

COMP311: *COMPUTER ORGANIZATION!*

Lecture 8: Karnaugh Maps

tinyurl.com/comp311-sp26

Logistics Updates

- WA2 due today!
- Support email!
 - comp311-sp26-cs@cs.unc.edu
 - *Email for logistics issues!*



KARNAUGH MAPS



Minimizing Logic

- Previously, we have discussed how to systematically convert logic specified as a truth table into sum-of-products (SOP).
- After that, we relied on cleverness, and luck to reduce these expressions via the properties and theorems of Boolean algebra.
- Today we will discuss a more systematic method for logic minimization, called a *Karnaugh map*, that is based on the *clustering* of terms.

Karnaugh Maps (K-Maps)

- Provide a graphical method to find simplified Boolean Equations
- K-maps make combinable terms (like the ones we just saw) easy to see by placing them next to each other on a grid!
- If you follow the procedure properly, you will end up with the most simplified equation!
- As an example consider the 2-variable Karnaugh map for the NAND function.

A	B	NAND
0	0	1
0	1	1
1	0	1
1	1	0

NAND truth table

B	0	1
A	0	1
1	1	0

NAND Karnaugh map

Because of this arrangement it is easier to see relationships between variables!



Karnaugh Maps (K-Maps)

A Karnaugh map is an *alternate truth-table layout* that places output terms such that shared variable states are adjacent. As an example consider the 2-variable Karnaugh map for the NAND function.

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NAND truth
table

A \ B	0	1
0	1	1
1	1	0

NAND Karnaugh
map

The first row of 1s is where

$$Y = \overline{A}$$

and the left column is
where

$Y = \overline{B}$, which together gives

$$Y = \overline{A} + \overline{B}$$

the De Morgan version of a
NAND gate



SOME K-MAP DEFINITIONS



K-Map Definitions

- Literal: A single variable, may be complemented
 - $A, \bar{A}$
- Product Term: AND/conjunction of literals
 - $\bar{A}\bar{B}C$
 - Not a product term: $\overline{ACB}$
- Minterm: Product term in which all variables appear once.
 - EX: if there are 3 input variables (A, B, C)
 - Minterms: $\bar{A}\bar{B}C, ABC$
 - Not minterms: $A, \bar{A}C$
- Maxterm: Sum term in which all variables appear once
 - Mixterm: $A + \bar{B} + C$
 - Not maxterm: $\bar{C}, A + \bar{C}$

$A, B, \bar{C}, C$

Minterms



A	B	C	Minterm	Minterm name
0	0	0	$\bar{A}\bar{B}\bar{C}$	m_0
0	0	1	$\bar{A}\bar{B}C$	m_1
0	1	0	$\bar{A}B\bar{C}$	m_2
0	1	1	$\bar{A}BC$	m_3
1	0	0	$A\bar{B}\bar{C}$	m_4
1	0	1	$A\bar{B}C$	m_5
1	1	0	$AB\bar{C}$	m_6
1	1	1	ABC	m_7

Deriving Equations Using Minterms

- Take the sum of the minterms of the rows whose output is 1

A	B	C	F	Minterm	Minterm Name
0	0	0	1	$\bar{A}\bar{B}\bar{C}$	m_0
0	0	1	0	$\bar{A}\bar{B}C$	m_1
0	1	0	1	$\bar{A}B\bar{C}$	m_2
0	1	1	0	$\bar{A}BC$	m_3
1	0	0	0	$A\bar{B}\bar{C}$	m_4
1	0	1	1	$A\bar{B}C$	m_5
1	1	0	0	$AB\bar{C}$	m_6
1	1	1	1	ABC	m_7

$$F = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}C + ABC$$

$$F(A, B, C) = \sum(m_0, m_2, m_5, m_7)$$

Recall: Sum of Products Form

When two or more products (AND) are summed (OR) together

Sum of Products

$$Y = AB + AC$$



Not Sum of Products

$$Y = (A + B)(C + A)$$

Recall: Sum of Products Form

When two or more products (AND) are summed (OR) together

Sum of Products

$$Y = AB + AC$$

Not Sum of Products

$$Y = (A + B)(C + A)$$

Canonical Sum of Products: A sum of products form in which each product contains all literals

