COMP 455
Models of Languages and Computation
Spring 2012
Rules of Inference for Operations on Languages

To show $a_1 a_2 \ldots a_n \in \Sigma^*$ if $n \geq 1$:
show $a_1 \in \Sigma$ and
show $a_2 \in \Sigma$ and ... and
show $a_n \in \Sigma$

To show $a_1 a_2 \ldots a_n \notin \Sigma^*$ if $n \geq 1$:
show $a_1 \notin \Sigma$ or
show $a_2 \notin \Sigma$ or ... or
show $a_n \notin \Sigma$

To show $e \in \Sigma^*$:
succeed.

To show $e \notin \Sigma^*$:
fail.

To show $x \in \overline{A}$:
show $x \in \Sigma^*$ and
show $x \notin A$

To show $x \notin \overline{A}$:
show $x \notin \Sigma^*$ or
show $x \in A$

To show $z \in L_1 L_2$:
show $z = xy$ and
show $x \in L_1$ and
show $y \in L_2$

To show $z \in L^*$ if $z \neq e$:
show $z = xy$ and
show $x \in L$ and
show $y \in L^*$

To show $e \in L^*$:
succeed.

To show $e \notin L^*$:
fail.

To show $z \in L^+$:
show $z = xy$ and
show $x \in L$ and
show $y \in L^*$

If $x \in L_1$ and $y \in L_2$ then $xy \in L_1 L_2$
If $x \in L$ and $y \in L^*$ then $xy \in L^*$
$e \in L^*$
If $x \in L$ and $y \in L^*$ then $xy \in L^+$

If $x \in L_1$ then $x \in L_1 \cup L_2$
If $x \in L_2$ then $x \in L_1 \cup L_2$

To show $a_1 a_2 \ldots a_n \in L_1 L_2$ if $n \geq 1$:
show ($e \in L_1$ and $a_1 a_2 \ldots a_n \in L_2$) or
show ($a_1 \in L_1$ and $a_2 \ldots a_n \in L_2$) or ... or
show ($a_1 a_2 \ldots a_{n-1} \in L_1$ and $a_n \in L_2$) or
show ($a_1 a_2 \ldots a_n \in L_1$ and $e \in L_2$)

To show $a_1 a_2 \ldots a_n \notin L_1 L_2$ if $n \geq 1$:
show ($e \notin L_1$ or $a_1 a_2 \ldots a_n \notin L_2$) and
show ($a_1 \notin L_1$ or $a_2 \ldots a_n \notin L_2$) and ... and
show ($a_1 a_2 \ldots a_{n-1} \notin L_1$ or $a_n \notin L_2$) and
show ($a_1 a_2 \ldots a_n \notin L_1$ or $e \notin L_2$)

To show $a_1 a_2 \ldots a_n \in L^*$ if $n \geq 1$:
show ($a_1 \in L$ and $a_2 \ldots a_n \in L^*$) or
show ($a_1 a_2 \in L$ and $a_3 \ldots a_n \in L^*$) or ... or
show ($a_1 a_2 \ldots a_{n-1} \in L$ and $a_n \in L^*$) or
show ($a_1 a_2 \ldots a_n \in L$ and $e \in L^*$)

To show $a_1 a_2 \ldots a_n \notin L^*$ if $n \geq 1$:
show ($a_1 \notin L$ or $a_2 \ldots a_n \notin L^*$) and
show ($a_1 a_2 \notin L$ or $a_3 \ldots a_n \notin L^*$) and ... and
show ($a_1 a_2 \ldots a_{n-1} \notin L$ or $a_n \notin L^*$) and
show ($a_1 a_2 \ldots a_n \notin L$ or $e \notin L^*$)

To show $a_1 a_2 \ldots a_n \in L^+$ if $n \geq 1$:
show ($e \in L$ and $a_1 a_2 \ldots a_n \in L^*$) or
show ($a_1 \in L$ and $a_2 \ldots a_n \in L^*$) or ... or
show ($a_1 a_2 \ldots a_{n-1} \in L$ and $a_n \in L^*$) or
show ($a_1 a_2 \ldots a_n \in L$ and $e \in L^*$)

To show $a_1 a_2 \ldots a_n \notin L^+$ if $n \geq 1$:
show ($e \notin L$ or $a_1 a_2 \ldots a_n \notin L^*$) and
show ($a_1 \notin L$ or $a_2 \ldots a_n \notin L^*$) and ... and
show ($a_1 a_2 \ldots a_{n-1} \notin L$ or $a_n \notin L^*$) and
show ($a_1 a_2 \ldots a_n \notin L$ or $e \notin L^*$)

Example proof in outline form:

$10001 \in \{1\}\{0\}^*\{0\}\{1\}^*$ because $100 \in \{1\}\{0\}^*$ and $01 \in \{0\}\{1\}^*$
- $100 \in \{1\}\{0\}^*$ because $1 \in \{1\}$ and $00 \in \{0\}^*$
  - $1 \in \{1\}$ by set theory
  - $00 \in \{0\}^*$ because $0 \in \{0\}$ and $0 \in \{0\}^*$
    - $0 \in \{0\}$ by set theory
    - $0 \in \{0\}^*$ because $0 \in \{0\}$ and $e \in \{0\}^*$
      - $0 \in \{0\}$ by set theory
- $01 \in \{0\}\{1\}^*$ because $0 \in \{0\}$ and $1 \in \{1\}^*$
  - $0 \in \{0\}$ by set theory
  - $1 \in \{1\}^*$ because $1 \in \{1\}$ and $e \in \{1\}^*$
    - $1 \in \{1\}$ by set theory