COMP435: *SECURITY CONCEPTS!*

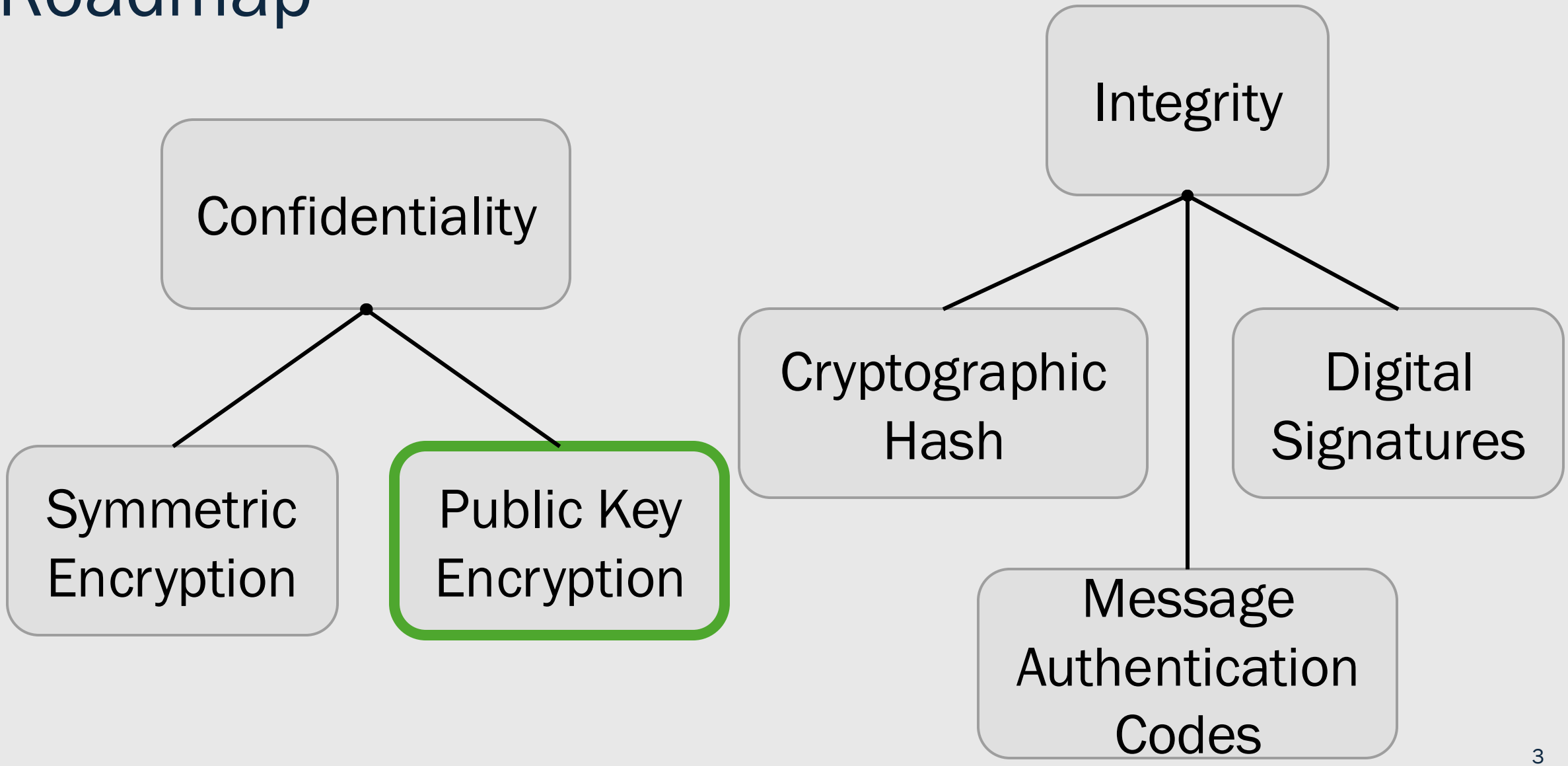
Lecture 9: Cryptographic Hash Functions

tinyurl.com/comp435-fa25

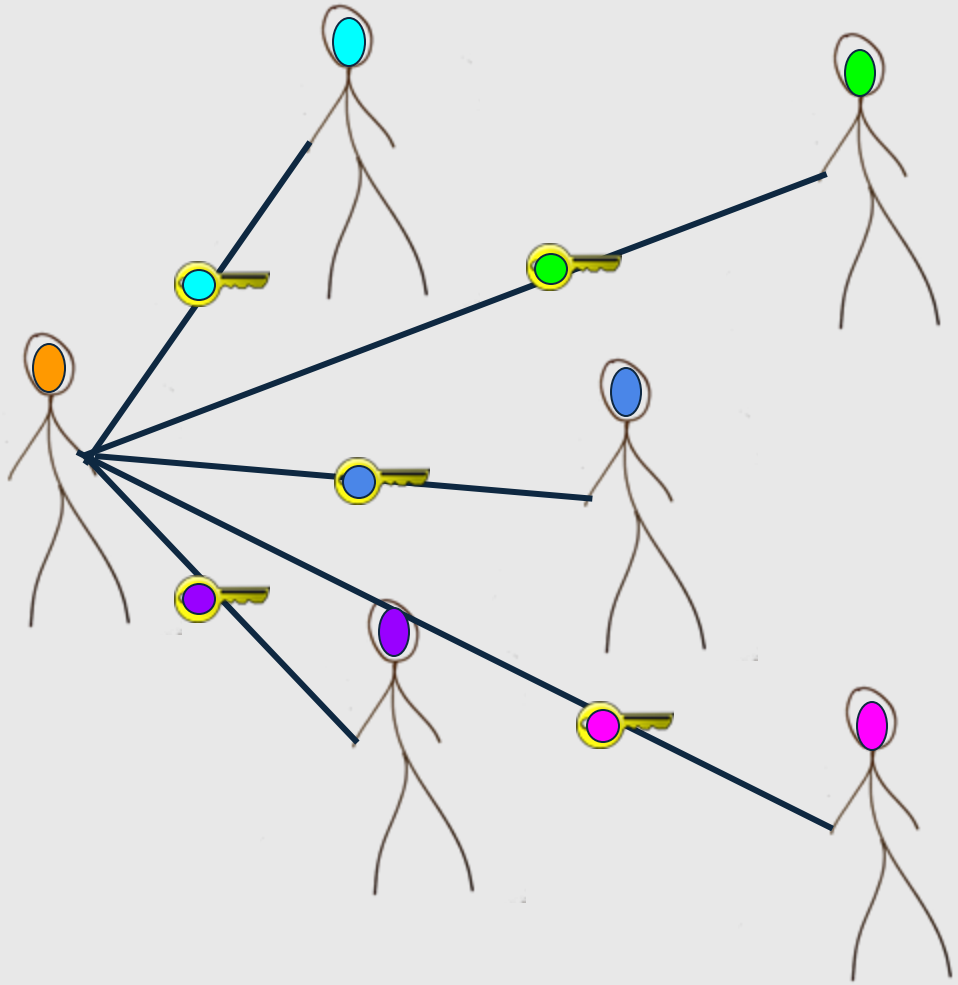
Logistics!

- Lab due tonight!
- Written Assignment due later this week
- Quiz on Wednesday. Review session tomorrow. 5pm in FB007
- Quiz Topics
  - *Passwords*
  - *Guessing Passwords, Keys, Lab 1*
  - *One-Time Pad Properties*
  - *Different Simple Ciphers (Mono/Poly Alphabetic, Caesar)*
  - *Block Ciphers, Modes of Operation – Encryption/Decryption*
  - *Biometrics, Templates & Reading Probability Density Curves*
  - *Symmetric vs. Public Key Cryptography*
  - *RSA: basic modular arithmetic*

