Page Replacement Algorithms

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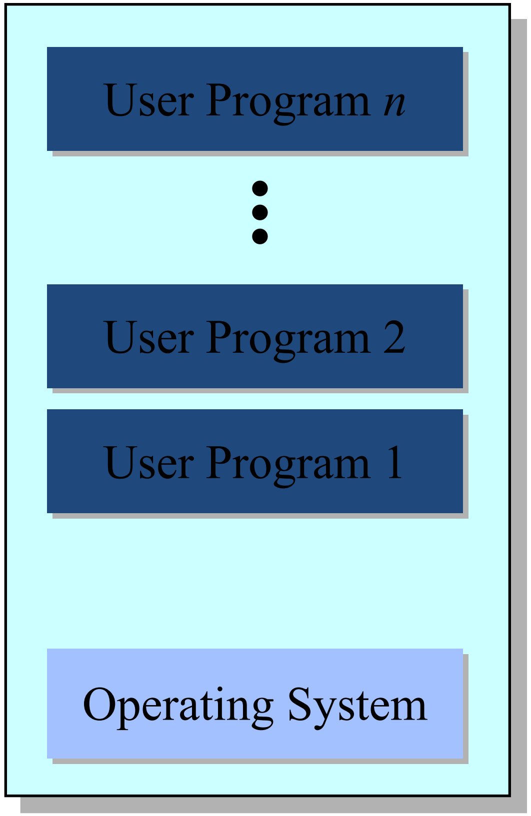
Portions courtesy Emmett Witchel and Kevin Jeffay

Virtual Memory Management: Recap

- Key concept: Demand paging
  - Load pages into memory only when a page fault occurs
- Issues:
  - Placement strategies
    - Place pages anywhere – no placement policy required
  - Replacement strategies
    - What to do when there exist more jobs than can fit in memory
  - Load control strategies
    - Determining how many jobs can be in memory at one time



Memory

Page Replacement Algorithms

- Typically Σ_i *VAS_i* >> *Physical Memory*
- With demand paging, physical memory fills quickly
- When a process faults & memory is full, some page must be swapped out
  - Handling a page fault now requires ***2*** disk accesses not 1!

Which page should be replaced?

Local replacement — Replace a page of the faulting process
Global replacement — Possibly replace the page of another process

Page Replacement: Eval. Methodology

- Record a *trace* of the pages accessed by a process
  - Example: (Virtual page, offset) address trace...
    (3,0), (1,9), (4,1), (2,1), (5,3), (2,0), (1,9), (2,4), (3,1), (4,8)
  - generates page trace
    3, 1, 4, 2, 5, 2, 1, 2, 3, 4 (represented as *c*, *a*, *d*, *b*, *e*, *b*, *a*, *b*, *c*, *d*)
- Hardware can tell OS when a new page is loaded into the TLB
  - Set a used bit in the page table entry
  - Increment or shift a register

Simulate the behavior of a page replacement algorithm on the trace and record the number of page faults generated

fewer faults ⇨ *better performance*

Optimal Strategy: Clairvoyant Replacement

- Replace the page that won't be needed for the longest time in the future

Initial allocation

Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0	*a*										
	1	*b*										
	2	*c*										
	3	*d*										
Faults												
Time page needed next												

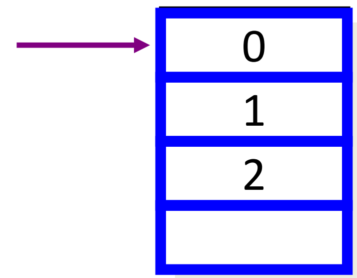
Optimal Strategy: Clairvoyant Replacement

- Replace the page that won't be needed for the longest time in the future

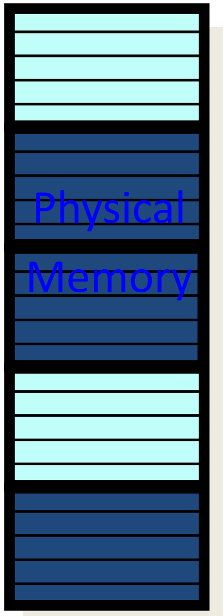
Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	***d***
	1	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*
	2	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*
	3	*d*	*d*	*d*	*d*	*d*	***e***	*e*	*e*	*e*	*e*	*e*
Faults							•					•
Time page needed next							*a* = 7 *b* = 6 *c* = 9 *d* = 10					*a* = 15 *b* = 11 *c* = 13 *d* = 14

