1.

Prove or disprove whether the following languages are regular. To prove that a language is regular, it is sufficient to construct a DFA or regular expression accepting the language.

a) [10 points] The set of all strings over \{0, 1\} that do not have three consecutive 1s.

b) [10 points] \( \{ x \mid x \in (0 + 1)^* \text{ and } x = x^R \} \)

c) [10 points] \( \{ xx^R \mid x \in (010)^* \} \)

d) [10 points] \( \{ 0^i \mid i \text{ is a power of 3} \} \). **Hint:** Try to come up with a pumped string with a length that is strictly between two consecutive powers of 3. This is similar to what we did in the example in class, on slide 12 of the “Properties of Regular Languages” presentation.

2. [20 points] If \( L \) is a language and \( a \) is a symbol, then \( L/a \) is the set of strings \( w \) such that \( wa \) is in \( L \). For example, if \( L = \{ a, cba, aab, baa \} \), then \( L/a = \{ \epsilon, cb, ba \} \). Prove that if \( L \) is regular, then \( L/a \) is also regular. **Hint:** Start with a DFA for \( L \) and consider the set of accepting states.

3. Let \( h \) be the homomorphism from \( \Sigma \rightarrow \Delta^* \), where \( \Sigma = \{ a, b \} \) and \( \Delta = \{ 0, 1 \} \):

   - \( h(a) = 01 \)
   - \( h(b) = 0 \)

   a) [10 points] Write a regular expression accepting \( h(L_1) \), where \( L_1 = b^*(a + ba)^* \).

   b) [15 points] Find \( h^{-1}(L_2) \), where \( L_2 = (10 + 1)^* \). Since \( L_2 \) is a regular language, you can resort to constructing a DFA for \( h^{-1}(L_2) \) if you have to.

   c) [15 points] Find \( h^{-1}(L_3) \), where \( L_3 \) is the set of all strings with an equal number of 0s and 1s. As shown in class, \( L_3 \) is not regular, so you will not be able to use the DFA construction to figure out \( h^{-1}(L_3) \). Instead, you will need to reason about the various possible cases like we did on slide 31 of the “Properties of Regular Languages” presentation.