COMP 550

Algorithms and Analysis

Spring 2008

Second Mid Semester Exam

Thursday, March 27, 2008

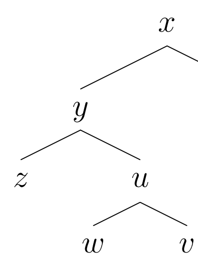
Closed Book - Closed Notes

Don't forget to write your name or ID and pledge on the exam sheet.

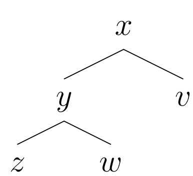
This exam has three pages.

1. (12 points) (a) Give an upper bound on the height of a red black tree having n internal nodes. ____________________ (b) Give an asymptotic bound on the worst case time to delete an element from a red-black tree as a function of n, the number of nodes in the tree. ____________________ (c) In a red black tree, what is the maximum number of red children that a black node can have? ____________________ (d) In a red black tree, what is the maximum number of red children that a red node can have? ____________________

2. (10 points) (a) Assume that the following tree is a binary search tree, where x, y, z, u, w, and v are nodes. Is the tree height balanced? Why or why not? ____________________



(b) Assume that the following tree is a binary search tree, where x, y, z, w, and v are nodes. Is this tree height balanced? Why or why not? ____________________



3. (10 points) Give a sequence of seven elements taken from the set $\{10, 15, 20, 25, 30, 35, 40\}$ which, when inserted into an initially empty binary search

tree, will produce a perfectly balanced tree. ____________________

4. (10 points) Suppose a hash table has a load factor of 9/10. (a) If open addressing is being used, assuming simple uniform hashing, what is the expected number of probes in an unsuccessful search? ____________________ (b) If hashing with chaining is being used, assuming simple uniform hashing, and the load factor is α, what is the asymptotic expected time for an unsuccessful search? ____________________

5. (10 points) (a) How fast can one sort n elements in the range $\{1, 2, \ldots, n\}$ using $O(n)$ storage? Give an asymptotic bound and a brief justification. ____________________ b). How fast can one sort n elements in the range $\{1, 2, \ldots, n^2\}$ using $O(n)$ storage? Give an asymptotic bound and a brief justification. ____________________

6. (10 points) Suppose one has $n = 10000$ real numbers uniformly distributed between 0 and 1. What is a good way to sort them? What is the asymptotic expected time for this sorting method, as a function of the number n of elements sorted? ____________________

7. (10 points) Consider the following set of elements, where $a : b$ denotes an element with key a and priority b. Construct a treap from these elements. $1 : 5, 2 : 3, 3 : 1, 4 : 6, 5 : 4, 6 : 2$.

8. (12 points) (a) How many comparisons are needed to find the minimum of n elements? Give a bound and be as precise as you can. ____________________ (b) How many comparisons are needed to find the median of n elements? Give an asymptotic bound. ____________________ (c) How many comparisons are needed to find the minimum and second smallest of n elements? Give a bound and be as precise as you can. ____________________

9. (10 points) For hashing by the multiplication method, which of the following values for A is best: (a) 0.35 (b) .618033988 (c) 0.4 (d) .618 ____________________

10. (10 points) For hashing by the division method, which of the following is the best value for the modulus m? Give a brief justification for your answer. (a) 128 (b) 100 (c) 199 (d) 99 ____________________

11. (10 points) Suppose one is using hashing by the division method and the table size m is 53 and the key k is 228. Which bin will this key hash to? ____________________

12. (10 points) Suppose one is using hashing by the multiplication method and A is .528 and the table size m is 20 and the key k is 17. Which bin will this key hash to? ____________________

13. (10 points) Suppose one is doing universal hashing and the table size m is 2000 and there are 40000 hash functions in all in the set H. Let x and y be two distinct keys. What is the maximum number of hash functions h in H such that $h(x) = h(y)$, according to universal hashing? ____________________

14. (6 points) If the hash table size is 150 and there are 80 elements in the table, what is the load factor? ____________________

EXTRA CREDIT: (5 points) Consider hashing by the multiplication method. Suppose the table size m is 2^p where p is an integer and the word size is w bits and $p \leq w$ and that k in binary has at most w bits. Suppose A is of the form $s/2^w$ where s is an integer in the range $0 < s < 2^w$. Give a formula for computing the hash function $h(k)$ and give an efficient method of computing it that minimizes the number of arithmetic operations.

EXTRA CREDIT: (5 points) State Markov's inequality. It gives an upper bound on the probability that a random variable will have a value far from the median of its probability distribution. For 5 extra points, derive Markov's inequality. ____________________

EXTRA CREDIT: (5 points) Let b_i be the number of binary trees with i nodes, so $b_0 = 1$ and $b_1 = 1$ and $b_2 = 2$, et cetera. Compute b_{10} as accurately as you can. ____________________