	0	0	0	$\bar{A} + B + C$	M_4
1	0	1	1	$\bar{A} + B + \bar{C}$	M_5
1	1	0	0	$\bar{A} + \bar{B} + C$	M_6
1	1	1	1	$\bar{A} + \bar{B} + \bar{C}$	M_7

Our equation, F, can be represented in the following ways:

$$F = (A + B + \bar{C})(A + \bar{B} + \bar{C})(\bar{A} + B + C)(\bar{A} + \bar{B} + C)$$

$$F = \Pi(M_1, M_3, M_4, M_6)$$

Why does this work?

A	B	C	F	$\overline{F}$
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

Let's create the canonical SOP equation for $\overline{F}$

$$\bar{F} = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C}$$

If we negate both sides, we get

$$F = \overline{\bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C}}$$

Applying DeMorgan's, we get the same equation

$$F = (A + B + \bar{C})(A + \bar{B} + \bar{C})(\bar{A} + B + C)(\bar{A} + \bar{B} + C)$$

Find the Boolean equation of this truth table in each of the following forms.

X	Y	Z	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

1. Canonical SOP form
2. Simplified SOP form
3. Canonical POS form

Find the Boolean equation of this truth table in each of the following forms.

X	Y	Z	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

1. Canonical SOP form

$$F = \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + XY\bar{Z} + XYZ$$

2. Simplified SOP form

$$F = \bar{X}\bar{Y} + XY$$

3. Canonical POS form

$$F = (X + \bar{Y} + Z)(X + \bar{Y} + \bar{Z})(\bar{X} + Y + Z)(\bar{X} + Y + \bar{Z})$$

K-Map Definitions

Definition: Implicant

Any product term that is a 1 for a given Boolean Equation (i.e. any group of 1s)

BC / A

A \ BC	00	01	11	10
0	0	0	1	0
1	0	1	1	0

Implicants

$\bar{A}BC$

$A\bar{B}C$

ABC

BC

AC

Definition: Prime Implicant

An implicant that is not a subset of any other implicant

Prime Implicant

An implicant that is not a subset of any other implicant

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this a prime implicant?

Prime Implicant

An implicant that is not a subset of any other implicant

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this a prime implicant?

Yes!

Prime Implicant

An implicant that is not a subset of any other implicant

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this a prime implicant?

Prime Implicant

An implicant that is not a subset of any other implicant

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this a prime implicant?

No, it can be covered by another group

Prime Implicant

An implicant that is not a subset of any other implicant

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this a prime implicant?

Prime Implicant

An implicant that is not a subset of any other implicant

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this a prime implicant?

No, it can be covered by another group

Definition: Essential Prime Implicant

A prime implicant with at least one element that is not covered by one or more prime implicants (i.e. we must use this group in our final solution in order to cover all 1s)

Essential Prime Implicant

A prime implicant with at least one element that is not covered by one or more prime implicants (i.e. we must use this group in our final solution in order to cover all 1s)

CD / AB

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this an essential prime implicant?

Essential Prime Implicant

A prime implicant with at least one element that is not covered by one or more prime implicants

CD / AB

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this an essential prime implicant?

Yes!

Essential Prime Implicant

A prime implicant with at least one element that is not covered by one or more prime implicants

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this an essential prime implicant?

Essential Prime Implicant

A prime implicant with at least one element that is not covered by one or more prime implicants

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Q: Is this an essential prime implicant?

No!

Definition: Non-Essential Prime Implicant

Prime implicant that has no element that cannot be covered by other prime implicant

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	1	1	1	0
10	1	1	0	0

Formal K-Map Procedure

1. Convert truth table to K-map
2. Include all essential primes
3. Include non-essential primes as needed to completely cover all ones

Revisiting ~~Q3~~ from Class

This was actually Q2.2 from class/the worksheet!!!
Starting from slide 72 on kmap day!

Let's apply this formal procedure to a problem we've seen in class.
We can skip step 1 since our k-map is already filled out.

AB \ CD	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	1	1

Revisiting ~~Q3~~ from Class

Q2.2

Next, include all essential prime implicants

CD

AB

AB \ CD	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	1	1

Revisiting ~~Q3~~ 2.2 from Class

Finally, to cover the remaining ones, we will choose one of these non-essential prime implicants

AB \ CD	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	1	1

AB \ CD	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	1	1

Both of these solutions give us the most simplified equation.