COMP 455
Models of Languages and Computation
Spring 2012
How does a Push-Down Automaton Work?

Suppose $((p, a, \beta), (q, \gamma))$ is a transition of a pda. The purpose of this handout is to describe how such a transition can fire. This depends on whether a, β, and γ are empty or not, making eight cases in all. Note that if β is not empty, then it can be written as $a_1 a_2 \ldots a_m$ for some $m \geq 1$, and if γ is not empty, then it can be written as $b_1 b_2 \ldots b_n$ for some $n \geq 1$, where the a_i and b_j are symbols from the stack alphabet. Each case is described as follows.

The transition $((p, a, a_1 a_2 \ldots a_m), (q, b_1 b_2 \ldots b_n))$, where a is not empty and $m, n \geq 1$, can fire if p is the current state and a is the input symbol being read and $a_1 a_2 \ldots a_m$ are the symbols on top of the stack. If this transition fires, then the state becomes q, the read head moves to the next symbol of the input, $a_1 a_2 \ldots a_m$ are removed from the top of the stack, and $b_1 b_2 \ldots b_n$ are put on the top of the stack.

The transition $((p, e, a_1 a_2 \ldots a_m), (q, b_1 b_2 \ldots b_n))$, where $m, n \geq 1$, can fire if p is the current state and $a_1 a_2 \ldots a_m$ are the symbols on top of the stack. If this transition fires, then the state becomes q, the read head does not move to the next symbol of the input, $a_1 a_2 \ldots a_m$ are removed from the top of the stack, and $b_1 b_2 \ldots b_n$ are put on the top of the stack.

The transition $((p, a, e), (q, b_1 b_2 \ldots b_n))$, where a is not empty and $n \geq 1$, can fire if p is the current state and a is the input symbol being read. If this transition fires, then the state becomes q, the read head moves to the next symbol of the input, and $b_1 b_2 \ldots b_n$ are put on the top of the stack.

The transition $((p, e, e), (q, b_1 b_2 \ldots b_n))$, where $n \geq 1$, can always fire if p is the current state. If this transition fires, then the state becomes q, the read head does not move to the next symbol of the input, and $b_1 b_2 \ldots b_n$ are put on the top of the stack.

The transition $((p, a, a_1 a_2 \ldots a_m), (q, e))$, where a is not empty and $m \geq 1$, can fire if p is the current state and a is the input symbol being read and $a_1 a_2 \ldots a_m$ are the symbols on top of the stack. If this transition fires, then the state becomes q, the read head moves to the next symbol of the input, and $a_1 a_2 \ldots a_m$ are removed from the top of the stack.

The transition $((p, e, a_1 a_2 \ldots a_m), (q, e))$, where $m \geq 1$, can fire if p is the current state and $a_1 a_2 \ldots a_m$ are the symbols on top of the stack. If this transition fires, then the state becomes q, the read head does not move to the next symbol of the input, and $a_1 a_2 \ldots a_m$ are removed from the top of the stack.

The transition $((p, a, e), (q, e))$, where a is not empty, can fire if p is the current state and a is the input symbol being read. If this transition fires, then the state becomes q and the read head moves to the next symbol of the input.

The transition $((p, e, e), (q, e))$, can always fire if p is the current state. If this transition fires, then the state becomes q and the read head does not move to the next symbol of the input.