COMP 181
Models of Languages and Computation
Spring 2001
Mid Semester Exam
Monday, Feb. 26, 2001
Closed Book - Closed Notes
Don't forget to write your name or ID and pledge on the exam sheet.

This exam has four pages.

1. (5 points)

a) Suppose R is the relation $\{(1,3),(2,3)\}$ and S is the relation $\{(3,4),(3,5)\}$. What is the relation $R \circ S$ obtained by composing R and S? List all ordered pairs in the relation.

b) Suppose that R and S are arbitrary relations having exactly four ordered pairs. (In part (a), both relations have two ordered pairs.) What is the *largest* possible number of ordered pairs in the relation $R \circ S$?

2. (5 points) If R is a relation, let R^T be R with all ordered pairs reversed. Thus if R is $\{(0,1),(0,4)\}$ then R^T is $\{(1,0),(4,0)\}$. True or false:

a) If a relation R is symmetric, then R^T is always symmetric.

b) If a relation R is reflexive, then R^T is always reflexive.

c) If a relation R is transitive, then R^T is always transitive.

3. Consider the following regular expressions:

a) $(0^*10^*10^*)^*(1^*0)$

b) $0^* \cup (0^*10^*10^*)^*$

c) $((0 \cup 1)(0 \cup 1))^*$

d) $0^*10^*10^*$

e) $((0 \cup 1)(0 \cup 1))^*(0 \cup 1)$

Which of these represent the following sets of strings?

A) The set of binary strings containing exactly two ones.

B) The set of binary strings of even length.

C) The set of binary strings of odd length.

D) The set of binary strings containing an even number of ones.

E) The set of binary strings containing an odd number of ones.

4. (10 points) Consider the following sets of strings:

a) $\{x \in \{0,1\}^* : x \text{ has an even number of zeroes}\}$

b) $\{x \in \{0,1\}^* : x \text{ has an odd number of zeroes}\}$

c) $\{x \in \{0,1\}^* : x \text{ has even length}\}$

d) $\{x \in \{0,1\}^* : x \text{ has odd length}\}$

For each of the following finite automata M, state which set of strings above is $L(M)$:

A)

B)

C)

D)

5. (10 points) Consider the following nondeterministic finite automaton M:

Which of the following automata are *both* deterministic *and* equivalent to M?

a)

b)

c)

d)

6. (10 points) Consider the following proof:

Theorem. Suppose M is a nondeterministic finite automaton. Let M' be identical to M except that the accepting and non-accepting states of M have been switched; that is, a state q is an accepting state of M' exactly when q is not an accepting state of M. Let Σ be the input alphabet of M and M'. Then $L(M') = \Sigma^* - L(M)$.

Proof. Consider a word $x \in \Sigma^*$. Suppose M accepts x. Then there is a computation starting from the start state of M, leading to an accepting state r of M. The same computation will lead to the state r of M', but r is not an accepting state of M'. Therefore M' does not accept x.

Similarly, if M does not accept x, then there is a computation leading from the start state of M to a non-accepting state t of M. Since t is a non-accepting state of M, t is an accepting state of M'. Therefore M' accepts x.

Therefore, M' accepts x exactly when M does not accept x, so $L(M') = \Sigma^* - L(M)$.

Is this proof correct? If so, say why. If not, say why not.

7.) Suppose M_1 and M_2 are deterministic finite automata. Is there always a deterministic finite automaton M such that $L(M) = L(M_1) - L(M_2)$, that is, M accepts a word exactly when M_1 accepts the word *and* M_2 does not accept the word. Justify your answer briefly.