COMP 455
Models of Languages and Computation
Spring 2012
The Pumping Lemma for Regular Languages

We illustrate the pumping lemma as a game. Suppose we are trying to show that a language L is not regular. The game is as follows:

The opponent chooses an integer n.

You choose a word w in L of length n or greater.

The opponent expresses w as xyz where x, y, and z are strings, y is not e, and $|xy| \leq n$.

You choose an integer i.

If the word xy^iz is in L, the opponent wins. If the word xy^iz is not in L, you win.

If you have a winning strategy in this game, then L is not regular.

If the opponent has a winning strategy, then L may or may not be regular.

Let's apply this to the language $\{a^nb^n : n \geq 0\}$. Suppose the opponent chooses $n = 3$. You choose $aaabbb$. The opponent chooses $x = a$, $y = aa$, and $z = bbb$.

If you choose $i = 1$, then the word xy^iz is in L and you lose. If you choose any other i, then the word xy^iz is not in L, and you win.

In order to show that L is not regular, you need to have a winning strategy, which tells you what moves to make in all games. Here is your strategy:

If the opponent chooses n, you choose the string a^nb^n. After the opponent chooses x, y, and z, you choose $i = 2$. This is a strategy because it tells what moves for you to make in all games. It is a winning strategy for you, because in all cases, xy^iz is not in L, as shown in the book, for example. If y contains only one letter, then xy^iz does not have the same number of a's and b's and is therefore not in L. If y contains two letters, then xy^iz contains a b before an a and is therefore not in l.

Thus L is not regular.