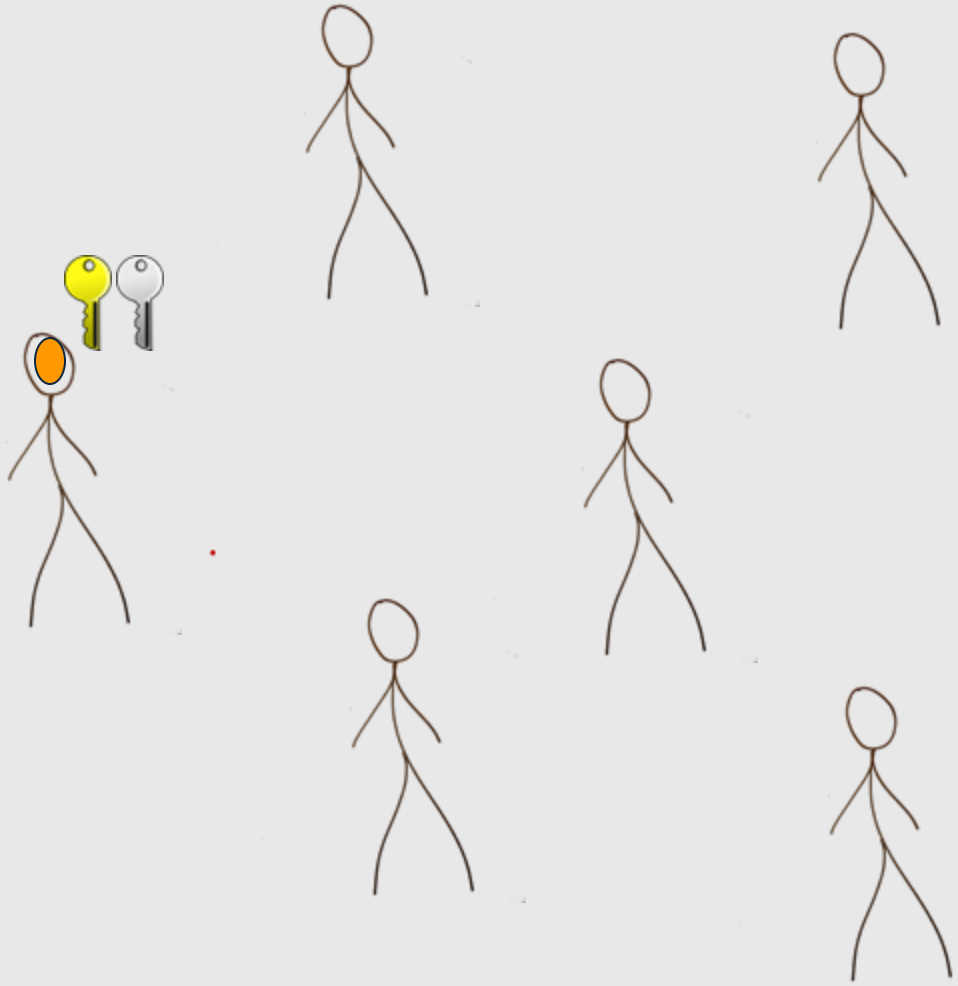
Roadmap


Confidentiality
Symmetric Encryption
Public Key Encryption
Integrity
Cryptographic Hash
Digital Signatures
Message Authentication Codes

Symmetric Encryption



Public Key Encryption



RSA

public key → $K_e = (e, N)$

private key → $K_d = (d, N)$

$ed \equiv 1 \mod \phi(N)$

RSA Encryption and Decryption

- Encryption: $c = msg^e \bmod N$
- Decryption: $msg = c^d \bmod N$

Given N, e and msg^e, it is difficult to compute msg

Given N, e, msg^e, and msg, it is difficult to compute d

RSA Security

- $c = msg^e \bmod N$
- The attacker's challenge:
  Given N, e, msg^e, find msg
- If the attacker can factor N, they win

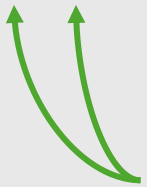
Given e and c, it is difficult to compute msg

Given e, c, and msg, it is difficult to compute d

RSA

Public Key K_e: (N, e) Private Key K_d: (N, d)

$N = pq$

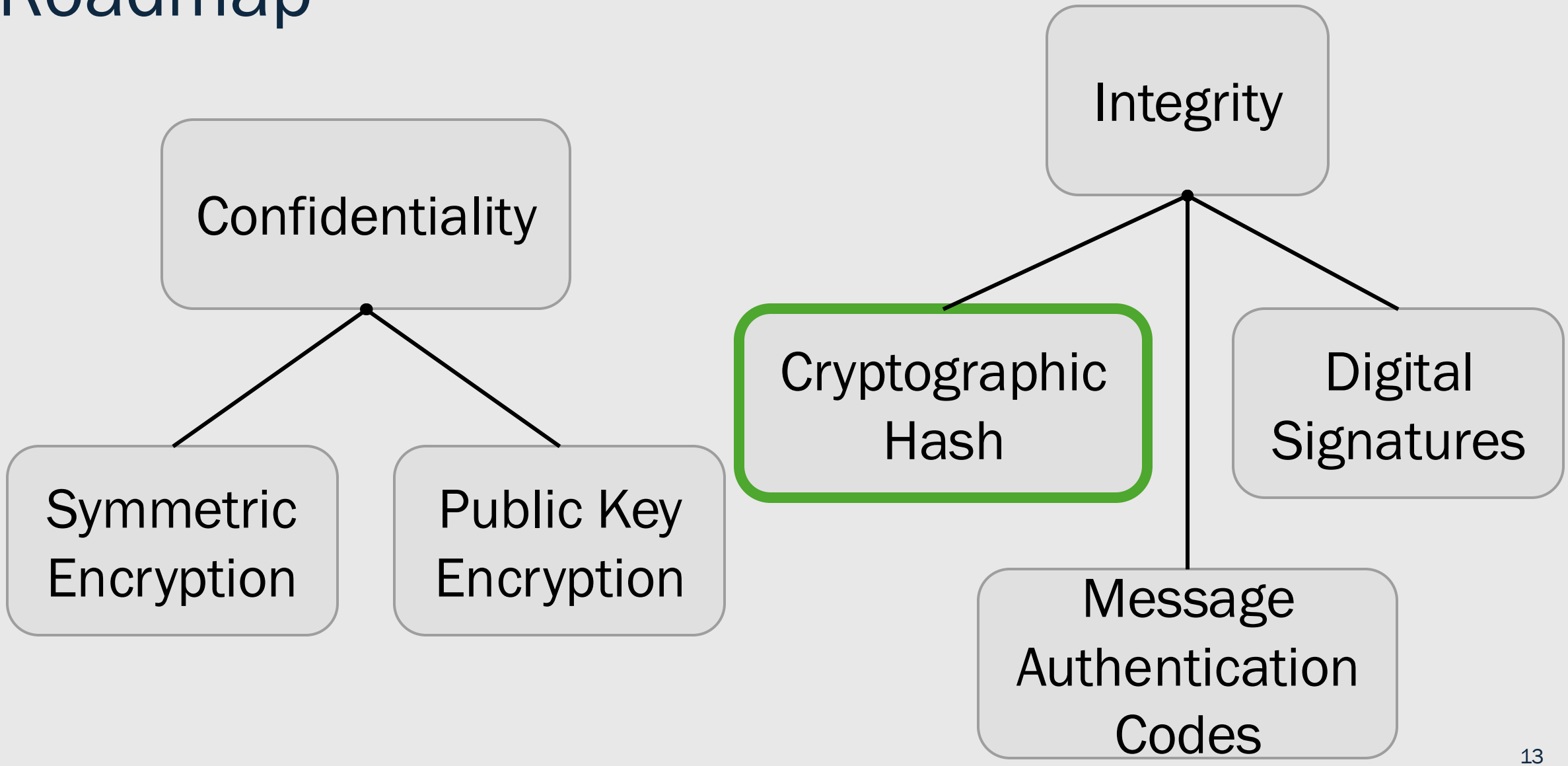


must remain secret!

RSA Practice

CRYPTOGRAPHIC HASH FUNCTIONS

Roadmap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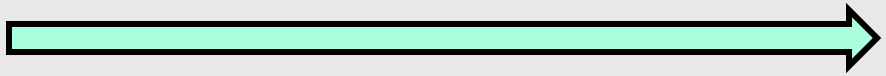
Alice

Bob

Dear Bob,
Do not trust Mallory!
Sincerely,
Alice

msg

msg



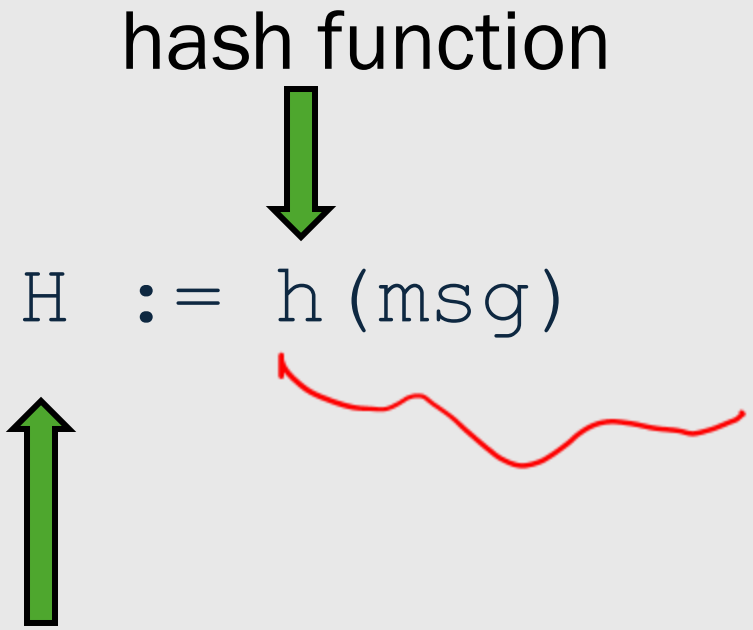
Dear Bob,
You can trust Mallory.
Sincerely,
Alice

msg

Goal: Bob would like to know that the message has been modified.

Tool: cryptographic hash functions!

