**COMP 455-002: Homework 5**

Due: April 29, 2019, By 10:00 AM, in my mailbox (my mailbox is outside SN107)

1. [20 points] Write a TM that increments a binary number $N$. Assume that the tape initially contains the number $N$ in binary, where $N$ is written with the most-significant bit on the left (note: this means the tape head will start by pointing at the most significant bit). Your TM should halt with $N + 1$, in binary, on its tape (for this question, it doesn’t matter if it accepts or not). For example, if the TM starts with the ID $q_0101$ (5 in binary), it should end with the ID $q_j110$ (6 in binary), for some state $q_j$. Give the transition function, $\delta$, of your TM as a transition diagram.

2. [20 points] Consider the following automaton, called a “queue machine”. A queue machine is similar to a PDA but instead of using a stack, it uses a (FIFO) queue to store symbols. Formally, $M = (Q, \Sigma, T, \delta, q_0, F)$. $Q$, $\Sigma$, and $F$ are the same as they are in a PDA, and $T$ is the queue alphabet (basically the same as a PDA’s stack alphabet $\Gamma$).

The transition function of the queue machine is $\delta : Q \times \Sigma \times T \rightarrow 2^{Q \times T}$. It depends on a current state, current input symbol, and the symbol at the head of the queue, and returns a symbol (or a string of symbols) to insert at the end of the queue. While transitioning, it removes the symbol at the head of the queue.

Describe in detail how this queue machine can simulate a TM. Your explanation must at least include how to read the symbol at the “tape head” of the simulated TM, and how to carry out left and right moves of the tape head.

3. [20 points] Which of the following properties of an arbitrary Turing Machine $M$ are algorithmically decidable? Justify your answer for each:
   i) $M$ has more than two states.
   ii) $M$ accepts the string 00.
   iii) $M$ uses more than 100 squares on its tape when started with a blank tape.

4. [20 points] Show that the Halting Problem is RE and not recursive. Use the definition of the Halting Problem from the “Halting Problem” box in the textbook near the end of Section 9.2.4.

5. [20 points] (Exercise 10.4.4a from the textbook.) Show that the subgraph isomorphism problem is NP-complete. The subgraph isomorphism problem is as follows:

   Given two graphs $G_1$ and $G_2$, does $G_1$ contain $G_2$ as a subgraph? That is, can we find a subset of the nodes of $G_1$ that, together with the edges among them in $G_1$, forms an exact copy of $G_2$? (Hint: Consider a reduction from the clique problem.)