CYK Algorithm: More Details

$S \to AB$
$A \to AA \mid AB \mid a$
$B \to CC$
$C \to b$
$x = aaabb \ (n = 5)$

Start by filling in the "top" row in the table, with the variables that directly produce the terminal symbol at the corresponding position.

	a	a	a	b	b
j ↓ \ i →	1	2	3	4	5
1	A	A	A	C	C
2					
3					
4					
5					

E.g. the single symbol at position 5 in the string is directly produced by variable C, so we put a C in the entry at column 5, row 1.

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

Fill in subsequent rows by looking at the cells directly above and the cells diagonally to the upper right. (How to do this isn't very clear using row 2 as an example, so I'll wait for row 3 to go over it in more detail.)

	a	a	a	b	b
j ↓ \ i →	1	2	3	4	5
1	A	A	A	C	C
2	A	A	$\emptyset$	B	
3					
4					
5					

CYK Algorithm: More Details

$S \rightarrow AB$
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$x = aaabb \ (n = 5)$

	a	a	a	b	b
	i → 1	2	3	4	5
j ↓ 1	A	A	A	C	C
2	A	A	$\emptyset$	B	
3					
4					
5					

Say we want to fill in this cell next.

CYK Algorithm: More Details

$S \to AB$
$A \to AA \mid AB \mid a$
$B \to CC$
$C \to b$
$x = aaabb \ (n = 5)$

We want to fill the red cell with the set of variables that produce the substring with length j that starts at position i (in this case, we want variables that produce aaa).

	a	a	a	b	b
	i → 1	2	3	4	5
j ↓ 1	A	A	A	C	C
2	A	A	$\emptyset$	B	
3					
4					
5					

CYK Algorithm: More Details

$S \to AB$
$A \to AA \mid AB \mid a$
$B \to CC$
$C \to b$
$x = aaabb \ (n = 5)$

We are going to look for ways to produce a substring of length 3 starting at position 1 by concatenating two shorter strings.
We know that the first of these shorter strings must start at the same position as the new, longer substring.

	a	a	a	b	b
	1	2	3	4	5
1	A	A	A	C	C
2	A	A	$\emptyset$	B	
3					
4					
5					

Substrings starting at position 1 are produced by variables in this column.

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

We know that, in order to have a substring of length 3, we need to choose the second of our two strings in such a way that the length of the combined string is 3.

	a	a	a	b	b
	i → 1	2	3	4	5
j ↓ 1	$\boxed{A}$	A	A	C	C
2	A	$\boxed{A}$	$\emptyset$	B	
3					
4					
5					

A string produced by a variable in cell 1,1 has a length of 1.

So, to get a string of length 3, we need to concatenate the first part with a second part that starts at position 2 and is of length 2. Such strings are produced by variables in cell 2,2 of the table.

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

So, now we know that we can get a substring of length 3 starting at position 1 by concatenating two strings that are produced by variable A. So, we need to look for productions that produce two A's concatenated together.

	a	a	a	b	b
j ↓ \ i →	1	2	3	4	5
1	$\boxed{A}$	A	A	C	C
2	A	$\boxed{A}$	$\emptyset$	B	
3					
4					
5					

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow \boxed{AA} \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

In this case, one such production exists: $A \rightarrow AA$. We will therefore add the "producer" variable to the list of variables in cell 1,3.

	a	a	a	b	b
j ↓ \ i →	1	2	3	4	5
1	$\boxed{A}$	A	A	C	C
2	A	$\boxed{A}$	$\emptyset$	B	
3	A				
4					
5					

CYK Algorithm: More Details

$S \to AB$
$A \to AA \mid AB \mid a$
$B \to CC$
$C \to b$
$x = aaabb \ (n = 5)$

We can continue checking all of the possible ways to produce strings of length 3 starting at position 1. However, in this case, we're just looking at two A's again so it's not very interesting.

	a	a	a	b	b
j ↓ \ i →	1	2	3	4	5
1	A	A	$\boxed{A}$	C	C
2	$\boxed{A}$	A	$\emptyset$	B	
3	A				
4					
5					

A string produced by a variable in cell 1,2 has a length of 2.

So, to get a string of length 3, we need a string that starts at position 3 and is of length 1. Such strings are produced by variables in cell 3,1 of the table.

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

Let's do the same exercise for the next cell in the table.

	a	a	a	b	b
j ↓ / i →	1	2	3	4	5
1	A	A	A	C	C
2	A	A	$\emptyset$	B	
3	A				
4					
5					

CYK Algorithm: More Details

$S \to AB$
$A \to AA \mid AB \mid a$
$B \to CC$
$C \to b$
$x = aaabb \ (n = 5)$

In this case, we want strings starting at position **2** of length 3.

	a	a	a	b	b
	i → 1	2	3	4	5
j ↓ 1	A	$\boxed{A}$	A	C	C
2	A	A	$\boxed{\emptyset}$	B	
3	A				
4					
5					

A string produced by a variable in cell 2,1 starts at position 2 and has length 1.