Terminology



hash value,
hash,
message digest,
digital fingerprint

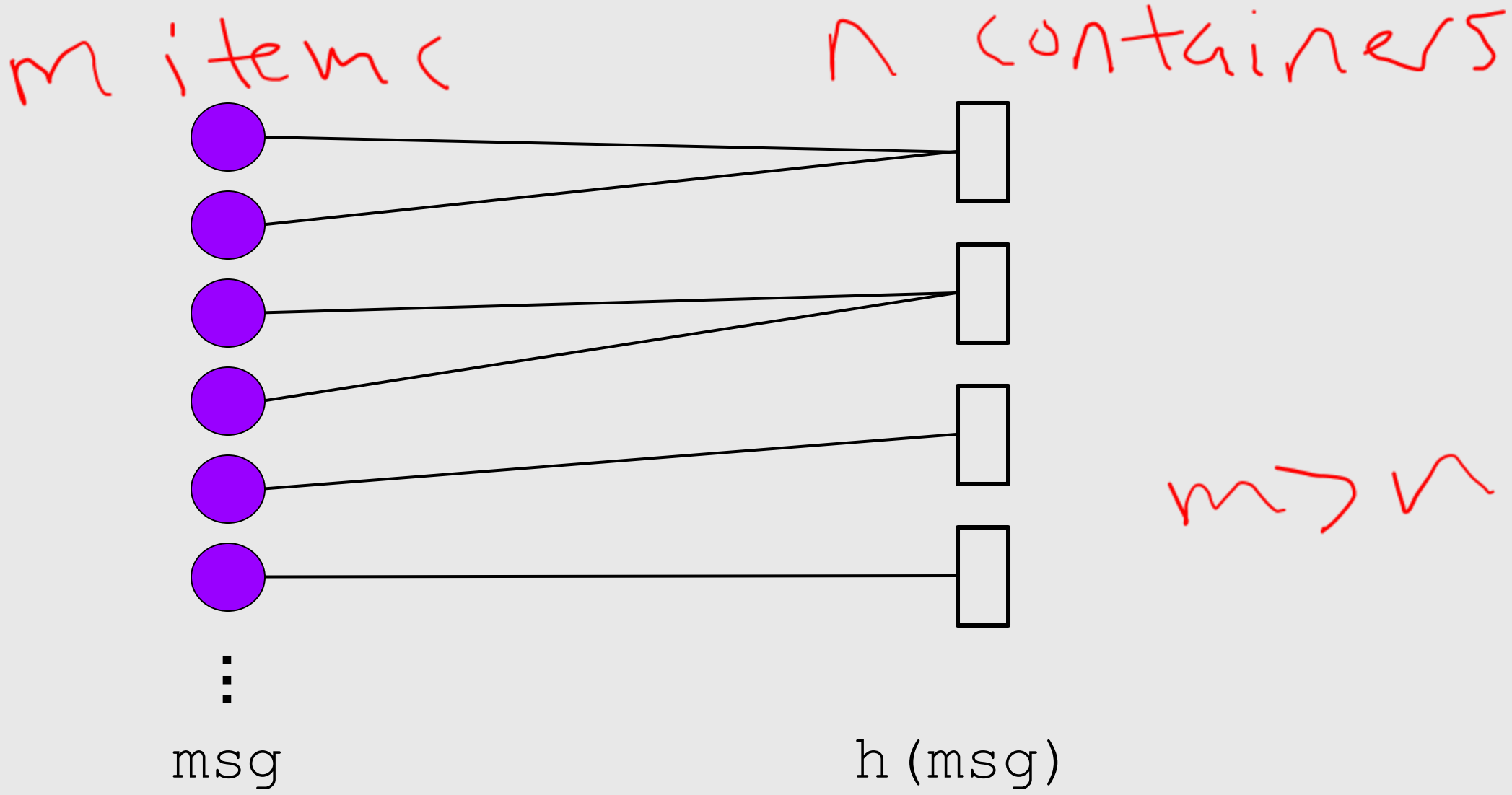
Hash Function

Def'n: a function that takes an arbitrary-length input and produces a fixed-length output