Canonical Sum of Products Form

$$F = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}C + ABC$$

Simplified Sum of Products Form

$$F = \bar{A}\bar{C} + AC$$

Maxterms

A	B	C	Maxterm	Maxterm Name
0	0	0	$A + B + C$	M_0
0	0	1	$A + B + \bar{C}$	M_1
0	1	0	$A + \bar{B} + C$	M_2
0	1	1	$A + \bar{B} + \bar{C}$	M_3
1	0	0	$\bar{A} + B + C$	M_4
1	0	1	$\bar{A} + B + \bar{C}$	M_5
1	1	0	$\bar{A} + \bar{B} + C$	M_6
1	1	1	$\bar{A} + \bar{B} + \bar{C}$	M_7

Maxterms

- Take the product of the maxterms of the rows whose output is 0

A	B	C	F	Maxterm	Maxterm Name
0	0	0	1	$A + B + C$	M_0
0	0	1	0	$A + B + \bar{C}$	M_1
0	1	0	1	$A + \bar{B} + C$	M_2
0	1	1	0	$A + \bar{B} + \bar{C}$	M_3
1	0	0	0	$\bar{A} + B + C$	M_4
1	0	1	1	$\bar{A} + B + \bar{C}$	M_5
1	1	0	0	$\bar{A} + \bar{B} + C$	M_6
1	1	1	1	$\bar{A} + \bar{B} + \bar{C}$	M_7

$$F = (A + B + \bar{C})(A + \bar{B} + \bar{C})(\bar{A} + B + C)(\bar{A} + \bar{B} + C)$$

$$F = \Pi(M_1, M_3, M_4, M_6)$$

Why does this work?

A	B	C	F	$\bar{F}$
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

Let's create the canonical SOP equation for $\bar{F}$

$$\bar{F} = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C}$$

If we negate both sides, we get

$$F = \overline{\bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C}}$$

Applying DeMorgan's, we get the same equation

$$F = (A + B + \bar{C})(A + \bar{B} + \bar{C})(\bar{A} + B + C)(\bar{A} + \bar{B} + C)$$



K-MAPS FROM TRUTH TABLES



Find the Equation

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$$\bar{A}\bar{B}C$$

$$ABC$$

Find the Equation

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$$\bar{A}\bar{B}C$$

$$\bar{A}BC$$

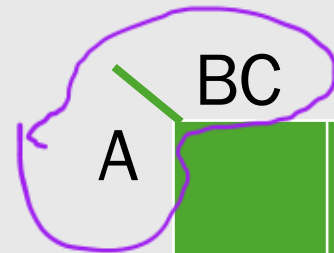
canonical SOP

$$\rightarrow Y = \bar{A}\bar{B}C + \bar{A}BC$$

$$Y = \bar{A}C$$

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0



BC	00	01	11	10
0				
1				

Each box in the K-Map corresponds to a single row in the truth table

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC					
A		00	01	11	10
	0	0			
	1				

Each box in the K-Map corresponds to a single row in the truth table

Row 0

$A = 0, B = 0, C = 0$

$Y = 0$

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A	BC			
	00	01	11	10
	0	0	1	
1				

Each box in the K-Map corresponds to a single row in the truth table

Row 1

$A = 0, B = 0, C = 1$

$Y = 1$

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

		BC				
			00	01	11	<u>10</u>
A	0	0	0	1		0
	1	1				

Each box in the K-Map corresponds to a single row in the truth table

Row 2

$A = 0, B = 1, C = 0$

$Y = 0$

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC

A

	00	01	11	10
0	0 ₀	1 ₁	1 ₃	0 ₂
1				

Each box in the K-Map corresponds to a single row in the truth table

Row 3
 $A = 0, B = 1, C = 1$
 $Y = 1$

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC		00	01	11	10
A					
0		0 ₀	1 ₁	1 ₃	0 ₂
1		0 ₄	0 ₅	0 ₇	0 ₆

Each box in the K-Map corresponds to a single row in the truth table

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

Gray Code

BC

A

	00	01	11	10
0	0 ₀	1 ₁	1 ₃	0 ₂
1	0 ₄	0 ₅	0 ₇	0 ₆

Gray Code

- Successive values differ by only one bit

→ Gray Code

00

01

11

10

Binary Code

00

01

10

11



In binary code, two bits flip when we go from 1 to 2

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC					
A		00	01	11	10
	0	0 ₀	1 ₁	1 ₃	0 ₂
	1	0 ₄	0 ₅	0 ₇	0 ₆

Each square in a K-map differs from an adjacent square by a change in a single literal

