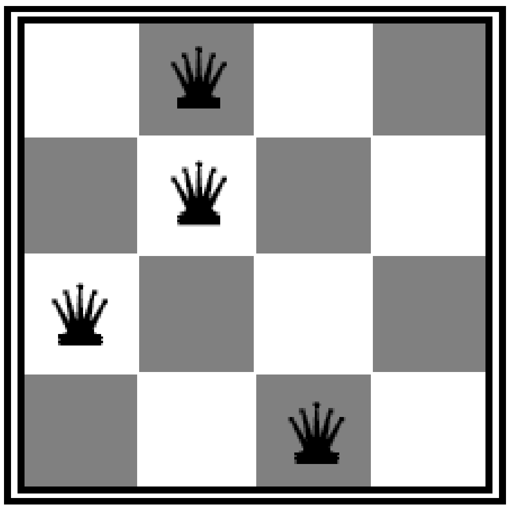
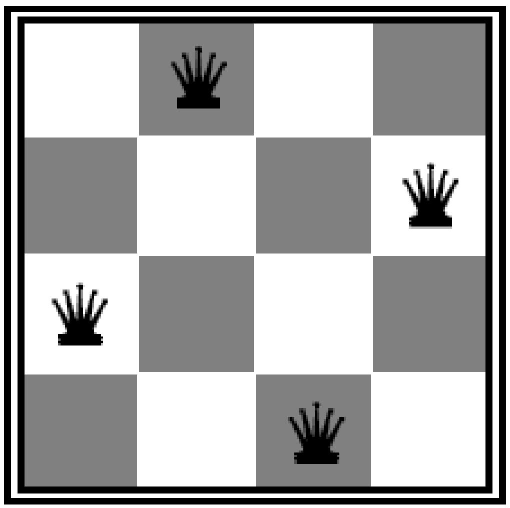
Local search algorithms

- In many optimization problems, the state space is the space of all possible *complete* solutions
- We have an **objective function** that tells us how “good” a given state is, and we want to find the solution (goal) by minimizing or maximizing the value of this function

Example: *n*-queens problem

- Put *n* queens on an *n* × *n* board with no two queens on the same row, column, or diagonal
- **State space:** all possible *n*-queen configurations
- What's the **objective function**?
  - Number of pairwise conflicts





Example: Traveling salesman problem

- Find the shortest tour connecting a given set of cities
- **State space:** all possible tours
- **Objective function:** length of tour