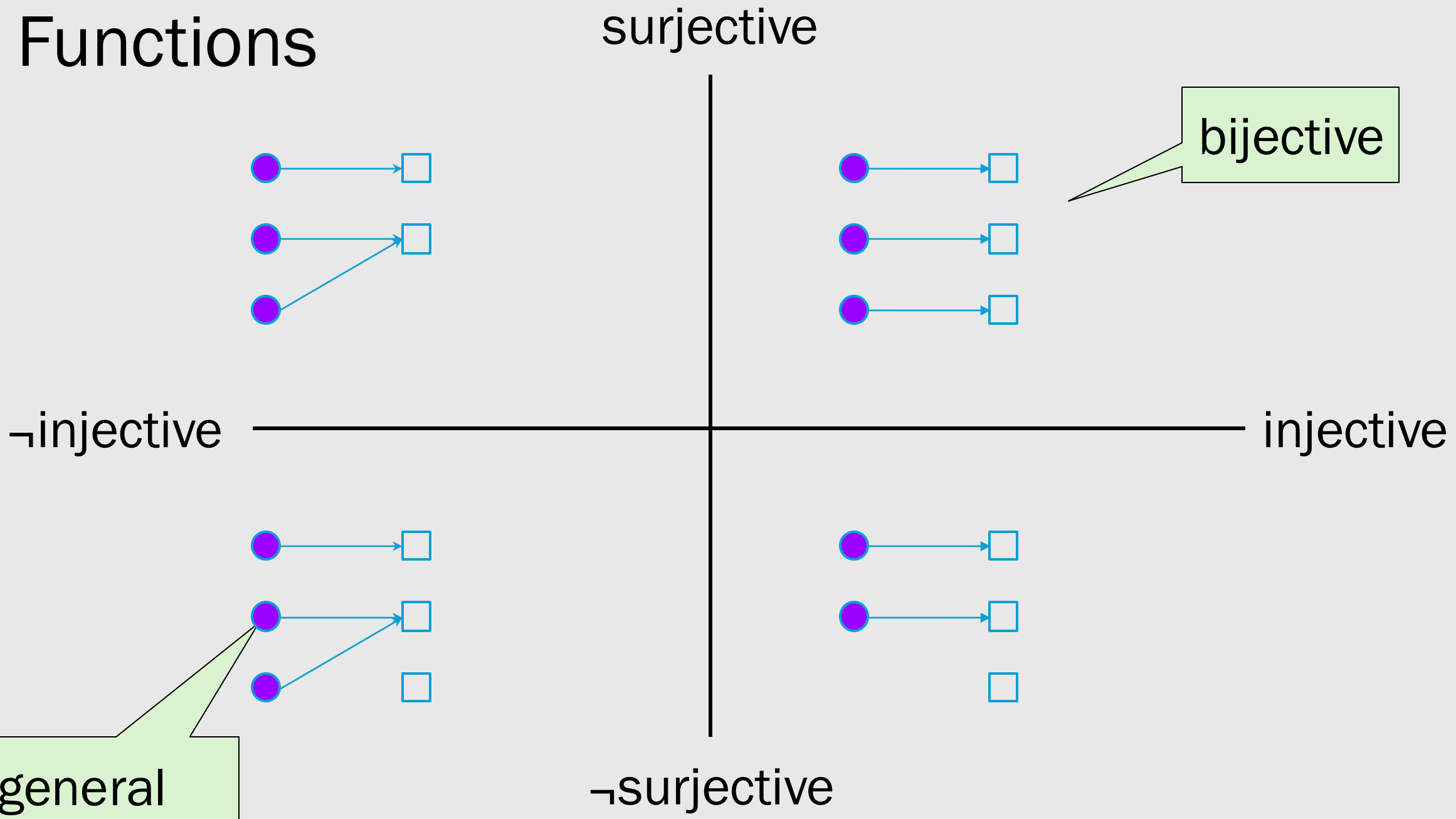
E.g., Modulo operation

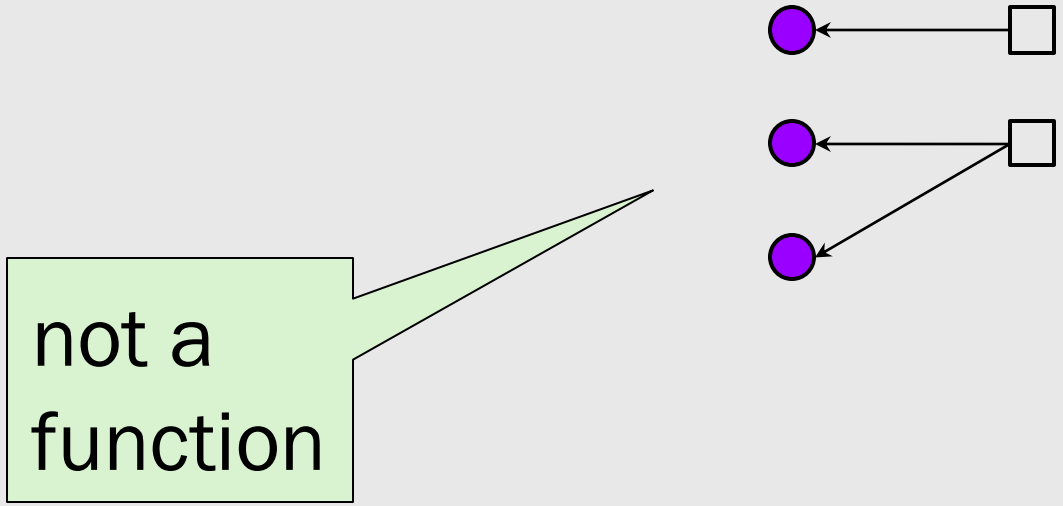
3 % 10 = 3
3590224723940253 % 10 = 3

Pigeonhole Principle



Functions


surjective
bijective
¬injective
injective
general
¬surjective

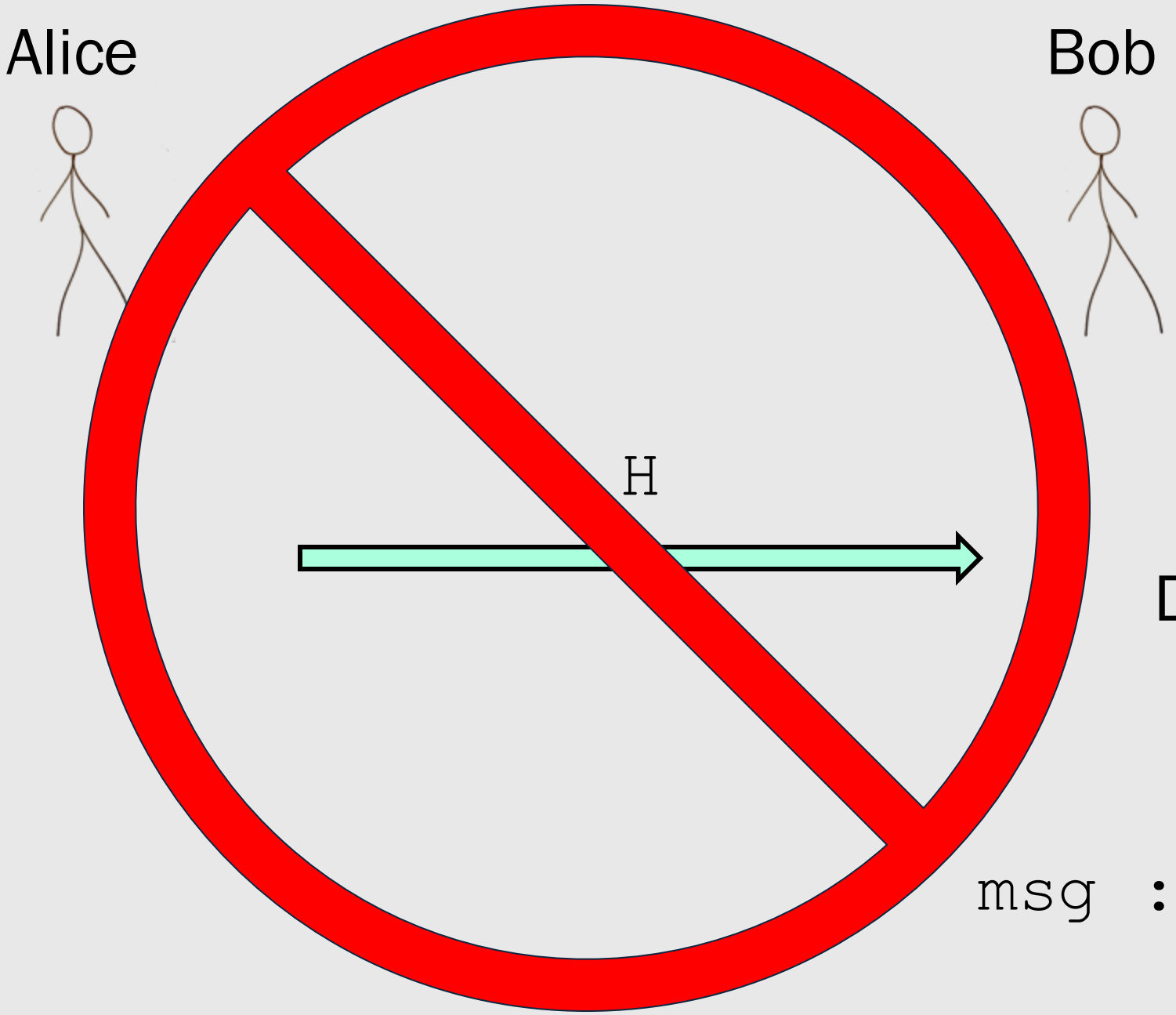


Hash functions not invertible!

One input mapping to two outputs.

Dear Bob,
Do not trust Mallory!
Sincerely,
Alice

msg



Does not exist!

$msg := h^{-1}(H)$

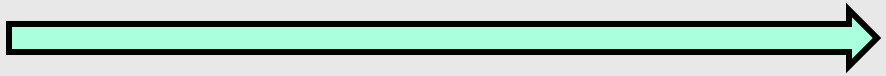
Alice

Bob

Dear Bob,
Do not trust Mallory!
Sincerely,
Alice

msg

(msg, H)



$H \overset{?}{=} h(msg)$

Alice

Bob

Dear Bob,
Do not trust Mallory!
Sincerely,
Alice

msg

(msg, H)

h ≜ msg % 256

Many collisons! ☹

H $\overset{?}{=}$ h(msg)

*** A hash can reveal modifications, but cannot guarantee that no modifications exist! ****

CRYPTOGRAPHIC HASH FUNCTIONS

Cryptographic Hash Functions

Def'n: a hash function that is strong enough to resist collisions

- collision resistant
  - *should be hard to find **any two different** inputs that hash to the same output*
- second pre-image resistant
  - *can't find a different input with same hash*
- pre-image resistant:
  - *can't go backwards*

Collision Resistant

Hard to find `msg`, `msg'` such that

`msg ≠ msg'` and

`h(msg) = h(msg')`

Second Pre-image Resistant

Given `msg`, hard to find `msg'` such that
`msg ≠ msg'` and
`h(msg) = h(msg')`

Pre-image Resistant

Let `H := h(msg)`
Given `H`, hard to find `msg'` such that
`H = h(msg')`

Cryptographic Hash Functions

Def’n: a hash function that is strong enough to resist collisions

- collision resistant
- second pre-image resistant
- pre-image resistant

COLLISION RESISTANT & THE BIRTHDAY PARADOX

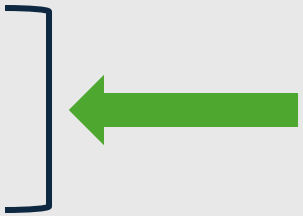
Number of people	Probability of a birthday collision
5	3%
20	41%
50	97%
100	99.99%

