COMP 455
Models of Languages and Computation
Spring 2012
The Pumping Theorem for Context-Free Languages Made "Simple"

Here is a simpler form for the pumping theorem, Theorem 3.5.3:

If L is a context-free language, then there is an integer N such that any string $w \in L$ of length larger than N can be written as $uvxyz$ such that ($v \neq e$ or $y \neq e$) and $uv^ixy^iz \in L$ for all $i \geq 0$.

This can be used to show that a language is *not* context-free as follows:

If L is a language and for all integers N, there is a string $w \in L$ of length greater than N such that for all ways of writing w as $uvxyz$ with ($v \neq e$ or $y \neq e$), there is an i such that uv^ixy^iz is not in L, then L is *not* context-free.

We illustrate the pumping lemma as a game. Suppose we are trying to show that a language L is not context-free. The game is as follows:

The opponent chooses an integer N.
You choose a word w in L of length larger than N.
The opponent expresses w as $uvxyz$ where u, v, x, y, and z are strings and v is not e or y is not e.
You choose an integer i.
If the word uv^ixy^iz is in L, the opponent wins.
If the word uv^ixy^iz is not in L, you win.

If you have a winning strategy in this game, then L is not context-free.
If the opponent has a winning strategy, then L may or may not be context-free.