3-input Function

	A	B	C	Y
0	0	0	0	0
→ 1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

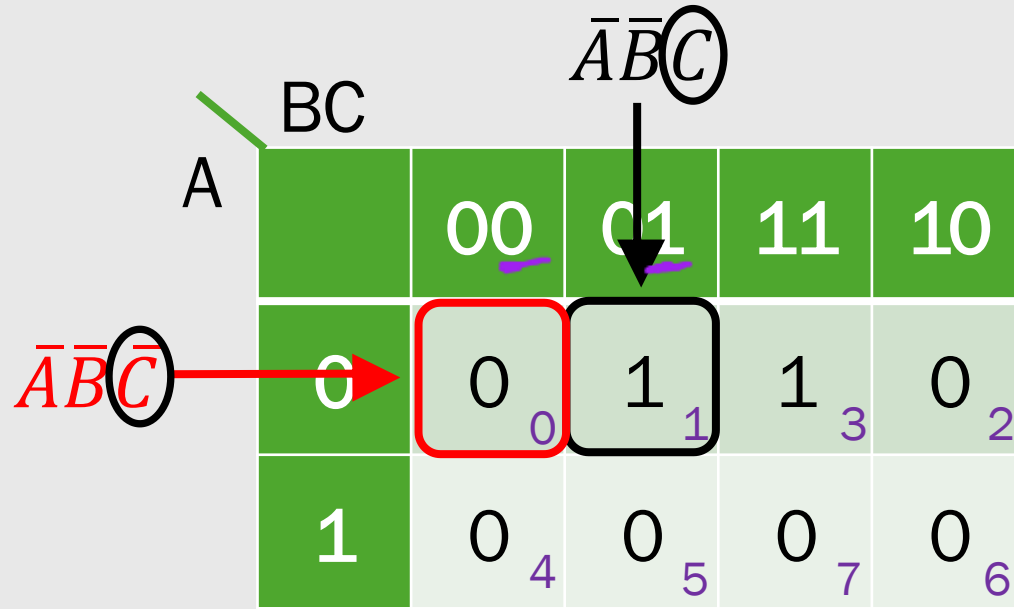
		$\bar{A}\bar{B}C$			
A	BC				
		00	01	11	10
0		0	1	1	0
1		0	0	0	0

Diagram illustrating a 3-input function table (Karnaugh map) for variables A, B, and C. The table shows the output Y for all combinations of inputs A, B, and C. The input combinations are labeled by the row (A) and column (BC). The output Y is shown in the rightmost column. The input combination 01 (A=0, B=0, C=1) is highlighted with a black box, and the output Y for this combination is 1. The input combination 01 is also labeled with the expression $\bar{A}\bar{B}C$ above it, with an arrow pointing to the box. The input combinations are also labeled with their decimal equivalents: 0, 1, 2, 3, 4, 5, 6, 7.

Let's examine the squares around box 1

3-input Function

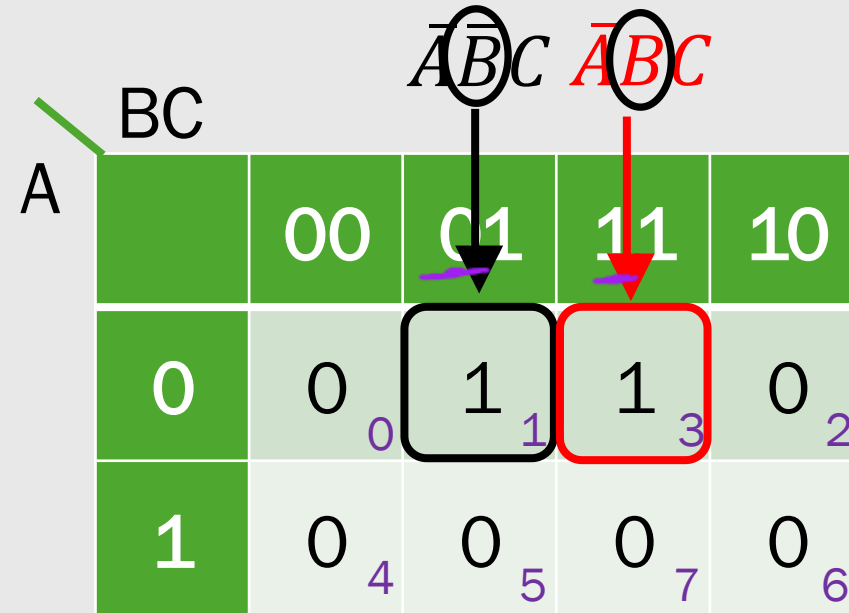
	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0



