Games and adversarial search
Why study games?

• Games can be a good model of many competitive activities
  – Military confrontations, negotiation, auctions, …
• Games are a traditional hallmark of intelligence
• Contrarian viewpoint (textbook):
  – “Chess is to AI as Grand Prix motor racing is to the car industry: state-of-the-art game programs are blindingly fast, highly optimized machines that incorporate the latest engineering advances, but they aren’t much use for doing the shopping or driving off-road.”
## Types of game environments

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<th>Stochastic</th>
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<td>Backgammon, monopoly</td>
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<td>Imperfect information</td>
<td>Battleships</td>
<td>Scrabble, poker, bridge</td>
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**Deterministic** games are those where the state of the game is always known to all players. **Stochastic** games involve elements of chance, where the next state of the game is not fully predictable.
Alternating two-player zero-sum games

• Players take turns
• Each game outcome or terminal state has a utility for each player (e.g., 1 for win, 0 for loss)
• The sum of both players’ utilities is a constant
Games vs. single-agent search

• We don’t know how the opponent will act
  – The solution is not a fixed sequence of actions from start state to goal state, but a strategy or policy (a mapping from state to best move in that state)

• Efficiency is critical to playing well
  – The time to make a move is limited
  – The branching factor, search depth, and number of terminal configurations are huge
    • In chess, branching factor $\approx 35$ and depth $\approx 100$, giving a search tree of $10^{154}$ nodes
  – This rules out searching all the way to the end of the game
Game tree

- A game of tic-tac-toe between two players, “max” and “min”
A more abstract game tree

Terminal utilities (for MAX)

A two-plex game
A more abstract game tree

- **Minimax value of a node**: the utility (for MAX) of being in the corresponding state, assuming perfect play on both sides.
- **Minimax strategy**: Choose the move that gives the best worst-case payoff.
Computing the minimax value of a state

- **Minimax**(state) =
  - Utility(state) if state is terminal
  - max **Minimax**(successors(state)) if player = MAX
  - min **Minimax**(successors(state)) if player = MIN
Computing the minimax value of a state

The minimax strategy is optimal against an optimal opponent

- If the opponent is sub-optimal, the utility can only be higher
- A different strategy may work better for a sub-optimal opponent, but it will necessarily be worse against an optimal opponent
More general games

- More than two players, non-zero-sum
- Utilities are now tuples
- Each player maximizes their own utility at each node
- Utilities get propagated (*backed up*) from children to parents
Alpha-beta pruning

- It is possible to compute the exact minimax decision without expanding every node in the game tree.

```
MAX

MIN

3 12 8 2 4 6 14 5 2
```
Alpha-beta pruning

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Alpha-beta pruning

- $\alpha$ is the value of the best choice for the MAX player found so far at any choice point above $n$
- We want to compute the MIN-value at $n$
- As we loop over $n$'s children, the MIN-value decreases
- If it drops below $\alpha$, MAX will never take this branch, so we can ignore $n$'s remaining children
- Analogously, $\beta$ is the value of the lowest-utility choice found so far for the MIN player
Alpha-beta pruning

• Pruning does not affect final result
• Amount of pruning depends on move ordering
  – Should start with the “best” moves (highest-value for MAX or lowest-value for MIN)
  – For chess, can try captures first, then threats, then forward moves, then backward moves
  – Can also try to remember “killer moves” from other branches of the tree
• With perfect ordering, branching factor can be cut in two, or depth of search effectively doubled
**Evaluation function**

- Cut off search at a certain depth and compute the value of an **evaluation function** for a state instead of its minimax value
  - The evaluation function may be thought of as the probability of winning from a given state or the *expected value* of that state
- A common evaluation function is a weighted sum of *features*:
  $$\text{Eval}(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$
  - For chess, $w_k$ may be the *material value* of a piece (pawn = 1, knight = 3, rook = 5, queen = 9) and $f_k(s)$ may be the advantage in terms of that piece
- Evaluation functions may be learned from game databases or by having the program play many games against itself
Cutting off search

- **Horizon effect**: you may incorrectly estimate the value of a state by overlooking an event that is just beyond the depth limit
  - For example, a damaging move by the opponent that can be delayed but not avoided

- **Possible remedies**
  - **Quiescence search**: do not cut off search at positions that are unstable – for example, are you about to lose an important piece?
  - **Singular extension**: a strong move that should be tried when the normal depth limit is reached
Additional techniques

- **Transposition table** to store previously expanded states
- **Forward pruning** to avoid considering all possible moves
- **Lookup tables** for opening moves and endgames
Chess playing systems

• Baseline system: 200 million node evalutions per move (3 min), minimax with a decent evaluation function and quiescence search
  – 5-ply \(\approx\) human novice

• Add alpha-beta pruning
  – 10-ply \(\approx\) typical PC, experienced player

• Deep Blue: 30 billion evaluations per move, singular extensions, evaluation function with 8000 features, large databases of opening and endgame moves
  – 14-ply \(\approx\) Garry Kasparov

• Recent state of the art (Hydra): 36 billion evaluations per second, advanced pruning techniques
  – 18-ply \(\approx\) better than any human alive?
Games of chance

• How to incorporate dice throwing into the game tree?
Games of chance

MAX

CHANCE

MIN

CHANCE

MAX

TERMINAL
Games of chance

• **Expectiminimax:** for chance nodes, average values weighted by the probability of each outcome
  – Nasty branching factor, defining evaluation functions and pruning algorithms more difficult

• **Monte Carlo simulation:** when you get to a chance node, simulate a large number of games with random dice rolls and use win percentage as evaluation function
  – Can work well for games like Backgammon
Partially observable games

• Card games like bridge and poker
• Monte Carlo simulation: deal all the cards randomly in the beginning and pretend the game is fully observable
  – “Averaging over clairvoyance”
  – Problem: this strategy does not account for bluffing, information gathering, etc.
Game playing algorithms today

- Computers are better than humans
  - **Checkers:** solved in 2007
  - **Chess:** IBM Deep Blue defeated Kasparov in 1997

- Computers are competitive with top human players
  - **Backgammon:** **TD-Gammon system** used reinforcement learning to learn a good evaluation function
  - **Bridge:** top systems use Monte Carlo simulation and alpha-beta search

- Computers are not competitive
  - **Go:** branching factor 361. Existing systems use Monte Carlo simulation and pattern databases
Origins of game playing algorithms

- Ernst Zermelo (1912): Minimax algorithm
- Claude Shannon (1949): chess playing with evaluation function, quiescence search, selective search (paper)
- John McCarthy (1956): Alpha-beta search
- Arthur Samuel (1956): checkers program that learns its own evaluation function by playing itself thousands of times