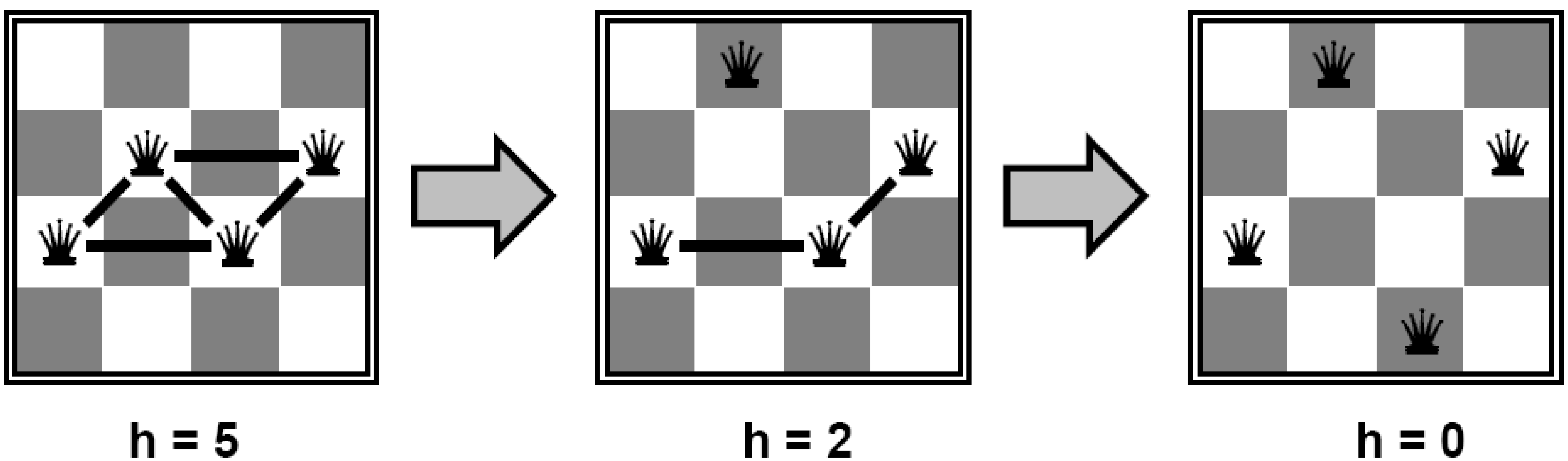
Local search algorithms

- In many optimization problems, the state space is the space of all possible *complete* solutions
- We have an **objective function** that tells us how “good” a given state is, and we want to find the solution (goal) by minimizing or maximizing the value of this function
- The start state may not be specified
- The path to the goal doesn’t matter

- In such cases, we can use local search algorithms that keep a single “current” state and gradually try to improve it

Example: *n*-queens problem

- Put *n* queens on an *n* × *n* board with no two queens on the same row, column, or diagonal
- **State space:** all possible *n*-queen configurations
- **Objective function:** number of pairwise conflicts
- What's a possible local improvement strategy?
  - Move one queen within its column to reduce conflicts



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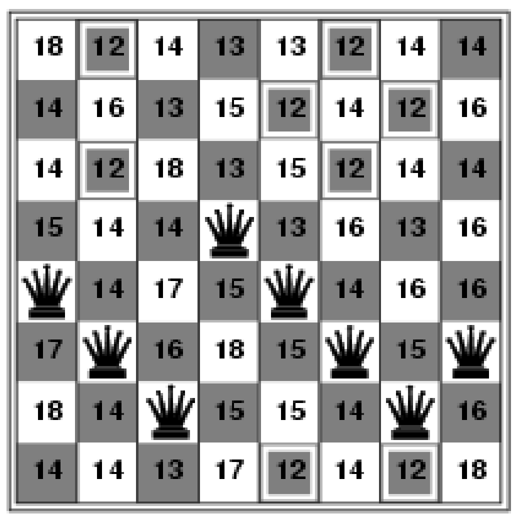
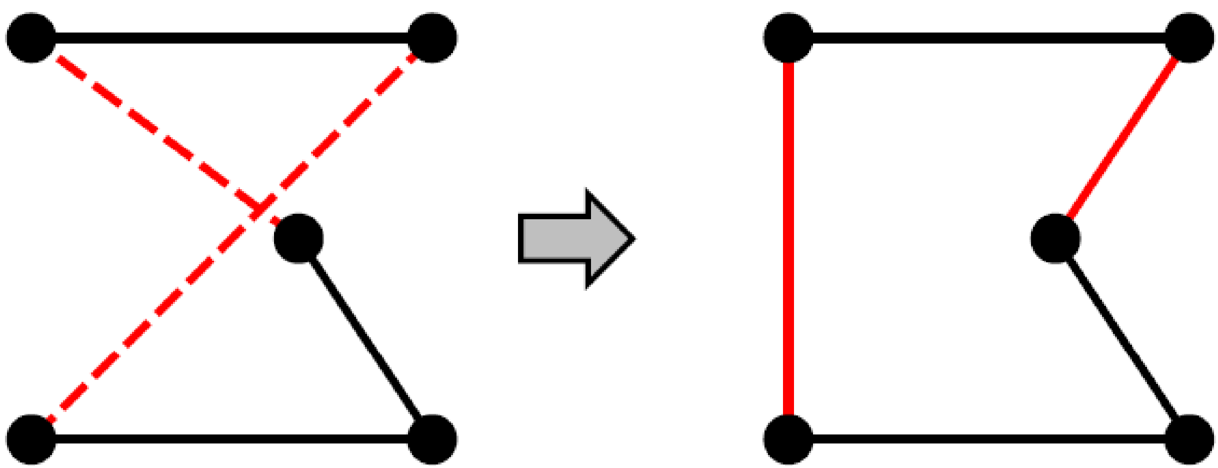