Cryptographic Hash Functions

- MD5
- SHA-1
- SHA-2
- SHA-3

Cryptographic Hash Functions

- MD5
- SHA-1
- SHA-2
- SHA-3



deprecated

Cryptographic Hash Functions

- MD5
- SHA-1
- SHA-2
- SHA-3

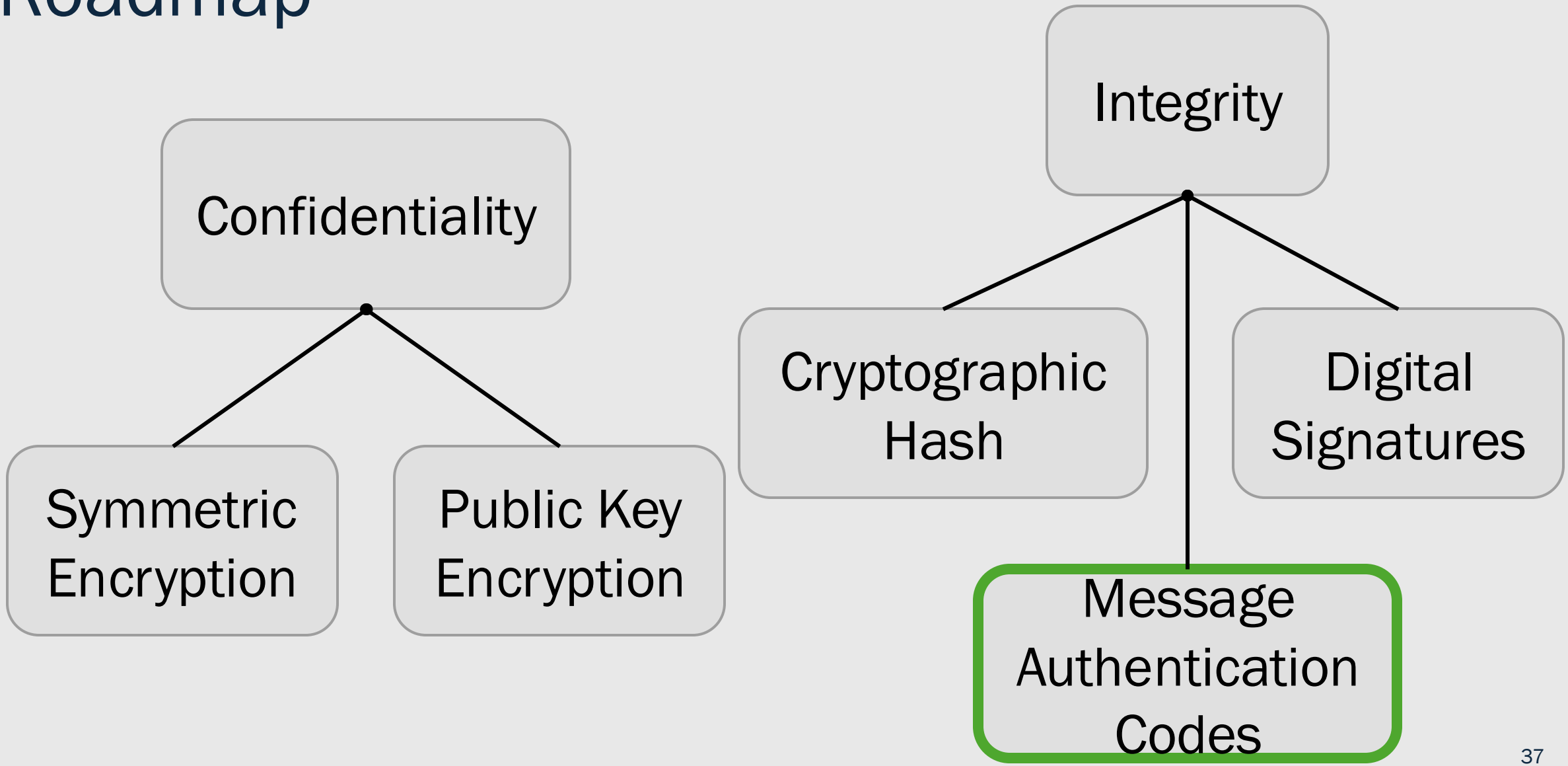
SHA-256

Cryptographic Hash Functions

- MD5
- SHA-1
- SHA-2
- SHA-3  Multiple Variants

MESSAGE AUTHENTICATION CODES

Roadmap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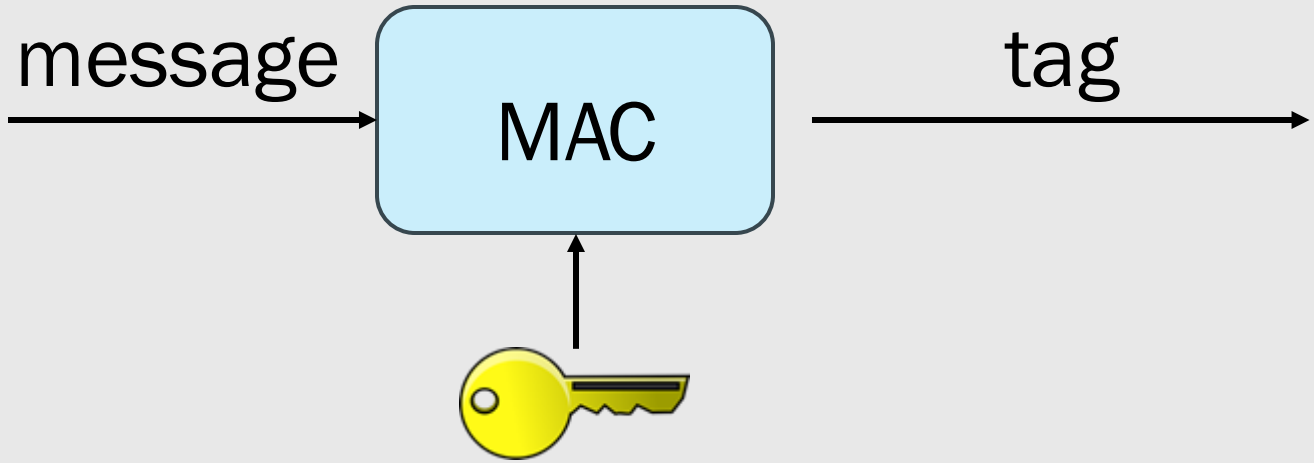
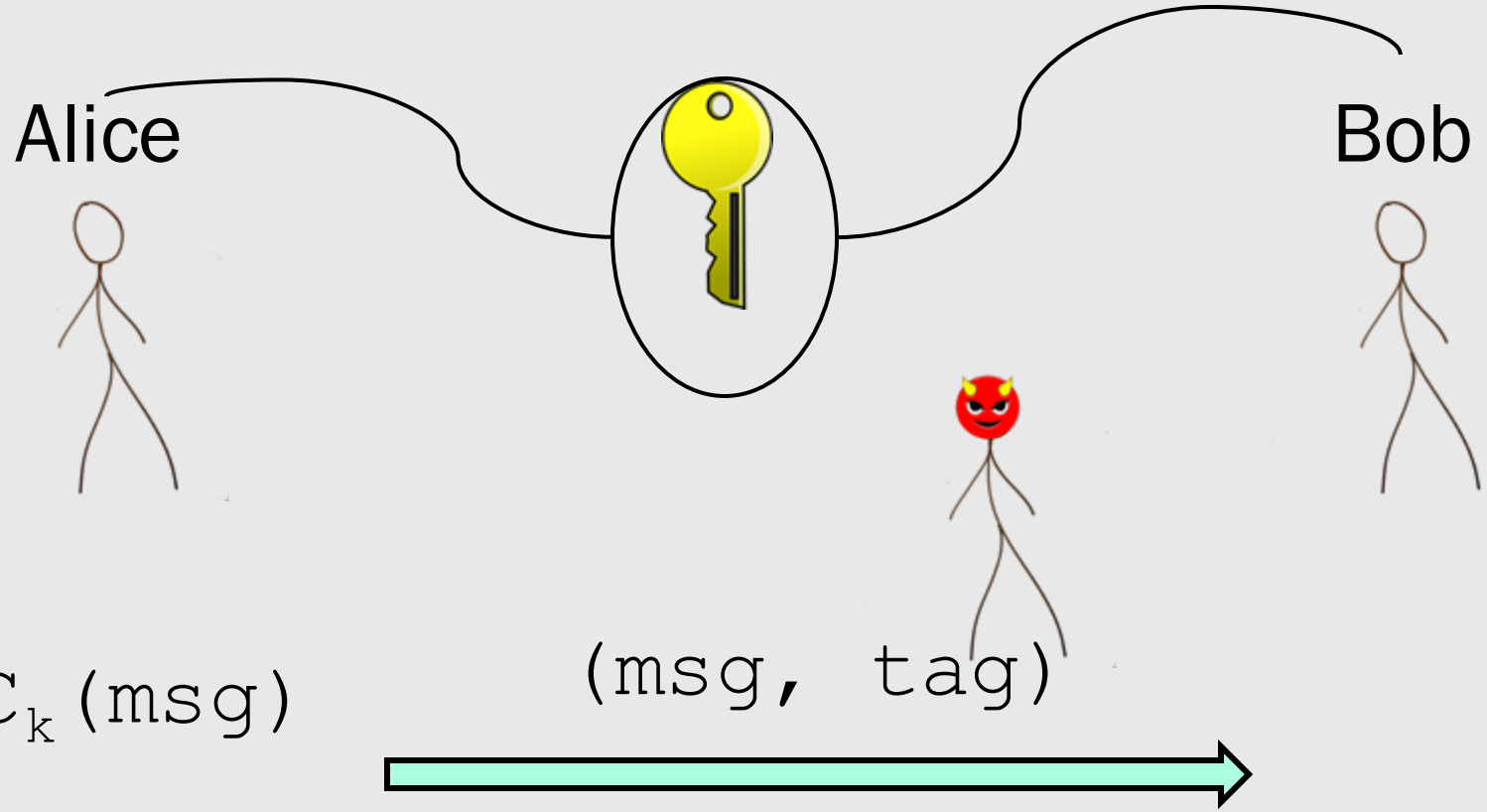
Message Authentication Code (MAC)

Def’n: a keyed cryptographic hash function

Message Authentication Codes





Dear Bob,
Do not trust
Mallory!
Sincerely,
Alice

msg

tag:= $MAC_k(msg)$

(msg, tag)

Dear Bob,
You can trust
Mallory.
Sincerely,
Alice

$tag \stackrel{?}{=} MAC_k(msg)$

Properties of MACs

- Unforgeability
- Integrity

Properties of MACs

- **Unforgeability**  *Authentication of data origin*
- Integrity

Properties of MACs

- Unforgeability
- **Integrity**  *Integrity of Data*

Message Authentication Codes

- HMAC
- CBC-MAC
- CMAC

Message Authentication Codes

- **HMAC** 
- CBC-MAC
- CMAC

a construction that keys an underlying collision-resistant cryptographic hash function

Message Authentication Codes

- HMAC
- **CBC-MAC**
- **CMAC**

combines a block cipher with a secret key

MESSAGE AUTHENTICATION & CONFIDENTIALITY

Message Authentication and Confidentiality

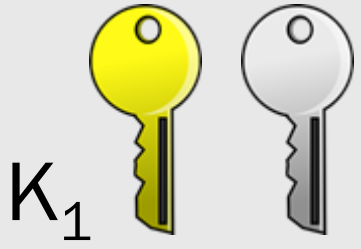
Encrypt-and-Authenticate

Authenticate-then-Encrypt

Encrypt-then-Authenticate

Encrypt and Authenticate

Alice



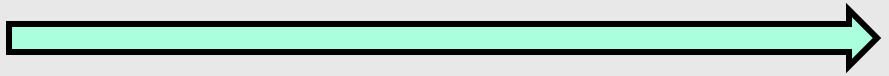
Bob

msg

$c := Enc_{k1}(msg)$

$t := MAC_{k2}(msg)$

(c,t)

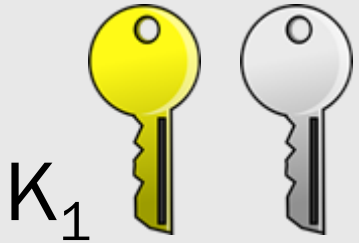


$msg := Dec_{k1}(c)$

$t \stackrel{?}{=} MAC_{k2}(msg)$

Authenticate-then-Encrypt

Alice



Bob

msg

$t := MAC_{k2}(msg)$

$c := Enc_{k1}(msg|t)$

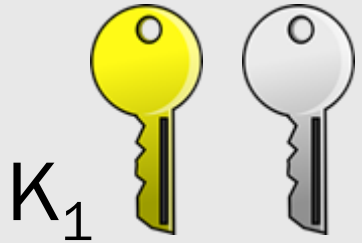
(c)

$msg|t := Dec_{k1}(c)$

$t \stackrel{?}{=} MAC_{k2}(msg)$

Encrypt-then-Authenticate

Alice



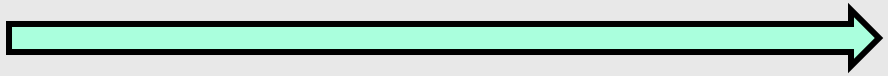
Bob

msg

$c := Enc_{k1}(msg)$

$t := MAC_{k2}(c)$

(c,t)



?