Box 0 differs by the C literal

3-input Function

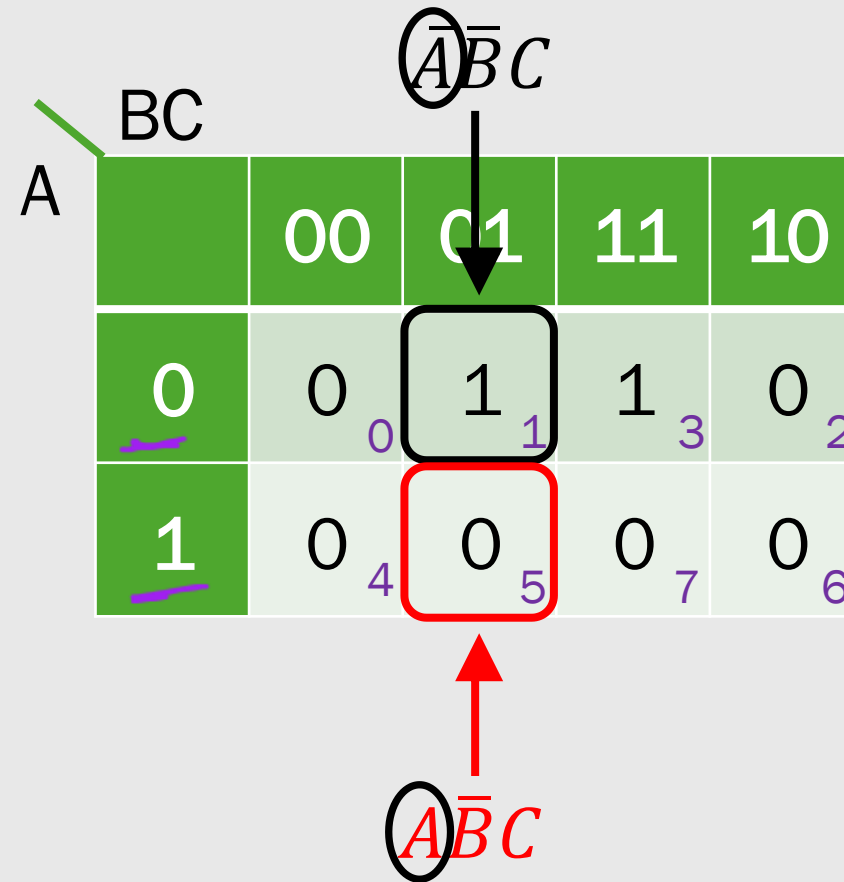
	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0



Box 3 differs by the B literal

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0



Box 5 differs by the A literal

3-input Function

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$\bar{A}\bar{B}C$

BC	00	01	11	10
A				
0	0	1	1	0
1	0	0	0	0

Diagram illustrating a 3-input function using a Karnaugh map. The map shows the output Y for inputs A, B, and C. The input combinations are labeled as BC (00, 01, 11, 10) and A (0, 1). The output Y is shown in the cells. The map highlights the diagonal entries (0, 1), (1, 0), (1, 1), and (0, 0) as not considered adjacent.

Diagonal entries are not considered adjacent

Simplification using the K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC		00	01	11	10
A	0	0	1 ₁	1 ₃	0 ₂
1	0 ₄	0 ₅	0 ₇	0 ₆	

To graphically find the simplified equation, first group the terms together. Then, find the literals that the group has in common.

Simplification using the K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC		00	01	11	10
A	0	0	1 ₁	1 ₃	0 ₂
1	0 ₄	0 ₅	0 ₇	0 ₆	

Which literals do these entries share?

Simplification using the K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC

A

		00	01	11	10
0	0	0	1	1	0
1	0	0	0	0	0

Diagram illustrating the K-Map for the function Y. The K-Map is a 2x5 grid with columns labeled BC (00, 01, 11, 10) and rows labeled A (0, 1). The entries are: Row 0: 0, 0, 1, 1, 0; Row 1: 0, 0, 0, 0, 0. A group of four cells (0, 01), (0, 11), (1, 01), (1, 11) is highlighted with a red rectangle, indicating a prime implicant. The cell (0, 00) is circled in black. A purple arrow points from the truth table to the K-Map.

