COMP 181
Models of Languages and Computation
Spring 2001
Mid Semester Exam
Monday, Feb. 26, 2001
Closed Book - Closed Notes
Don't forget to write your name or ID and pledge on the exam sheet.

This exam has four pages.

1. (5 points)

a) Suppose R is the relation $\{(1,3),(2,3)\}$ and S is the relation $\{(3,4),(3,5)\}$. What is the relation $R \circ S$ obtained by composing R and S? List all ordered pairs in the relation.

{(1,4),(1,5),(2,4),(2,5)}

b) Suppose that R and S are arbitrary relations having exactly four ordered pairs. (In part (a), both relations have two ordered pairs.) What is the *largest* possible number of ordered pairs in the relation $R \circ S$?

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2. (5 points) If R is a relation, let R^T be R with all ordered pairs reversed. Thus if R is $\{(0,1),(0,4)\}$ then R^T is $\{(1,0),(4,0)\}$. True or false:

a) If a relation R is symmetric, then R^T is always symmetric. T

b) If a relation R is reflexive, then R^T is always reflexive. T

c) If a relation R is transitive, then R^T is always transitive. T

3. Consider the following regular expressions:

a) $(0^*10^*10^*)^*(1^*0)$

b) $0^* \cup (0^*10^*10^*)^*$

c) $((0 \cup 1)(0 \cup 1))^*$

d) $0^*10^*10^*$

e) $((0 \cup 1)(0 \cup 1))^*(0 \cup 1)$

Which of these represent the following sets of strings?

A) The set of binary strings containing exactly two ones. D

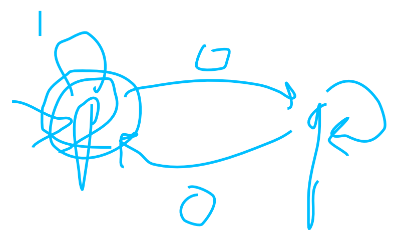
B) The set of binary strings of even length. C

C) The set of binary strings of odd length. E

D) The set of binary strings containing an even number of ones. B

E) The set of binary strings containing an odd number of ones. None

4. (10 points) Consider the following sets of strings:



a) $\{x \in \{0,1\}^* : x \text{ has an even number of zeroes}\}$

b) $\{x \in \{0,1\}^* : x \text{ has an odd number of zeroes}\}$

c) $\{x \in \{0,1\}^* : x \text{ has even length}\}$

d) $\{x \in \{0,1\}^* : x \text{ has odd length}\}$

For each of the following finite automata M, state which set of strings above is $L(M)$:

A)

B)

C)

D)

5. (10 points) Consider the following nondeterministic finite automaton M:

Which of the following automata are *both* deterministic *and* equivalent to M?

a)

b)

c)

d)

6. (10 points) Consider the following proof:

Theorem. Suppose M is a nondeterministic finite automaton. Let M' be identical to M except that the accepting and non-accepting states of M have been switched; that is, a state q is an accepting state of M' exactly when q is not an accepting state of M. Let Σ be the input alphabet of M and M'. Then $L(M') = \Sigma^* - L(M)$.

Proof. Consider a word $x \in \Sigma^*$. Suppose M accepts x. Then there is a computation starting from the start state of M, leading to an accepting state r of M. The same computation will lead to the state r of M', but r is not an accepting state of M'. Therefore M' does not accept x.

Similarly, if M does not accept x, then there is a computation leading from the start state of M to a non-accepting state t of M. Since t is a non-accepting state of M, t is an accepting state of M'. Therefore M' accepts x.

Therefore, M' accepts x exactly when M does not accept x, so $L(M') = \Sigma^* - L(M)$.

Is this proof correct? If so, say why. If not, say why not.

No. May be more than one computation path.

7.) Suppose M_1 and M_2 are deterministic finite automata. Is there always a deterministic finite automaton M such that $L(M) = L(M_1) - L(M_2)$, that is, M accepts a word exactly when M_1 accepts the word *and* M_2 does not accept the word. Justify your answer briefly.

Yes intersect L(M1) with complement of L(M2)