COMP 181
Models of Languages and Computation
Fall 2005
Mid Semester Exam
Tuesday, November 8, 2005
Closed Book - Closed Notes
This exam has four pages.
Don't forget to write your name or ID and pledge on the exam sheet.

1. (10 points) Consider the context free grammar $G = (V, \Sigma, R, S)$ where V is $\{S, A, B, a, b, c\}$, Σ is $\{a, b, c\}$, and R consists of the following rules:

$$S \to ASB \quad S \to BSA \quad S \to CSC \quad S \to c$$
$$A \to a \quad B \to b \quad C \to c$$

(a) Which of the following strings are in $L(G)$? Circle all that are in $L(G)$.

$$aaaacbb,\ abcab,\ cacbc,\ abacaba,\ ccccc$$

(b) Show parse trees for all strings that are in $L(G)$ in part (a).

2. (10 points) Consider the push-down automaton $M = (K, \Sigma, \Gamma, \Delta, s, F)$ where $K = \{s, f\}$, $\Sigma = \{a, b, c\}$, $\Gamma = \{a, b\}$, $F = \{f\}$, and Δ consists of the following transitions:

$$((s, a, e), (s, b)),\ ((s, b, e), (s, a)),\ ((s, c, e), (f, e)),\ ((f, a, b), (f, e)),\ ((f, b, a), (f, e)).$$

(a) Which of the following strings are in $L(M)$? Circle all that are.

abcab, *aca*, *aa*, *bbbacabbb*, *aabcabb*, *aaaabb*

(b) Describe the language accepted by M in simpler terms (that is, without reference to a push-down automaton).

3. (10 points) Consider the deterministic finite automaton M with states $\{q, r, s, t\}$, input alphabet $\{0, 1\}$, start state q, accepting states q and s, and with the following transitions:

Construct an equivalent minimal deterministic finite automaton.

For each multiple choice question, choose the best answer.

4. (4 points) The language $\{a^n b a^m : n \leq m\}$ is

a) finite
b) regular
c) context-free but not regular
d) not context free

5. (4 points) Suppose L is a context-free language. Then

a) There is a deterministic finite automaton M such that $L = L(M)$
b) There is a nondeterministic finite automaton M such that $L = L(M)$
c) There is a (nondeterministic) push-down automaton M such that $L = L(M)$
d) L is a finite set.

6. (4 points) Suppose L is the language represented by the regular expression $(ab)^*$. True or false:

a) $a \approx_L aba$
b) $a \approx_L bbb$
c) $b \approx_L bab$
d) $babab \approx_L b$

7. (10 points) Consider the context free grammar $G = (V, \Sigma, R, S)$ where V is $\{S, A, a, b\}$, Σ is $\{a, b\}$, and R consists of the following rules:

$$S \to AB \quad B \to A \quad A \to S$$
$$S \to a \quad S \to b$$

Is this grammar ambiguous? Justify your answer.

8. (10 points) Consider the language $L = \{(aa)^k c(bb)^m : k, m \geq 0\}$. This language contains the strings c, $aacbb$, $aaaacbb$, et cetera. Consider the following theorem and proof:

Theorem: L is regular.

Proof: We show that in the regular expression game, A (the opponent) can always win. Suppose A picks the integer $n = 50$, B picks any string $(aa)^k c(bb)^m$ of length larger than 50, then if $k \geq 1$ A picks $x = e$, $y = aa$,

$z = (aa)^{k-1}c(bb)^m$. Now whatever value of i B picks, the string xy^iz is in L because xy^iz is $(aa)^i(aa)^{k-1}c(bb)^m$. If $k = 0$ then (because the string has length larger than 50), $m > 2$ and A picks $x = c$, $y = bb$, and $z = (bb)^{m-1}$. This is possible because $m > 2$. Whatever value of i B picks, the string xy^iz is in L because xy^iz is $c(bb)^i(bb)^{m-1}$. Therefore the opponent (A) can always win, so L is regular.

(a) Is the theorem correct? Justify your answer.

(b) Is the proof correct? Justify your answer.

9. (4 points) Suppose $((p, a, \beta), (q, \gamma))$ is a production in a push-down automaton. True or false:

a) γ is popped from the stack if this production is used.
b) γ is pushed onto the stack if this production is used.
c) β is popped from the stack if this production is used.
d) β is pushed onto the stack if this production is used.

EXTRA CREDIT: (14 points) True or false:

(a) The context free languages are closed under concatenation.
(b) The context free languages are closed under intersection.
(c) The context free languages are closed under complementation.
(d) The context free languages are closed under Kleene star.
(e) If L is context free and R is regular then $L \cup R$ is context free.
(f) If L is context free and R is regular then $L \cap R$ is context free.
(g) A language L is context-free if there is a deterministic push-down automaton M such that $L = L(M)$.