Local Replacement: FIFO

- Simple to implement
  - A single pointer suffices
- Performance with 4 page frames:



Frame List



Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0	*a*										
	1	*b*										
	2	*c*										
	3	*d*										
Faults												

Local Replacment: FIFO

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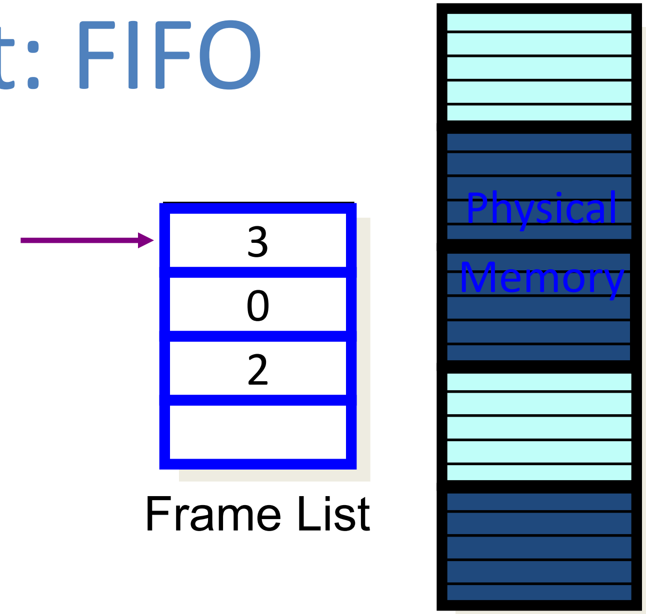
  - A single pointer suffices
- Performance with 4 page frames:



		Time	0	1	2	3	4	5	6	7	8	9	10
Requests				*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0		*a*	*a*	*a*	*a*	*a*	***e***	*e*	*e*	*e*	*e*	***d***
	1		*b*	*b*	*b*	*b*	*b*	*b*	*b*	***a***	*a*	*a*	*a*
	2		*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	***b***	*b*	*b*
	3		*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	***c***	*c*
Faults								•		•	•	•	•

Least Recently Used (LRU) Replacement

- Use the recent past as a predictor of the near future
- Replace the page that hasn't been referenced for the longest time

Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0	*a*										
	1	*b*										
	2	*c*										
	3	*d*										
Faults												
Time page last used												

Least Recently Used (LRU) Replacement

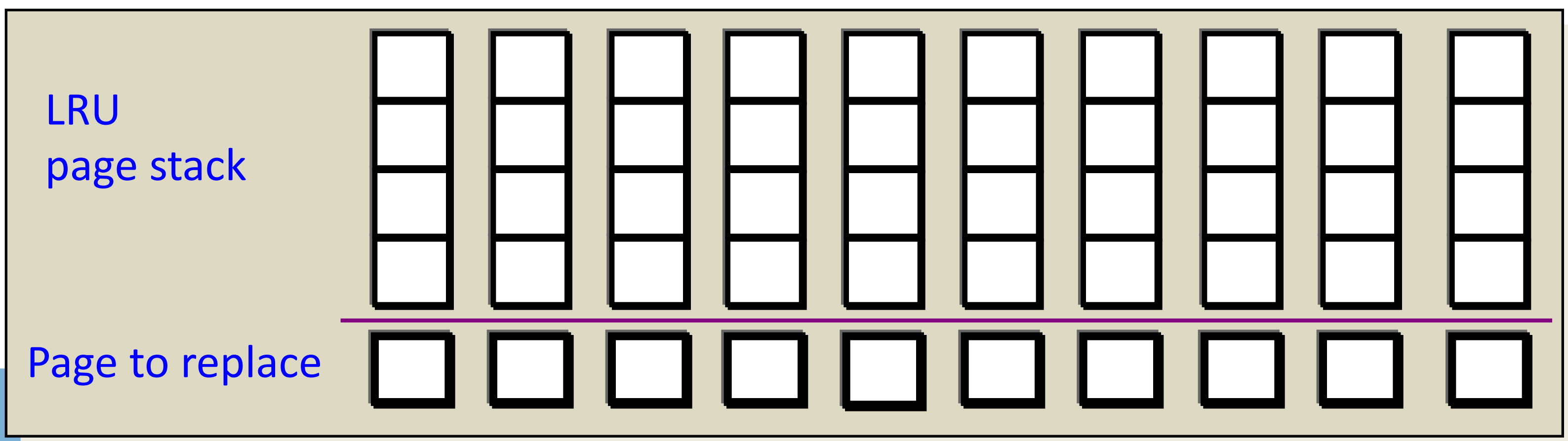
- Use the recent past as a predictor of the near future
- Replace the page that hasn't been referenced for the longest time

Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*
	1	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*
	2	*c*	*c*	*c*	*c*	*c*	***e***	*e*	*e*	*e*	*e*	***d***
	3	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	***c***	*c*
Faults							•				•	•
Time page last used						*a* = 2 *b* = 4 *c* = 1 *d* = 3				*a* = 7 *b* = 8 *e* = 5 *d* = 3	*a* = 7 *b* = 8 *e* = 5 *c* = 9	

How to Implement LRU?

- Maintain a “stack” of recently used pages

		Time	0	1	2	3	4	5	6	7	8	9	10
Requests				*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0		*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*
	1		*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*
	2		*c*	*c*	*c*	*c*	*c*	***e***	*e*	*e*	*e*	*e*	***d***
	3		*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	***c***	*c*
Faults								•				•	•



How to Implement LRU?

- Maintain a “stack” of recently used pages

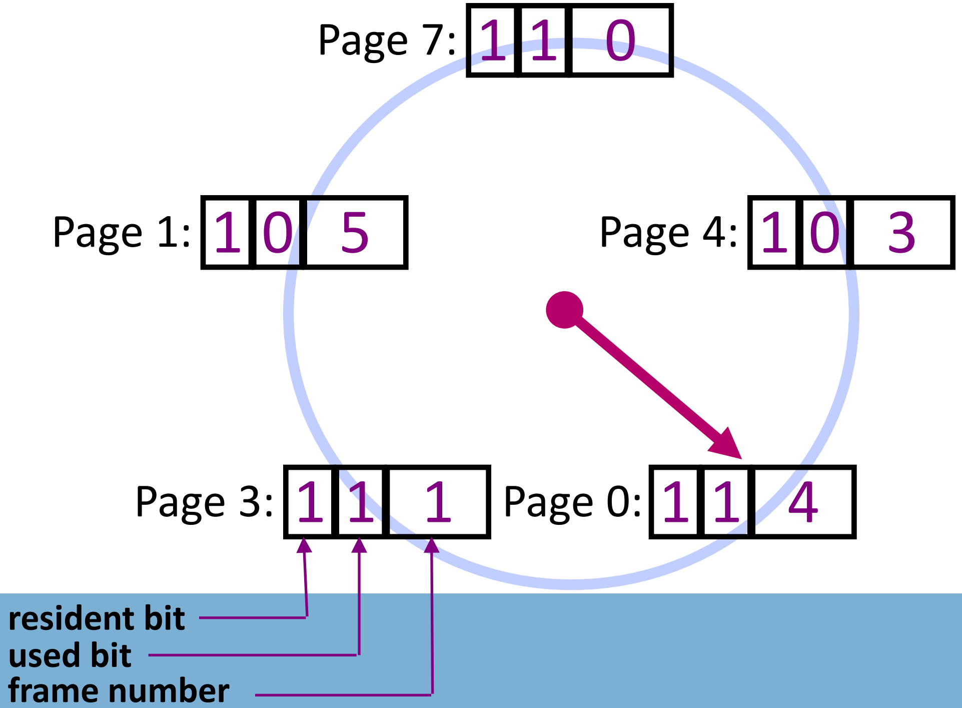
		0	1	2	3	4	5	6	7	8	9	10
Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*
	1	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*
	2	*c*	*c*	*c*	*c*	*c*	***e***	*e*	*e*	*e*	*e*	***d***
	3	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	***c***	*c*
Faults							•				•	•

LRU page stack	*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
		c	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*
			c	*a*	*d*	*d*	*e*	*e*	*a*	*b*
				c	*a*	*a*	*d*	*d*	*e*	*a*
Page to replace					***c***				***d***	***e***

- What is the goal of a page replacement algorithm?
  - A. Make life easier for OS implementer
  - B. Reduce the number of page faults
  - C. Reduce the penalty for page faults when they occur
  - D. Minimize CPU time of algorithm

Approximate LRU: The Clock Algorithm

- Maintain a circular list of pages resident in memory
  - Use a *clock* (or *used/referenced*) bit to track how often a page is accessed
  - The bit is set whenever a page is referenced
- Clock hand sweeps over pages looking for one with *used* bit = 0
  - Replace pages that haven't been referenced for one complete revolution of the clock




```
func Clock_Replacement
begin
   while (victim page not found) do
      if (used bit for current page = 0) then
         replace current page
      else
         reset used bit
      end if
      advance clock pointer
   end while
end Clock_Replacement
```