$t = MAC_{k2}(c)$

$msg := Dec_{k1}(c)$

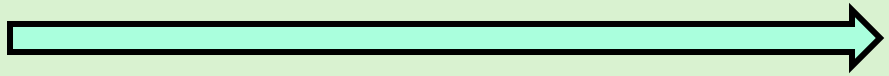
Alice

Bob

msg

H:= h(msg)

(msg, H)



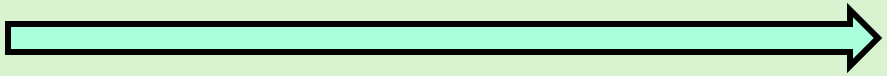
H ?= h(msg)

---

msg

tag:= MAC_k(msg)

(msg, tag)



tag ?= MAC_k(msg)

Alice

Bob

MAC with a shared key: gives **integrity + authenticity !**

msg

H:= h(msg)

? = h(msg)

msg

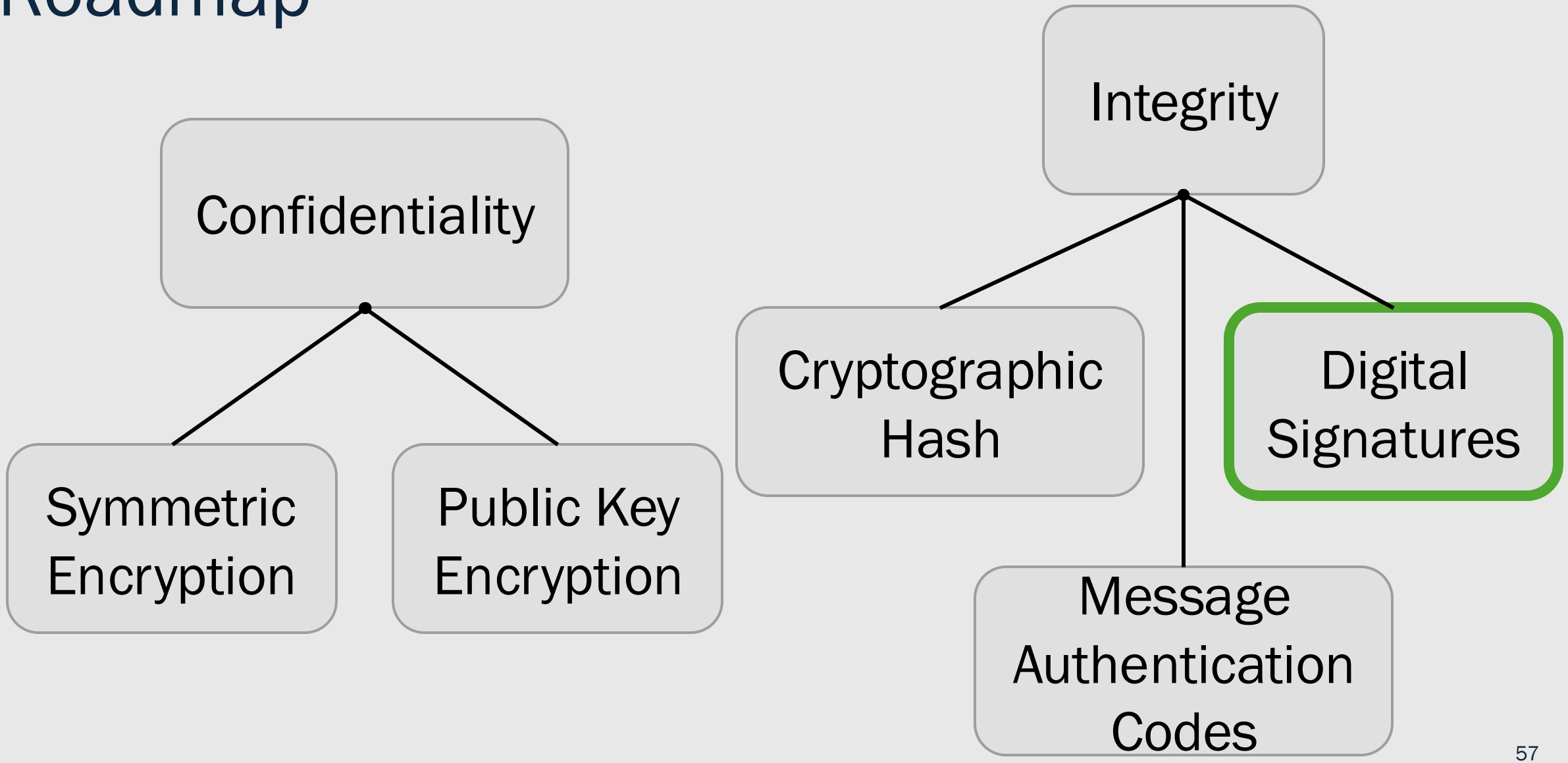
tag:= MAC_k(msg)

(msg, tag)

? tag = MAC_k(msg)

DIGITAL SIGNATURES

Roadmap


Confidentiality
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Digital Signature

Def’n: a means to ensure a given message comes from a given source.

Properties of a Digital Signature

- Authentication
- Integrity
- Accountability

Properties of a Digital Signature

- **Authentication**  Authentication of data origin
- Integrity
- Accountability

Properties of a Digital Signature

- Authentication
- **Integrity**  Integrity of data
- Accountability

Properties of a Digital Signature

- Authentication
- Integrity
- **Accountability**

Non-repudiation. A signer cannot deny having signed.

Digital Signature Requirements

- Unforgeable
- Authentic
- Unalterable
- Not reusable

Digital Signature Requirements

- **Unforgeable** 
- Authentic
- Unalterable
- Not reusable

No party can produce a viable signature by outside means

Digital Signature Requirements

- Unforgeable
- **Authentic**
- Unalterable
- Not reusable

A signature is tied to a single party

Digital Signature Requirements

- Unforgeable
- Authentic
- **Unalterable**
- Not reusable

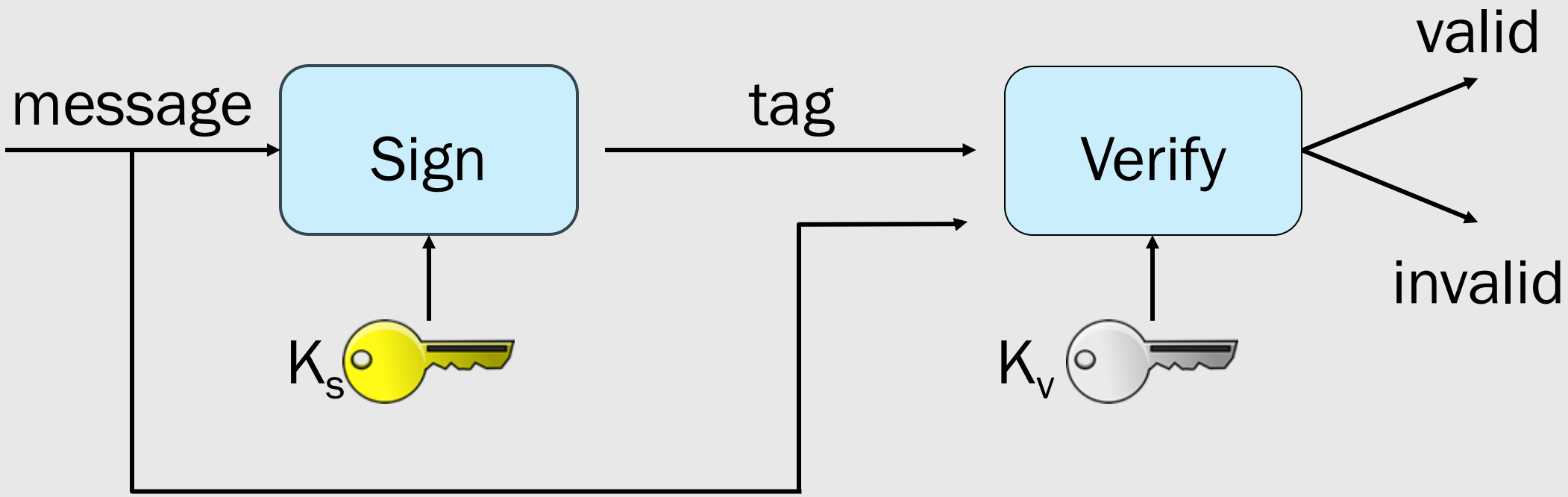
A signed message cannot be altered without invalidating the signature