We can see that the entries inside of the group each contain $\bar{A}$ and C .

$$Y = \bar{A}C$$

Simplification using the K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC		00	01	11	10
A	0	0	1	1	0
1	1	0	0	0	0

Diagram illustrating the K-Map simplification process. The K-Map is a 2x4 grid with rows labeled A (0, 1) and columns labeled BC (00, 01, 11, 10). The values in the cells are: Row 0: 0, 1, 1, 0; Row 1: 1, 0, 0, 0. The cells (0, 01) and (0, 11) are circled in black. The cells (1, 00) and (1, 01) are circled in black. The cells (0, 01) and (0, 11) are grouped together by a red rectangle. The cells (1, 00) and (1, 01) are grouped together by a red rectangle. The simplified equation is $Y = \bar{A}C$.

The literals that the group share form the simplified equation.

$$Y = \bar{A}C$$

Simplification using the K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

		BC			
A		00	01	10	11
0	0 ₀	1 ₁	0 ₂	1 ₃	
1	0 ₄	0 ₅	0 ₆	0 ₇	

If we had used binary code instead of gray code, this visual simplification would not have been possible!

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

		BC			
A		00	01	11	10
0		0	1	3	2
1		4	5	7	6

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC		00	01	11	10
A	0	1 ₀	1 ₁	1 ₃	1 ₂
1		0 ₄	0 ₅	0 ₇	0 ₆

Step 1: Fill in the K-map

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC		00	01	11	10
A					
0		1 ₀	1 ₁	1 ₃	1 ₂
1		0 ₄	0 ₅	0 ₇	0 ₆

Step 1: Fill in the K-map

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

		BC			
		00	01	11	10
A	0	1 ₀	1 ₁	1 ₃	1 ₂
	1	0 ₄	0 ₅	0 ₇	0 ₆

Step 2: Group the terms together

bigger group $\Rightarrow$
smaller term

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

		BC			
		00	01	11	10
A	0	1 ₀	1 ₁	1 ₃	1 ₂
	1	0 ₄	0 ₅	0 ₇	0 ₆

Step 2: Group the terms together

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC		00	01	11	10
A					
0		1 ₀	1 ₁	1 ₃	1 ₂
1		0 ₄	0 ₅	0 ₇	0 ₆

Step 3: Find the literals that the group has in common

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC					
A		00	01	11	10
0	0	1 ₀	1 ₁	1 ₃	1 ₂
1	1	0 ₄	0 ₅	0 ₇	0 ₆

Step 3: Find the literals that the group has in common

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC					
A		00	01	11	10
0	1	1 ₀	1 ₁	1 ₃	1 ₂
1	0	0 ₄	0 ₅	0 ₇	0 ₆

Step 4: Derive the equation.

$$Y = \overline{A}$$

Example 2

	A	B	C	Y
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC					
A		00	01	11	10
0	0	1 ₀	1 ₁	1 ₃	1 ₂
1	1	0 ₄	0 ₅	0 ₇	0 ₆

Step 4: Derive the equation.