Clock Example

Time		0	1	2	3	4	5	6	7	8	9	10
Requests			c	a	d	b	e	b	a	b	c	d
Page Frames	0	a	a	a	a	a						
	1	b	b	b	b	b						
	2	c	c	c	c	c						
	3	d	d	d	d	d						
Faults												

Page table entries for resident pages:

1	a
1	b
1	c
1	d

Clock Example

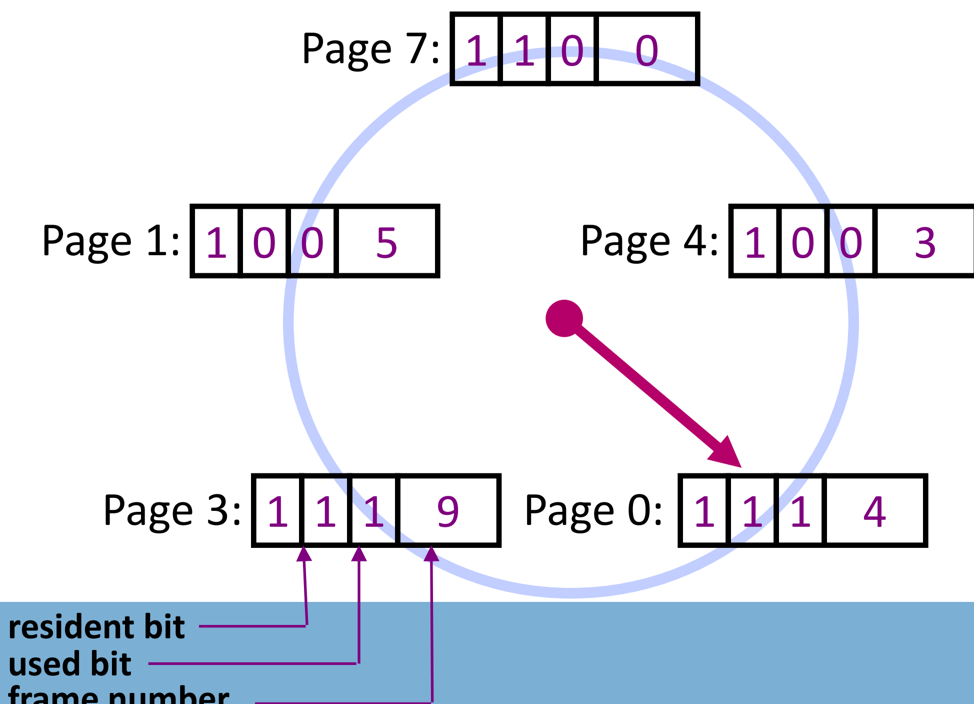
Time		0	1	2	3	4	5	6	7	8	9	10
Requests			c	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0	a	a	*a*	*a*	*a*	***e***	*e*	*e*	*e*	*e*	***d***
	1	b	b	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*
	2	c	c	*c*	*c*	*c*	*c*	*c*	***a***	*a*	*a*	*a*
	3	d	d	*d*	*d*	*d*	*d*	*d*	*d*	*d*	***c***	*c*
Faults							•		•		•	•

Page table entries for resident pages:

1 *a*	1 *e*	1 *e*	1 *e*	1 *e*	1 *e*	1 *d*
1 *b*	0 *b*	1 *b*	1 *b*	1 *b*	1 *b*	0 *b*
1 *c*	0 *c*	0 *c*	1 *a*	1 *a*	1 *a*	0 *a*
1 *d*	0 *d*	0 *d*	0 *d*	0 *d*	1 *c*	0 *c*

Optimization: Second Chance Algorithm

- There is a significant cost to replacing "dirty" pages
  - Why?
    - Must write back contents to disk before freeing!
- Modify the Clock algorithm to allow dirty pages to always survive one sweep of the clock hand
  - Use both the *dirty bit* and the *used bit* to drive replacement



Second Chance Algorithm

Before clock sweep

used	*dirty*
0	0
0	1
1	0
1	1

After clock sweep

used	*dirty*
replace page	
0	0
0	0
0	1

Second Chance Example

Time		0	1	2	3	4	5	6	7	8	9	10
Requests			c	a^w	d	b^w	e	b	a^w	b	c	d
Page Frames	0	a	a	a	a	a						
	1	b	b	b	b	b						
	2	c	c	c	c	c						
	3	d	d	d	d	d						
Faults												

