COMP 181

Models of Languages and Computation

Fall 2005

Mid Semester Exam

Tuesday, Oct. 4, 2005

Closed Book - Closed Notes

Don't forget to write your name or ID and pledge on the exam sheet.

This exam has four pages.

1. (5 points)

a) Suppose R is the relation $\{(3,1),(3,2)\}$ and S is the relation $\{(2,4),(2,5)\}$. What is the relation $R \circ S$ obtained by composing R and S? List all ordered pairs in the relation.

b) Suppose that R is a set having exactly four elements and S is a set having exactly two elements. What is the number of ordered pairs in the Cartesian product $R \times S$?

2. (5 points) Suppose $L_1 = \{01, 10\}$ and $L_2 = \{11, 00\}$.

a) List all strings in $L_1 L_2$.

b) List all strings in $L_1 \cup L_2$.

c) Is the string 011010 in L_1^*?

d) Is the string 10 in L_1^*?

e) Is the string 0 in L_1^*?

3. (5 points) Consider the following regular expressions:

a) $((0 \cup 1)(0 \cup 1))^*$

b) $(0^*10^*1)^*(0^*10^*)$

c) $0^*10^*10^*$

d) $((0 \cup 1)(0 \cup 1))^*(0 \cup 1)$

e) $0^*(10^*10^*)^*$

Which of these represent the following sets of strings?

A) The set of binary strings of odd length.

B) The set of binary strings containing an even number of ones.

C) The set of binary strings containing exactly two ones.

D) The set of binary strings containing an odd number of ones.

E) The set of binary strings of even length.

4. (10 points) Consider the following sets of strings:

a) $\{x \in \{0,1\}^* : x \text{ has an even number of ones}\}$

b) $\{x \in \{0,1\}^* : x \text{ has an odd number of ones}\}$

c) $\{x \in \{0,1\}^* : x \text{ has even length}\}$

d) $\{x \in \{0,1\}^* : x \text{ has odd length}\}$

For each of the following finite automata M, state which set of strings above is $L(M)$:

A)

B)

C)

D)

5. (5 points) Give a regular expression for the set of binary numbers having at most two one's. Thus "001010" and "1000" are in the set but "0111" is not.

6. (5 points) True or false:

a) For every n-state nondeterministic finite automaton M with e arrows there is an equivalent n-state nondeterministic finite automaton M'' without e arrows.

b) Every finite automaton accepts at most a finite number of input stings.

c) For every n state nondeterministic finite automaton there is an equivalent deterministic finite automaton having at most $2n$ states.

d) For every n state nondeterministic finite automaton there is an equivalent deterministic finite automaton having at most 2^n states.

e) For every regular expression E there is a nondeterministic finite automaton M such that $\mathcal{L}(E) = L(M)$.

7. (10 points) Consider the following proof:

Theorem. Suppose $M = (K, \Sigma, \Delta, s, F)$ is a nondeterministic finite automaton having n states. Suppose M accepts at least one string w of length $n-1$ or more. Then M accepts an infinite number of strings.

Proof. Consider the sequence $q_0, q_1, q_2, \ldots,$ of states through which M passes as it accepts w. This sequence has at least $n+1$ entries in it, but M has only n states, so there must be i and j such that i and j are distinct and $q_i = q_j$. Suppose without loss of generality that $i < j$. Let w be split up into three strings x,y,z such that $w = xyz$ and when M reads x, it goes from the start state to state q_i, when M reads y, it goes from state q_i to state q_j, and when M reads z, it goes from state q_j to an accepting state. Because $q_i = q_j$, M will also accept the strings xy^2z, xy^3z, xy^4z, et cetera, which is an infinite set of strings in all.

a) Is the theorem correct? Justify your answer briefly.

b) Is the proof of the theorem correct? Justify your answer briefly.

8.) (5 points) Suppose M is a deterministic finite automaton over an input alphabet Σ. Is there always a deterministic finite automaton M_1 such that $L(M_1) = L(M) \cap A$ where A is the set of strings in Σ^* having even length? Justify your answer briefly. Recall that $L(M) \cap A = \{u : u \in L(M) \text{ and } u \in A\}$.