$$Y = \bar{A}$$



K-MAP GROUPING RULES



K-Map Grouping Rules

All squares in each circle must contain 1's

	B		
A		0	1
	0	0	0
	1	1	1

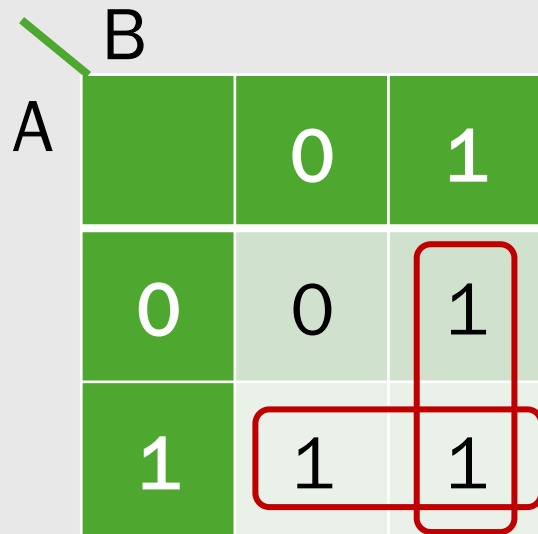
Right

	B		
A		0	1
	0	0	0
	1	1	1

Wrong

K-Map Grouping Rules

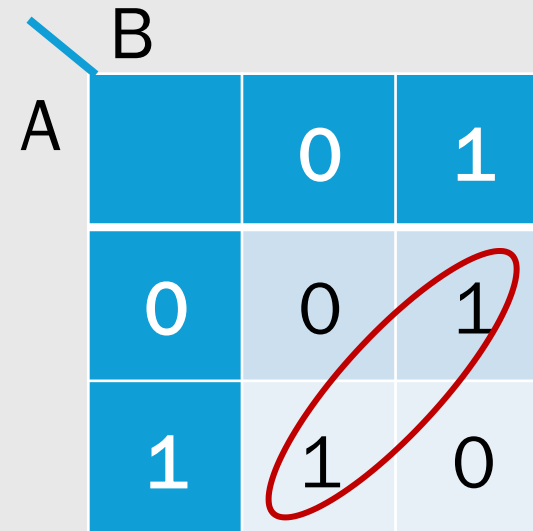
Groups may be horizontal or vertical, but not diagonal



A 3x3 Karnaugh map for variables A and B. The map is divided into two regions: a green region (top-left and bottom-left) and a light green region (top-right and bottom-right). The green region contains the values 0, 1, 0, 1. The light green region contains the values 0, 0, 1, 1. Two red rectangles highlight valid groupings: one horizontal group of two 1s in the bottom row, and one vertical group of two 1s in the rightmost column.

	B		
A		0	1
	0	0	1
	1	1	1

Right



A 3x3 Karnaugh map for variables A and B. The map is divided into two regions: a blue region (top-left and bottom-left) and a light blue region (top-right and bottom-right). The blue region contains the values 0, 1, 0, 1. The light blue region contains the values 0, 0, 1, 0. A red oval highlights an invalid diagonal grouping of two 1s across the boundary between the blue and light blue regions.

	B		
A		0	1
	0	0	1
	1	1	0

Wrong

K-Map Grouping Rules

Groups must contain 2^n cells where $n = 0, 1, 2$, etc.

2^0

	B		
A		0	1
	0	0	0
	1	1	0

Right

	BC				
A		00	01	11	10
0	0	0	0	0	0
1	1	1	1	1	0

Wrong

	BC				
A		00	01	11	10
0	0	0	0	0	0
1	1	1	1	1	0

Right

2^2

	BC				
A		00	01	11	10
0	1	1	0	0	
1	1	1	1	0	

Right

K-Map Grouping Rules

Each group must be as large as possible

BC

A

	00	01	11	10
0	1	1	1	1
1	0	0	1	1

Right

BC

A

	00	01	11	10
0	1	1	1	1
1	0	0	1	1

Wrong

K-Map Grouping Rules

Each cell containing a 1 must be in at least one group

BC

A

	00	01	11	10
0	0	1	0	1
1	0	0	0	1

Right

BC

A

	00	01	11	10
0	0	1	0	1
1	1	0	0	1

Wrong

K-Map Grouping Rules

Groups may wrap around the table

A Karnaugh Map (K-Map) for a 2-variable function with variables A and BC. The map is a 2x5 grid. The columns are labeled with BC values: 00, 01, 11, 10. The rows are labeled with A values: 0, 1. The cells contain the function values (0 or 1). Two groups of 1s are circled in red, illustrating the wrap-around rule. The first group consists of the cells (A=0, BC=00), (A=0, BC=10), (A=1, BC=00), and (A=1, BC=10). The second group consists of the cells (A=0, BC=11), (A=0, BC=01), (A=1, BC=11), and (A=1, BC=01).

A \ BC	BC				
		00	01	11	10
0	1	1	0	0	1
1	1	1	0	0	1

K-Map Grouping Rules

Every circle should contain at least one unique 1



A \ BC				
	00	01	11	10
0	1	1	1	1
1	0	0	1	1

Right



A \ BC				
	00	01	11	10
0	1	1	1	1
1	0	0	1	1

Wrong

Ex: Reading a K-Map

BC

A

	00	01	11	10
0	0 ₀	0 ₁	0 ₃	1 ₂
1	0 ₄	1 ₅	1 ₇	1 ₆

Ex: Reading a K-Map

BC

A

	00	01	11	10
0	0 ₀	0 ₁	0 ₃	1 ₂
1	0 ₄	1 ₅	1 ₇	1 ₆

$$Y = AC + B\bar{C}$$

① ②

Ex: Reading a K-Map

BC

A

	00	01	11	10
0	0 ₀	0 ₁	0 ₃	1 ₂
1	0 ₄	1 ₅	1 ₇	1 ₆

Wrong – the blue group does not contain a unique 1. This grouping would produce a logically equivalent equation, but it would not be the most simplified.