$h = 17$

Example: Traveling Salesman Problem

- Find the shortest tour connecting n cities
- **State space:** all possible tours
- **Objective function:** length of tour
- What's a possible local improvement strategy?
  - Start with any complete tour, perform pairwise exchanges

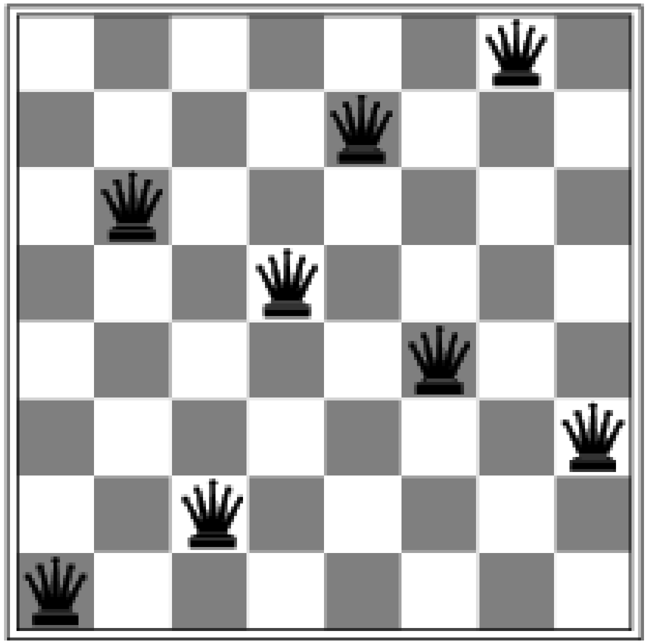


Hill-climbing search

- Initialize *current* to starting state
- Loop:
  - Let *next* = highest-valued successor of *current*
  - If value(*next*) < value(*current*) return *current*
  - Else let *current* = *next*
- Variants: choose first better successor, randomly choose among better successors
- “Like climbing mount Everest in thick fog with amnesia”

Hill-climbing search

- Is it complete/optimal?
  - No – can get stuck in local optima
  - Example: local optimum for the 8-queens problem