Digital Signature Requirements

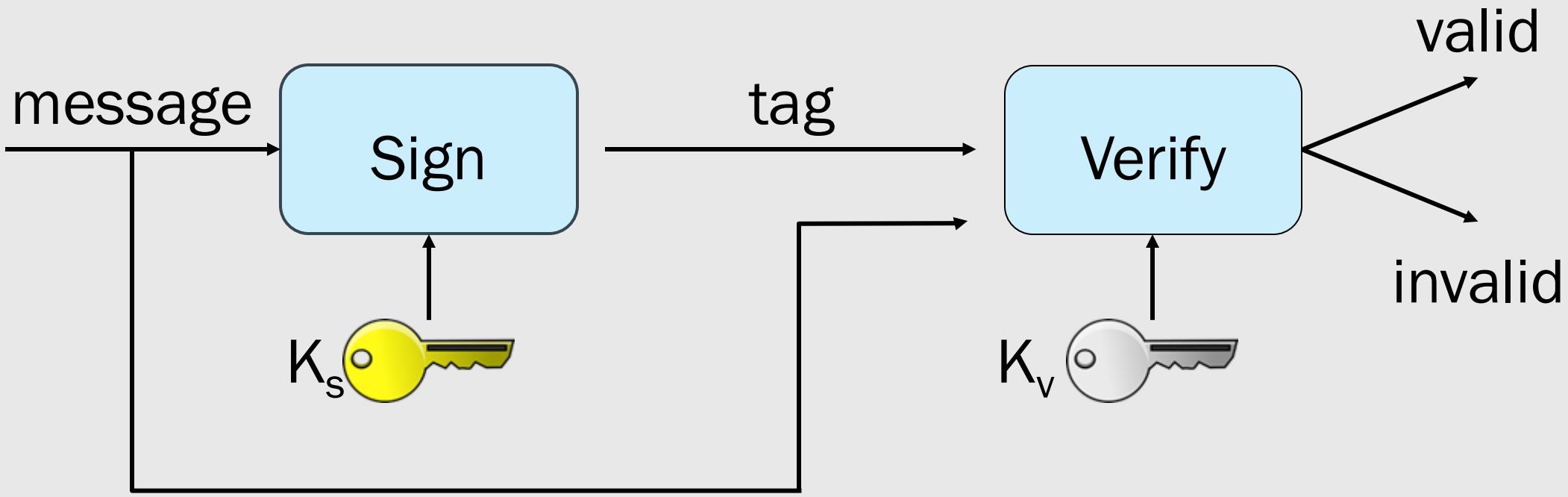
- Unforgeable
- Authentic
- Unalterable
- **Not reusable**

A signature is tied to the signed message

Public Key Signatures



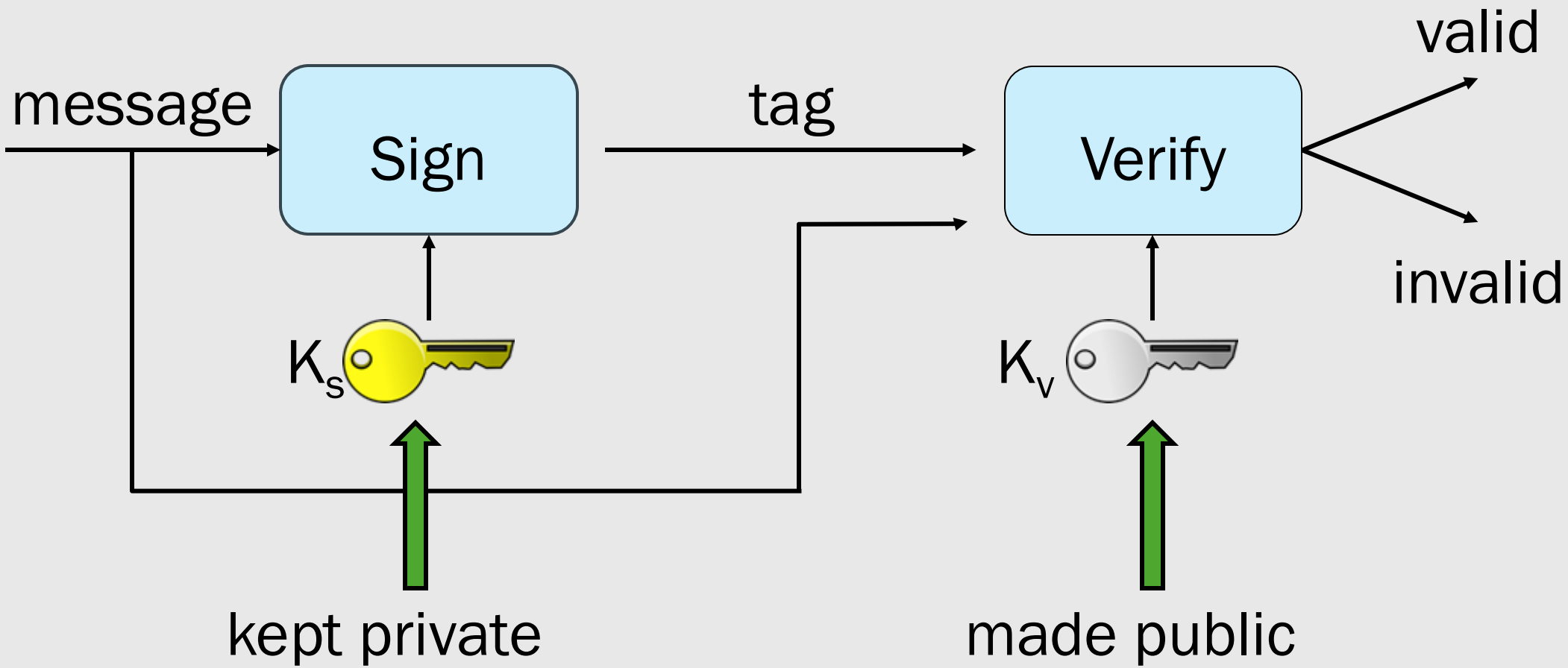
Public Key Signatures

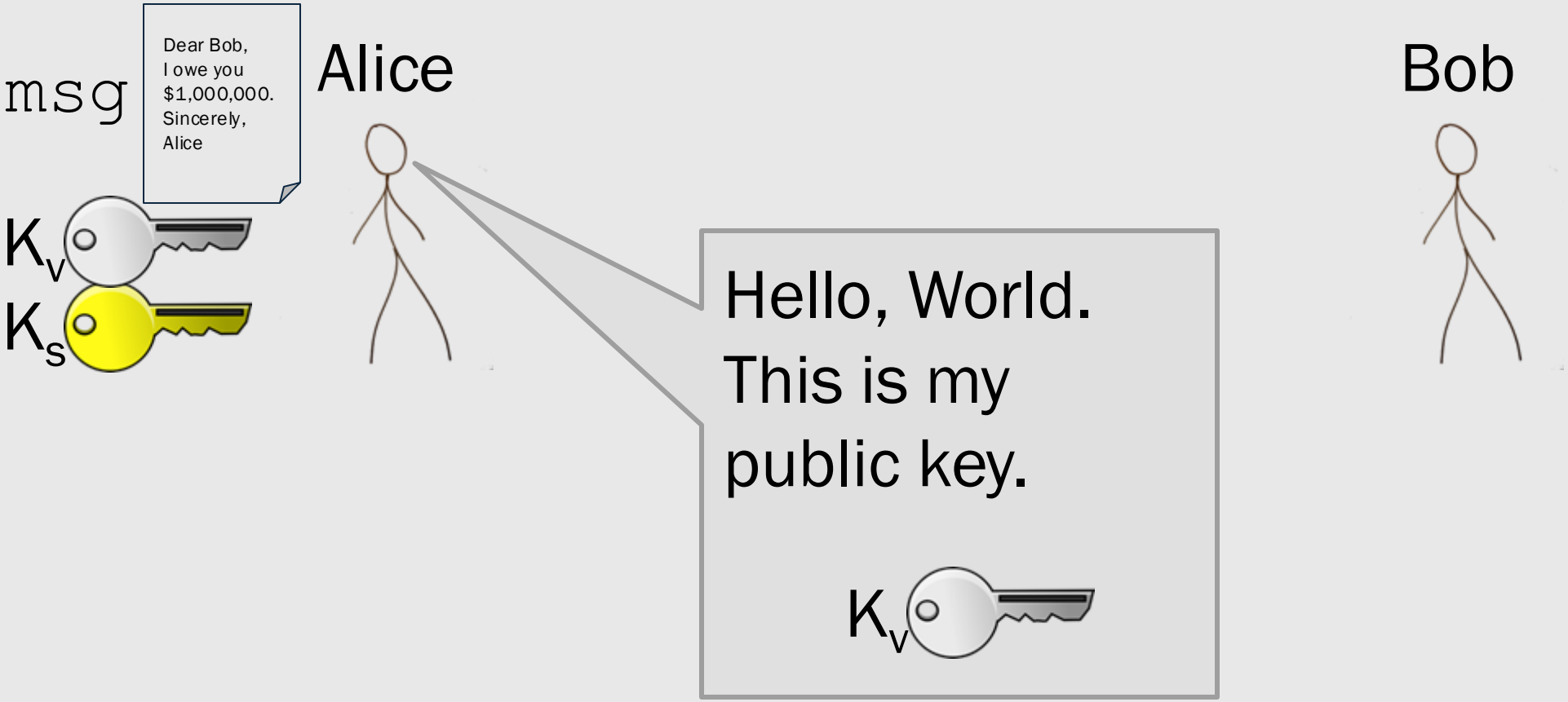


$$tag = Sign_{Ks}(msg)$$

$$isValid = Verify_{Kv}(msg, tag)$$

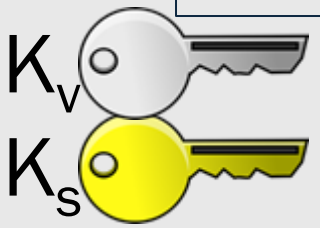
Public Key Signatures




msg
Dear Bob,
I owe you
$1,000,000.
Sincerely,
Alice
Alice
Bob
K_v
K_s
Hello, World.
This is my
public key.
K_v

msg

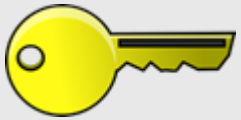
Dear Bob,
I owe you
$1,000,000.
Sincerely,
Alice