BC

A

	00	01	11	10
0	0 ₀	0 ₁	0 ₃	1 ₂
1	0 ₄	1 ₅	1 ₇	1 ₆

Wrong – the blue group is not a power of 2. We cannot derive a term for this group.



K-MAP GROUPS



Ex1: Reading A K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

BC

A

	00	01	11	10
0	0	0	0	0
1	0	0	1	1

Handwritten annotations: Red circles around the '11' and '10' cells in the first row of the K-map. A red box around the '1' and '1' cells in the third row. A red arrow pointing from the first row to the third row.

$$y = AB$$

Ex1: Reading A K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

BC					
A		00	01	11	10
	0	0 ₀	0 ₁	0 ₃	0 ₂
	1	0 ₄	0 ₅	1 ₇	1 ₆

$$Y = AB$$

Ex2: Reading A K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

BC		00	01	11	10
A	0	0 ₀	1 ₁	0 ₃	0 ₂
1	0 ₄	0 ₅	0 ₇	0 ₆	

$$Y = \overline{A} \overline{B} C$$

Ex2: Reading A K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

A	BC				
		00	01	11	10
0	0	0 ₀	1 ₁	0 ₃	0 ₂
1	1	0 ₄	0 ₅	0 ₇	0 ₆

$$Y = \bar{A}\bar{B}C$$

Ex3: Reading a K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

BC

A

	00	01	11	10
0				
1	0	0	0	0
2				
3	1	1	1	0

$$Y = A\bar{B} + AC$$

Ex3: Reading a K-Map

	A	B	C	Y
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

BC		00	01	11	10
A	0	0 ₀	0 ₁	0 ₃	0 ₂
1	1 ₄	1 ₅	1 ₇	0 ₆	

$$Y = A\bar{B} + AC$$

Use a K-Map to find the simplified SOP Equation

	A	B	C	Y
0	0	0	0	<u>1</u>
1	0	0	1	0
2	0	1	0	<u>1</u>
3	0	1	1	<u>1</u>
4	1	0	0	<u>1</u>
5	1	0	1	0
6	1	1	0	<u>1</u>
7	1	1	1	1

		BC			
A		00	01	11	10
	0	1 ₀	0 ₁	1 ₃	1 ₂
	1	1 ₄	0 ₅	1 ₇	1 ₆

Handwritten annotations on the K-Map:
- Red circles around the 1s in the first row (00, 11, 10) and the first column (0, 1).
- A red line connecting the 1s in the first row and first column.
- A purple circle around the 1 in the first row, first column.
- A purple circle around the 1 in the first row, last column.
- A purple circle around the 1 in the first column, last row.
- A purple circle around the 1 in the last row, last column.
- A purple circle around the 1 in the last row, first column.

$$y = B + \overline{C}$$

Use a K-Map to find the simplified SOP Equation

	A	B	C	Y
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

BC					
A		00	01	11	10
	0	1 ₀	0 ₁	1 ₃	1 ₂
	1	1 ₄	0 ₅	1 ₇	1 ₆

$$Y = B + \bar{C}$$

4-variable K-Map

		CD			
AB		00	01	11	10
	00				
	01				
	11				
	10				

Use Gray code order so that adjacent cells only differ by one literal.

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

4-variable K-Map

	CD				
AB		00	01	11	10
00					
01					
11					
10					

This cell is 0000, so it corresponds to row 0



	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

4-variable K-Map

		CD			
AB		00	01	11	10
	00				
	01				
	11				
	10				

This cell is 0001, so it corresponds to row 1



	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

4-variable K-Map

		CD			
AB		00	01	11	10
	00				
	01				
	11				
	10				

This cell is 0110, so it corresponds to row 6

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

4-variable K-Map

		CD			
AB		00	01	11	10
	00				
	01				
	11				
	10				

This cell is 1111, so it corresponds to row 15

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

4-variable K-Map

		CD			
AB		00	01	11	10
	00				
	01				
	11				
	10				
		0	1	3	2
		4	5	7	6
		12	13	15	14
		8	9	11	10

Use Gray code order so that adjacent cells only differ by one literal.

