Comp181 Spring 2002

Additional Context-Free Languages Exercises

1) Given the following languages, show that they are context-free by constructing context-free grammars that generate them:
   a. $\{ ab^ncd^nf \}$
   b. $\{ a^nb^mc^p : n \leq m+p \}$
   c. $\{ wc^*w^R : w \in \{a,b\}^* \}$
   d. $\{ \{a,b\}^*$: the number of a's = the number of b's $\}$

2) Given the following grammar:

   V = { a,b,c,(,),+,*,S,T }
   Σ = { a,b,c,(,),+,* }
   R = {
   S → T + S | T
   T → T * T | (S)
   T → a | b | c
   }

   a. show a derivation for:
      i. a*(b+c)
      ii. a+(b*c)
      iii. ((a+b)*(b+c))

   b. give a parse tree for each of the above

3) Given the following grammar:

   V = { Sentence, Subject, Predicate, Noun, Verb, Object, SubordinateClause, Adjective, her, I, duck, saw}
   Σ = { saw, duck, I, her }
   R = {
   Sentence → Subject Predicate
   Subject → Noun
   Predicate → Verb Object | Verb Object SubordinateClause
   Object → Adjective Noun | Noun
   SubordinateClause → Verb
   Adjective → her
   Noun → I, her, duck
   Verb → saw, duck
   }

   Show that the statement "I saw her duck" is ambiguous by constructing two non-equivalent parse trees.

4) Construct a PDA that recognizes the following grammars:
   a. $\{ \{a,b\}^* :$ the number of b's = the number of a's $\}$
   b. $\{ \{a,b\}^* :$ the number of a's $\neq$ the number of b's $\}$

5) Give an intuitive description of the following grammars, and construct a PDA that recognizes it:

   a.

   $V = \{ S, A, B, a, b \}$
   $\Sigma = \{ a,b \}$
   $R = \{$
   $S \rightarrow \varepsilon$
   $S \rightarrow ASB$
   $A \rightarrow a$
   $B \rightarrow b$
   $\}$

   b.

   $V = \{ S, A, B, a, b \}$
   $\Sigma = \{ a,b \}$
   $R = \{$
   $S \rightarrow \varepsilon$
   $S \rightarrow SASBS$
   $S \rightarrow SBSAS$
   $A \rightarrow a$
   $B \rightarrow b$
   $\}$

   c.

   $V = \{ S, S_1, S_2, A, B, a, b \}$
   $\Sigma = \{ a,b \}$
   $R = \{$
   $S \rightarrow \varepsilon$
   $S \rightarrow S_1$
   $S \rightarrow S_2$
   $S_1 \rightarrow \varepsilon$
   $S_1 \rightarrow AS_1B$
   $S_2 \rightarrow \varepsilon$
   $S_2 \rightarrow S_2AS_2BS_2$
   $S_2 \rightarrow S_2BS_2AS_2$
   $A \rightarrow a$
   $B \rightarrow b$
   $\}$

d.

$V = \{ S, A, a, b \}$

$\Sigma = \{ a,b \}$

$R = \{$

$S \rightarrow \varepsilon$

$S \rightarrow ASb$

$A \rightarrow a \mid aa$

$\}$

6) Use the pumping lemma for context free grammars to show that the following is not a context free grammar:

$\{ \{a,b\}^n\{c,d\}^n$: the number of a's = the number of c's $\}$