Alice

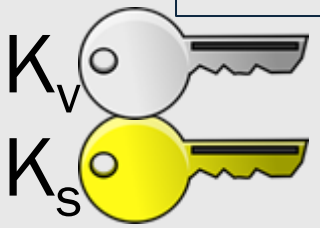
Bob

$tag := Sign_{Ks}(msg)$



msg

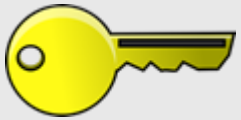
Dear Bob,
I owe you
$1,000,000.
Sincerely,
Alice



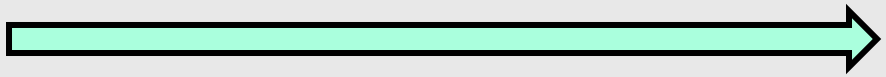
Alice

Bob

$tag := Sign_{Ks}(msg)$



(tag, msg)

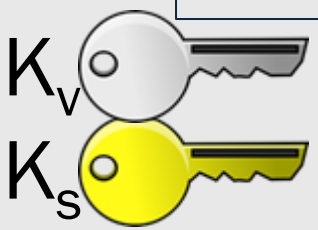


msg

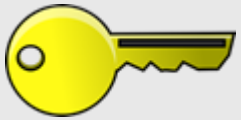
Dear Bob,
I owe you
$1,000,000.
Sincerely,
Alice

Alice

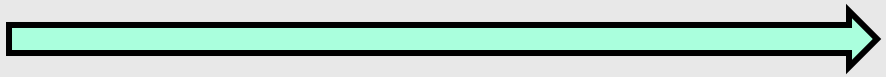
Bob



$tag := Sign_{Ks}(msg)$



(tag, msg)



$isValid = Verify_{Kv}(msg, tag)$

RSA FOR DIGITAL SIGNATURES

RSA for digital signatures

sign: $sig = m^s \bmod N$

verify: $m \overset{?}{=} sig^v \bmod N$

RSA for digital signatures

sign: $sig = m^s \bmod N$

verify: $m \overset{?}{=} sig^v \bmod N$

(N, v) → K_v

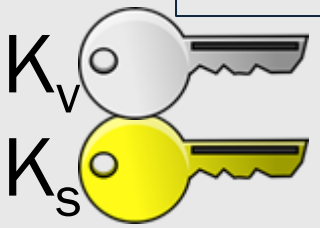
(N, s) → K_s

```
sig := Rand(N)
msg := sig^v mod N
```

RSA does not provide unforgeability

msg

Dear Bob,
I owe you
$1,000,000.
Sincerely,
Alice

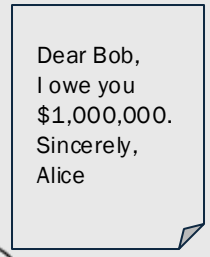


Alice

Hello, World.
This is my
public key.

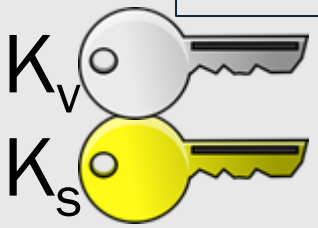
Bob

msg

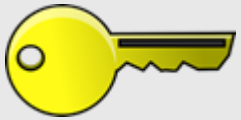


Alice

Bob

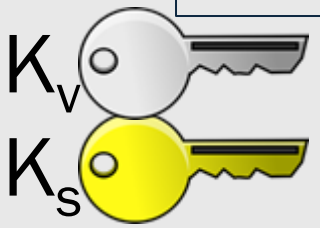


$tag := Sign_{K_s}(h(msg))$



msg

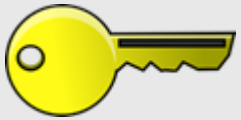
Dear Bob,
I owe you
$1,000,000.
Sincerely,
Alice



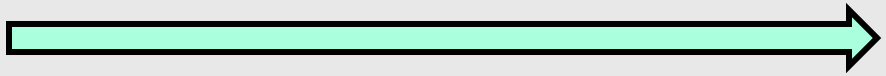
Alice

Bob

$tag := Sign_{Ks}(h(msg))$



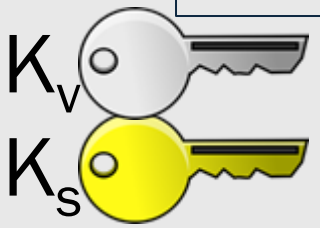
(tag, msg)



msg

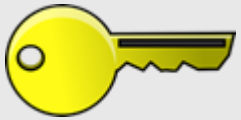
Dear Bob,
I owe you
$1,000,000.
Sincerely,
Alice

Alice

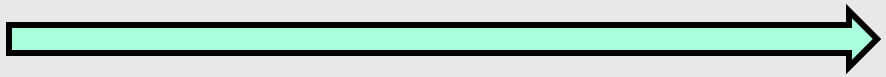


Bob

$tag := Sign_{Ks}(h(msg))$

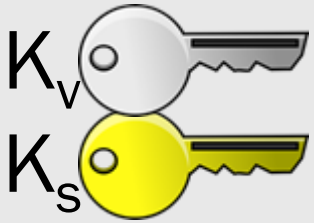


(tag, msg)



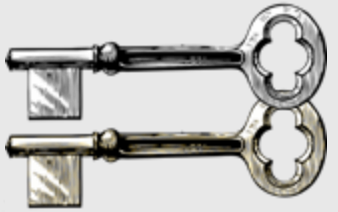
$isValid = Verify_{Kv}(h(msg), tag)$

Encrypt then Sign



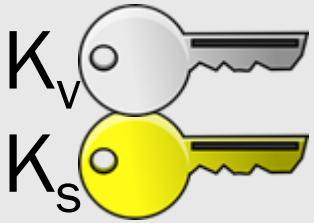
Alice

Bob



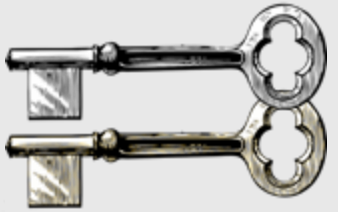
Hello, World. This is my public key for verifying my signature.

K_v 

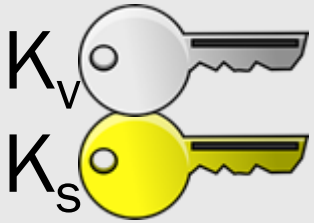


Alice

Bob

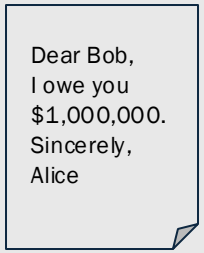


Hello, World.
This is my public key for encrypting.

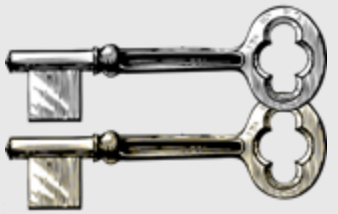


Alice

msg



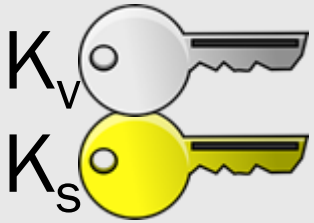
Bob



K_e
K_d

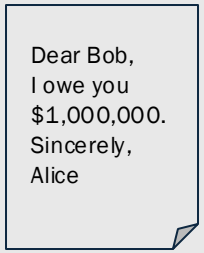
$$c := Enc_{Ke}(msg)$$

$$tag := Sign_{ks}(c)$$

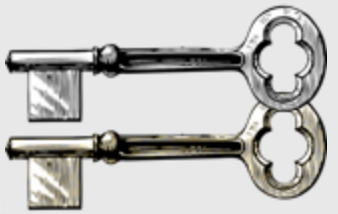


Alice

msg



Bob

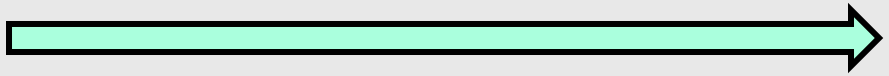


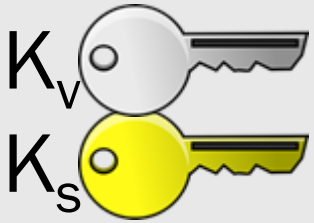
K_e
K_d

$c := Enc_{Ke}(msg)$

$tag := Sign_{ks}(c)$

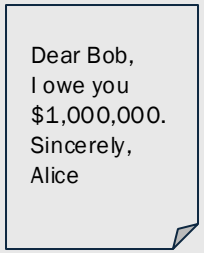
(c,tag)



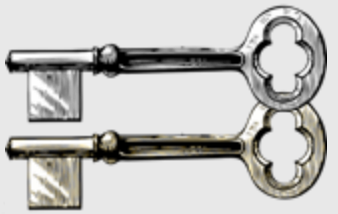


Alice

msg



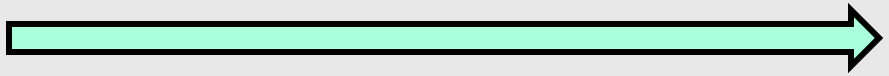
Bob



$c := Enc_{Ke}(msg)$

$tag := Sign_{ks}(c)$

(c,tag)



$isValid := Verify_{Kv}(c,tag)$

$msg := Dec_{kd}(c)$

A Note of Caution

- Beware of determinism
- Use unique keys for encryption, signing, generating MACs
- Use appropriate algorithms for the desired property
- Do not roll your own