	A	B	C	D	Y
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

Ex: 4-variable K-Map

		CD			
AB		00	01	11	10
	00				
	01				
	11				
	10				

	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

Ex: 4-variable K-Map

		CD			
AB		00	01	11	10
	00	0 ₀	0 ₁	1 ₃	1 ₂
	01	1 ₄	1 ₅	0 ₇	0 ₆
	11	1 ₁₂	1 ₁₃	0 ₁₅	0 ₁₄
	10	0 ₈	0 ₉	0 ₁₁	1 ₁₀

	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

Ex: 4-variable K-Map

	CD				
AB		00	01	11	10
	00	0 ₀	0 ₁	1 ₃	1 ₂
	01	1 ₄	1 ₅	0 ₇	0 ₆
	11	1 ₁₂	1 ₁₃	0 ₁₅	0 ₁₄
	10	0 ₈	0 ₉	0 ₁₁	1 ₁₀

$$Y = \underline{BC} + \underline{\bar{A}\bar{B}C} + \underline{\bar{B}C\bar{D}}$$

	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

Q2.1: Reading a K-Map

AB \ CD				
	00	01	11	10
00	1	0	0	0
01	1	1	1	0
11	1	1	1	1
10	1	0	0	0

$$y = \overline{C}\overline{D} + AB +$$

BD

pls do both questions!

WS Q2.1: Reading a K-Map

		CD			
AB		00	01	11	10
	00	1	0	0	0
	01	1	1	1	0
	11	1	1	1	1
	10	1	0	0	0

$$Y = \bar{C}\bar{D} + BD + AB$$

Q2.2: Reading a K-Map

CD

AB

	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	1	1

Diagram illustrating a K-Map for a 4-variable function. The map is a 5x5 grid with columns labeled 00, 01, 11, 10 and rows labeled 00, 01, 11, 10. The values are: (00,00)=0, (00,01)=0, (00,11)=1, (00,10)=1; (01,00)=1, (01,01)=1, (01,11)=1, (01,10)=1; (11,00)=1, (11,01)=1, (11,11)=0, (11,10)=0; (10,00)=0, (10,01)=0, (10,11)=1, (10,10)=1. Red and blue groupings are shown, with a double-headed arrow indicating a relationship between the two maps.

$$Y = \underline{B\bar{C}} + \underline{\bar{B}C} + \underline{\bar{A}C}$$

CD

AB

	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	1	1

Diagram illustrating a K-Map for a 4-variable function. The map is a 5x5 grid with columns labeled 00, 01, 11, 10 and rows labeled 00, 01, 11, 10. The values are: (00,00)=0, (00,01)=0, (00,11)=1, (00,10)=1; (01,00)=1, (01,01)=1, (01,11)=1, (01,10)=1; (11,00)=1, (11,01)=1, (11,11)=0, (11,10)=0; (10,00)=0, (10,01)=0, (10,11)=1, (10,10)=1. Red and blue groupings are shown.

$$Y = \underline{B\bar{C}} + \underline{\bar{B}C} + \underline{\bar{A}B}$$

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

		CD			
AB		00	01	11	10
	00				
	01				
	11				
	10				

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

CD

AB

	00	01	11	10
00	1			
01	1			
11	1			
10	1			

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + \underbrace{ABC\bar{D}} + \underbrace{ACD} + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

		CD			
AB		00	01	11	<u>10</u>
	00	1			
	01	1			
	<u>11</u>	1			1
	10	1			

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + \textcolor{red}{ACD} + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

AB \ CD	CD				
	00	01	11	10	
00	1				
01	1				
11	1		1	1	
10	1		1		

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

		CD				
AB		00	01	11	10	
	00	1			1	
	01	1			1	
	11	1		1	1	
	10	1		1		

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + \textcolor{red}{A}\bar{B}C\bar{D}$$

AB	CD				
		00	01	11	10
	00	1			1
	01	1			1
	11	1		1	1
	10	1		1	1

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

		CD			
AB		00	01	11	10
	00	1	0	0	1
	01	1	0	0	1
	11	1	0	1	1
	10	1	0	1	1

$$Y = \bar{D} + AC$$

Simplifying an Equation Using a K-Map

$$Y = \bar{C}\bar{D} + ABC\bar{D} + ACD + \bar{A}C\bar{D} + A\bar{B}C\bar{D}$$

		CD			
AB		00	01	11	10
	00	1	0	0	1
	01	1	0	0	1
	11	1	0	1	1
	10	1	0	1	1

$$Y = AC + \bar{D}$$