Page table entries for resident pages:

10	a
10	b
10	c
10	d

Second Chance Example

Time		0	1	2	3	4	5	6	7	8	9	10
Requests			c	a^w	d	b^w	e	b	a^w	b	c	d
Page Frames	0	a	a	a	a	a	a	a	a	a	a	a
	1	b	b	b	b	b	b	b	b	b	b	**d**
	2	c	c	c	c	c	**e**	e	e	e	e	e
	3	d	d	d	d	d	d	d	d	d	**c**	c
Faults							•				•	•

Page table entries for resident pages:

Time 0	Time 4	Time 5	Time 6	Time 7	Time 9	Time 10
10 a	11 a	00 a^*	00 a	11 a	11 a	00 a^*
10 b	11 b	00 b^*	10 b	10 b	10 b	10 d
10 c	10 c	10 e	10 e	10 e	10 e	00 e
10 d	10 d	00 d	00 d	00 d	10 c	00 c

Local Replacement and Memory Sensitivity

Time	0	1	2	3	4	5	6	7	8	9	10	11	12
Requests		*a*	*b*	*c*	*d*	*a*	*b*	*c*	*d*	*a*	*b*	*c*	*d*

Page Frames	0	1	2	3	4	5	6	7	8	9	10	11	12
0	*a*												
1	*b*												
2	*c*												
Faults													

Page Frames	0	1	2	3	4	5	6	7	8	9	10	11	12
0	*a*												
1	*b*												
2	*c*												
3	–												
Faults													

Local Replacement and Memory Sensitivity

Time	0	1	2	3	4	5	6	7	8	9	10	11	12
Requests		*a*	*b*	*c*	*d*	*a*	*b*	*c*	*d*	*a*	*b*	*c*	*d*

Page Frames													
0	*a*	*a*	*a*	*a*	***d***	*d*	*d*	***c***	*c*	*c*	***b***	*b*	*b*
1	*b*	*b*	*b*	*b*	*b*	***a***	*a*	*a*	***d***	*d*	*d*	***c***	*c*
2	*c*	*c*	*c*	*c*	*c*	*c*	***b***	*b*	*b*	***a***	*a*	*a*	***d***
Faults					•	•	•	•	•	•	•	•	•

Page Frames													
0	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*
1	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*
2	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*
3	–				***d***	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*
Faults					•								

Page Replacement Performance

- Local page replacement
  - LRU — Ages pages based on when they were last used
  - FIFO — Ages pages based on when they're brought into memory
- Towards global page replacement ... with variable number of page frames allocated to processes

The principle of locality

- 90% of the execution of a program is sequential
- Most iterative constructs consist of a relatively small number of instructions
- When processing large data structures, the dominant cost is sequential processing on individual structure elements
- Temporal vs. physical locality

Optimal Replacement with a Variable Number of Frames

- *VMIN* — Replace a page that is not referenced in the *next* τ accesses

- Example: τ = 4

Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*c*	*d*	*b*	*c*	*e*	*c*	*e*	*a*	*d*
Pages in Memory	Page *a*	• t = 0										
	Page *b*	-										
	Page *c*	-										
	Page *d*	• t = -1										
	Page *e*	-										
Faults												

Optimal Replacement with a Variable Number of Frames

- *VMIN* — Replace a page that is not referenced in the *next* τ accesses
- Example: $\tau = 4$

		0	1	2	3	4	5	6	7	8	9	10
Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*c*	*d*	*b*	*c*	*e*	*c*	*e*	*a*	*d*
Pages in Memory	Page *a*	• $t = 0$	-	-	-	-	-	-	-	-	*F*	-
	Page *b*	-	-	-	-	*F*	-	-	-	-	-	-
	Page *c*	-	*F*	•	•	•	•	•	•	-	-	-
	Page *d*	• $t = -1$	•	•	•	-	-	-	-	-	-	*F*
	Page *e*	-	-	-	-	-	-	*F*	•	•	-	-
Faults			•			•		•			•	•

The Working Set Model

- Assume recently referenced pages are likely to be referenced again soon…
- ... and *only* keep those pages recently referenced in memory (called *the working set*)
  - Thus pages may be removed even when no page fault occurs
  - The number of frames allocated to a process will vary over time
- A process is allowed to execute only if its working set fits into memory
  - The working set model performs implicit load control