So, to get a string of length 3, we need a string that starts at position 3 and is of length 2. Such strings are produced by variables in cell 3,2 of the table.

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

No possible productions produce a variable in cell 2,1 followed by a variable in cell 3,2, simply because cell 3,2 doesn't contain any variables.

	a	a	a	b	b
j ↓ \ i →	1	2	3	4	5
1	A	$\boxed{A}$	A	C	C
2	A	A	$\boxed{\emptyset}$	B	
3	A				
4					
5					

CYK Algorithm: More Details

$S \to AB$
$A \to AA \mid AB \mid a$
$B \to CC$
$C \to b$
$x = aaabb \ (n = 5)$

Now, we are looking for productions that produce an A followed by a C. However, there aren't any.

	a	a	a	b	b
j ↓ \ i →	1	2	3	4	5
1	A	A	A	$\boxed{C}$	C
2	A	$\boxed{A}$	$\emptyset$	B	
3	A				
4					
5					

A string produced by a variable in cell 2,2 starts at position 2 and has length 2.

So, to get a string of length 3, we need a string that starts at position 4 and is of length 1. Such strings are produced by variables in cell 4,1 of the table.

CYK Algorithm: More Details

$S \to AB$
$A \to AA \mid AB \mid a$
$B \to CC$
$C \to b$
$x = aaabb \ (n = 5)$

We weren't able to find any productions for this cell, so we'll just indicate that the "set of variables producing substrings of length 3 starting at position 2" is empty.

	a	a	a	b	b
	i → 1	2	3	4	5
j ↓ 1	A	A	A	C	C
2	A	A	$\emptyset$	B	
3	A	$\emptyset$			
4					
5					

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

Following the same pattern, for cell 3,3, we'll first look for productions that produce an A followed by a B.

	a	a	a	b	b
j ↓ / i →	1	2	3	4	5
1	A	A	$\boxed{A}$	C	C
2	A	A	$\emptyset$	$\boxed{B}$	
3	A	$\emptyset$			
4					
5					

CYK Algorithm: More Details

$S \rightarrow \boxed{AB}$
$A \rightarrow AA \mid \boxed{AB} \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

There are two variables that produce this: S, and A. So, we'll put both of these variables into the set in cell 3,3.

	a	a	a	b	b
j ↓ / i →	1	2	3	4	5
1	A	A	$\boxed{A}$	C	C
2	A	A	$\emptyset$	$\boxed{B}$	
3	A	$\emptyset$	S,A		
4					
5					

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

The next entries we check once again involves an empty cell, so we don't add anything else to cell 3,3.

	a	a	a	b	b
j ↓ / i →	1	2	3	4	5
1	A	A	A	C	$\boxed{C}$
2	A	A	$\boxed{\emptyset}$	B	
3	A	$\emptyset$	S,A		
4					
5					

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

Now, we'll move on to cell 4,1, corresponding to substrings of length 4 starting at position 1.

	a	a	a	b	b
	i → 1	2	3	4	5
j ↓ 1	A	A	A	C	C
2	A	A	$\emptyset$	B	
3	A	$\emptyset$	S,A		
4					
5					

CYK Algorithm: More Details

$S \to AB$
$A \to AA \mid AB \mid a$
$B \to CC$
$C \to b$
$x = aaabb \ (n = 5)$

First, we'll check these two cells, one of which is empty so we clearly won't find any productions for this combination.

	a	a	a	b	b
	i → 1	2	3	4	5
j ↓ 1	$\boxed{A}$	A	A	C	C
2	A	A	$\emptyset$	B	
3	A	$\boxed{\emptyset}$	S,A		
4					
5					

Reminder: produces substrings of length 1 starting at position 1.

Produces substrings of length 3 starting at position 2 (basically this is saying that no variable in this grammar can produce the string aab.)

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

Next, there's another production involving an empty set, so we won't find anything here.

	a	a	a	b	b
	i →				
	1	2	3	4	5
j ↓ 1	A	A	A	C	C
2	$\boxed{A}$	A	$\boxed{\emptyset}$	B	
3	A	$\emptyset$	S,A		
4					
5					

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

Finally, we'll check for productions that produce an A followed by a C, and there aren't any.

	a	a	a	b	b
	i → 1	2	3	4	5
j ↓ 1	A	A	A	$\boxed{C}$	C
2	A	A	$\emptyset$	B	
3	$\boxed{A}$	$\emptyset$	S,A		
4					
5					

CYK Algorithm: More Details

So, cell 1,4 is empty.