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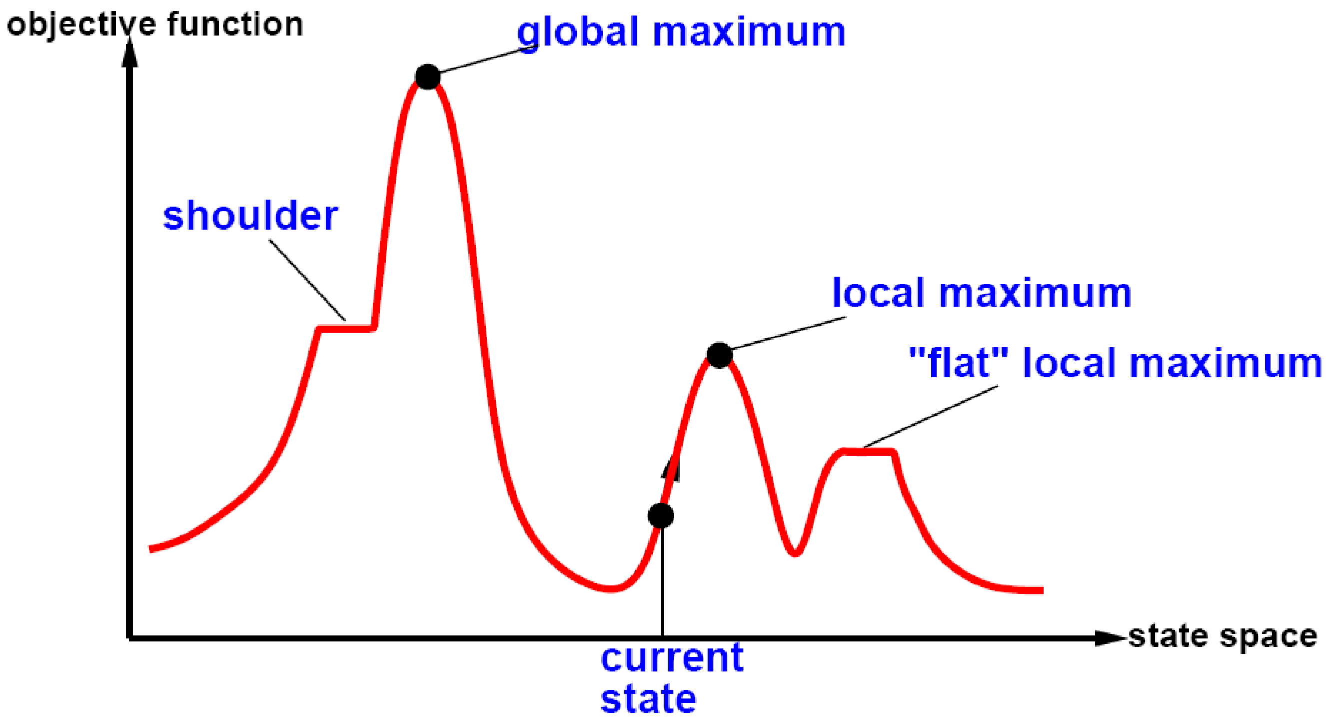
The state space “landscape”



- How to escape local maxima?
  - Random restart hill-climbing
- What about “shoulders”?
- What about “plateaux”?