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

	a	a	a	b	b
	i → 1	2	3	4	5
j ↓ 1	A	A	A	C	C
2	A	A	$\emptyset$	B	
3	A	$\emptyset$	S,A		
4	$\emptyset$				
5					

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

Next, we'll check cell 2,4. The first combination we check has two possible variables corresponding to the second part of the substring, so we'll look for productions of AS and AA (using both possible variables in the second cell).

	a	a	a	b	b
	i →				
j ↓	1	2	3	4	5
1	A	$\boxed{A}$	A	C	C
2	A	A	$\emptyset$	B	
3	A	$\emptyset$	$\boxed{S,A}$		
4	$\emptyset$				
5					

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow \boxed{AA} \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

Nothing produces an S, however A produces AA, so we can add A to cell 2,4.

	a	a	a	b	b
	i →				
j ↓	1	2	3	4	5
1	A	$\boxed{A}$	A	C	C
2	A	A	$\emptyset$	B	
3	A	$\emptyset$	$\boxed{S,A}$		
4	$\emptyset$	A			
5					

CYK Algorithm: More Details

$S \rightarrow \boxed{AB}$
$A \rightarrow AA \mid \boxed{AB} \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

Next, we'll look for productions that produce AB, and we see that S and A produce AB. Cell 2,4 already contains A, so we only need to add the S.

	a	a	a	b	b
j ↓ / i →	1	2	3	4	5
1	A	A	A	C	C
2	A	$\boxed{A}$	$\emptyset$	$\boxed{B}$	
3	A	$\emptyset$	S,A		
4	$\emptyset$	S,A			
5					

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

Finally, we know we won't get any new productions from these two cells, so we're done with cell 2,4.

	a	a	a	b	b
	i → 1	2	3	4	5
j ↓ 1	A	A	A	C	$\boxed{C}$
2	A	A	$\emptyset$	B	
3	A	$\boxed{\emptyset}$	S,A		
4	$\emptyset$	S,A			
5					

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

Finally, we need to fill in the last cell, corresponding to a string starting at position 1 of length 5. (In other words, the entire string.) We'll just work through the possibilities like before.

	a	a	a	b	b
	i →				
	1	2	3	4	5
j ↓ 1	A	A	A	C	C
2	A	A	$\emptyset$	B	
3	A	$\emptyset$	S,A		
4	$\emptyset$	S,A			
5					

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow \boxed{AA} \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

Once again, we're looking for productions that produce either AS or AA. A produces AA, so add it to cell 1,5.

	a	a	a	b	b
	i → 1	2	3	4	5
j ↓ 1	$\boxed{A}$	A	A	C	C
2	A	A	$\emptyset$	B	
3	A	$\emptyset$	S,A		
4	$\emptyset$	$\boxed{S,A}$			
5	A				

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow \boxed{AA} \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

In this example, we ended up already considering the next combination, so we've already added the variables we need to.

	a	a	a	b	b
	i → 1	2	3	4	5
j ↓ 1	A	A	A	C	C
2	$\boxed{A}$	A	$\emptyset$	B	
3	A	$\emptyset$	$\boxed{S,A}$		
4	$\emptyset$	S,A			
5	A				

CYK Algorithm: More Details

$S \rightarrow \boxed{AB}$
$A \rightarrow AA \mid \boxed{AB} \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

We next look for productions producing AB. We already added A to cell 1,5, so we only need to add the S.

	a	a	a	b	b
	i → 1	2	3	4	5
j ↓ 1	A	A	A	C	C
2	A	A	$\emptyset$	$\boxed{B}$	
3	$\boxed{A}$	$\emptyset$	S,A		
4	$\emptyset$	S,A			
5	S,A				

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

And finally, we have a combination involving an empty cell—no productions from this.

	a	a	a	b	b
j ↓ \ i →	1	2	3	4	5
1	A	A	A	C	$\boxed{C}$
2	A	A	$\emptyset$	B	
3	A	$\emptyset$	S,A		
4	$\boxed{\emptyset}$	S,A			
5	S,A				

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

So now we're finished filling out the table. To know if the string is in the language produced by the CFG, we only need to see if the start symbol is in the last cell.

	a	a	a	b	b
	i →				
	1	2	3	4	5
j ↓ 1	A	A	A	C	C
2	A	A	$\emptyset$	B	
3	A	$\emptyset$	S,A		
4	$\emptyset$	S,A			
5	S,A				

CYK Algorithm: More Details

$S \rightarrow AB$
$A \rightarrow AA \mid AB \mid a$
$B \rightarrow CC$
$C \rightarrow b$
$x = aaabb \ (n = 5)$

In this case, cell 1,5 does contain the start symbol S, so the string $aaabb$ is in the language produced by the grammar.

	a	a	a	b	b
	i → 1	2	3	4	5
j ↓ 1	A	A	A	C	C
2	A	A	$\emptyset$	B	
3	A	$\emptyset$	S,A		
4	$\emptyset$	S,A			
5	$\boxed{S}$,A				

Recall that this is saying that the start symbol is able to produce the "substring" with a length of 5 starting at position 1.