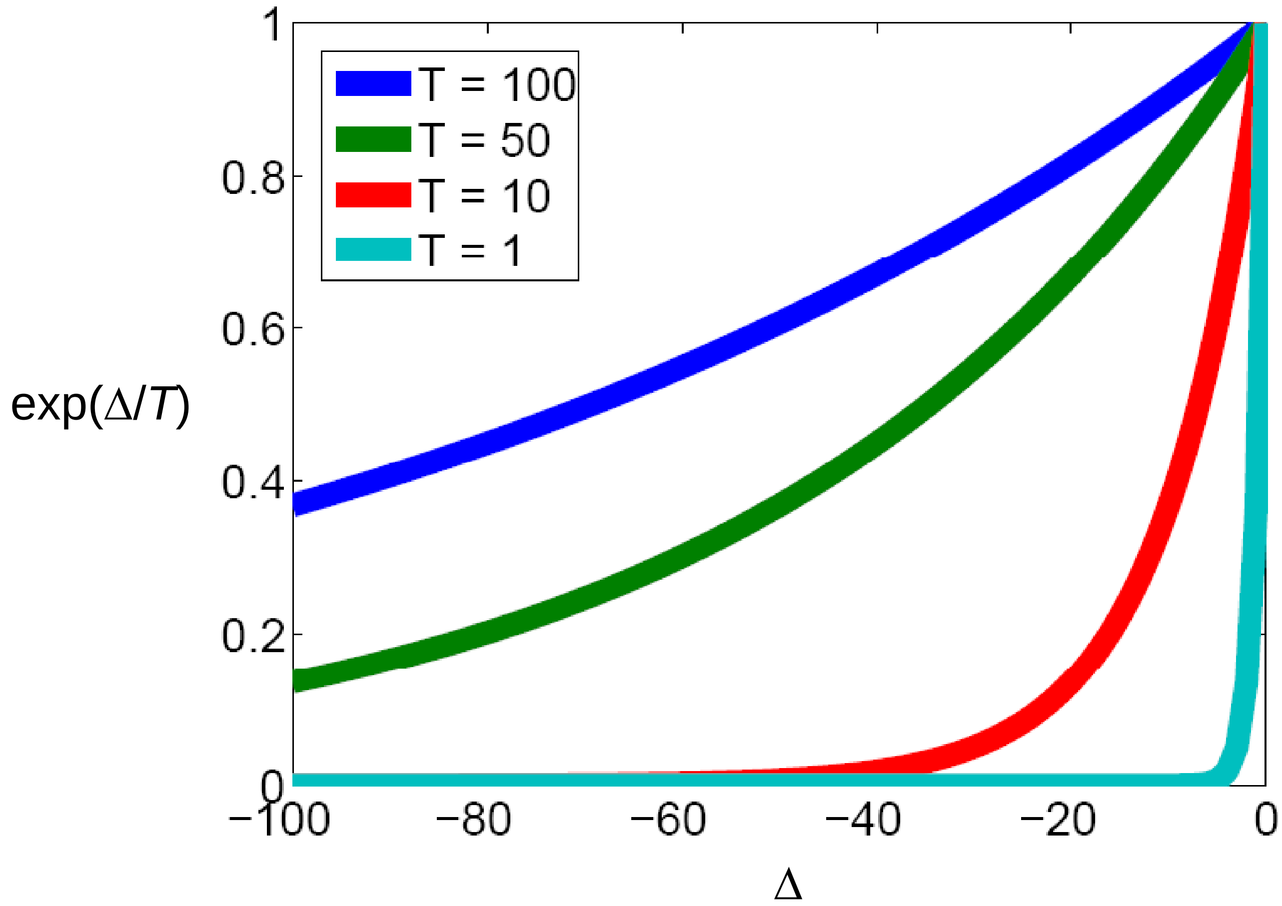
Simulated annealing search

- Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency
  - Probability of taking downhill move decreases with number of iterations, steepness of downhill move
  - Controlled by *annealing schedule*
- Inspired by tempering of glass, metal

Simulated annealing search

- Initialize *current* to starting state
- For i = 1 to ∞
  - If $T(i) = 0$ return *current*
  - Let *next* = random successor of *current*
  - Let Δ = value(*next*) – value(*current*)
  - If $\Delta > 0$ then let *current* = *next*
  - Else let *current* = *next* with probability $\exp(\Delta/T(i))$

Effect of temperature



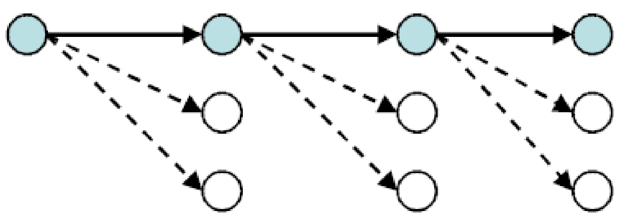
Simulated annealing search

- One can prove: If temperature decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching one
- However:
  - This usually takes impractically long
  - The more downhill steps you need to escape a local optimum, the less likely you are to make all of them in a row
- More modern techniques: general family of *Markov Chain Monte Carlo* (MCMC) algorithms for exploring complicated state spaces

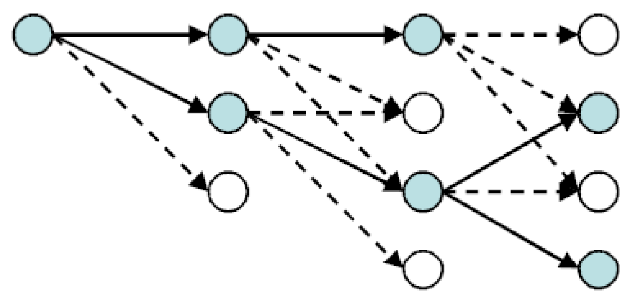
Local beam search

- Start with *k* randomly generated states
- At each iteration, all the successors of all *k* states are generated
- If any one is a goal state, stop; else select the *k* best successors from the complete list and repeat

- Is this the same as running *k* greedy searches in parallel?



Greedy search



Beam search