Working Set Page Replacement

- Keep track of the last τ references (including faulting reference)
  - The pages referenced during the last τ memory accesses are the working set
  - τ is called the *window size*
- Example: Working set computation, $\tau = 4$ references:

		0	1	2	3	4	5	6	7	8	9	10
Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*c*	*d*	*b*	*c*	*e*	*c*	*e*	*a*	*d*
Pages in Memory	Page *a*	• $t = 0$										
	Page *b*	-										
	Page *c*	-										
	Page *d*	• $t = -1$										
	Page *e*	• $t = -2$										
Faults												

Working Set Page Replacement

- Keep track of the last τ references
  - The pages referenced during the last τ memory accesses are the working set
  - τ is called the *window size*
- Example: Working set computation, τ = 4 references:

		Time	0	1	2	3	4	5	6	7	8	9	10
		Requests		*c*	*c*	*d*	*b*	*c*	*e*	*c*	*e*	*a*	*d*
Pages in Memory	Page *a*		• *t* = 0	•	•	•	-	-	-	-	-	*F*	•
	Page *b*		-	-	-	-	*F*	•	•	•	-	-	-
	Page *c*		-	*F*	•	•	•	•	•	•	•	•	•
	Page *d*		• *t* = -1	•	•	•	•	•	•	-	-	-	*F*
	Page *e*		• *t* = -2	•	-	-	-	-	*F*	•	•	•	•
Faults				•			•		•			•	•

Page-Fault-Frequency Page Replacment

- An alternate approach to computing working set
- Explicitly attempt to minimize page faults
  - When page fault frequency is high — *increase working set*
  - When page fault frequency is low — *decrease working set*

Algorithm:

Keep track of the rate at which faults occur

When a fault occurs, compute the time since the last page fault

Record the time, t_{last}, of the last page fault

If the time between page faults is "large" then reduce the working set

If $t_{current} - t_{last} > \tau$, then remove from memory all pages not referenced in $[t_{last}, t_{current}]$

If the time between page faults is "small" then increase working set

If $t_{current} - t_{last} \leq \tau$, then add faulting page to the working set

Page Fault Frequency Replacement

- Example, window size = 2
- If $t_{current} - t_{last} > 2$, remove pages not referenced in $[t_{last}, t_{current}]$ from the working set
- If $t_{current} - t_{last} \leq 2$, just add faulting page to the working set

	Time	0	1	2	3	4	5	6	7	8	9	10
	Requests		*c*	*c*	*d*	*b*	*c*	*e*	*c*	*e*	*a*	*d*
Pages in Memory	Page *a*	•										
	Page *b*	-										
	Page *c*	-										
	Page *d*	•										
	Page *e*	•										
	Faults											
	$t_{cur} - t_{last}$											

Page Fault Frequency Replacement

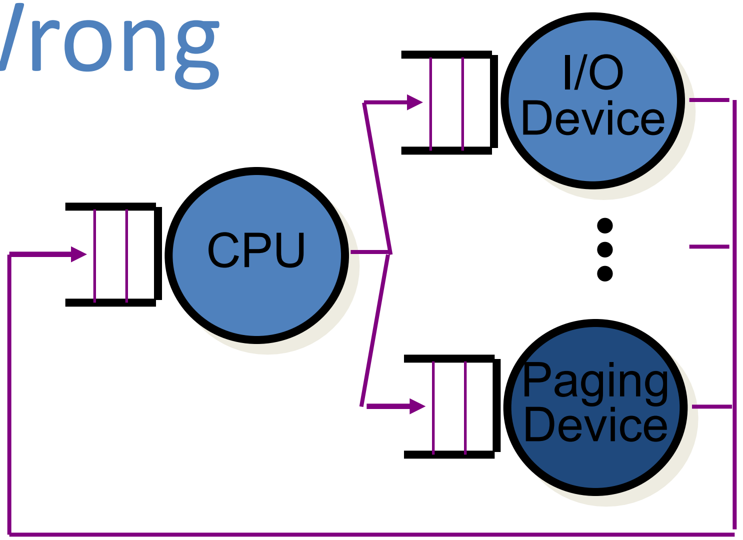
- Example, window size = 2
- If $t_{current} - t_{last} > 2$, remove pages not referenced in $[t_{last}, t_{current}]$ from the working set
- If $t_{current} - t_{last} \le 2$, just add faulting page to the working set

		0	1	2	3	4	5	6	7	8	9	10
Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*c*	*d*	*b*	*c*	*e*	*c*	*e*	*a*	*d*
Pages in Memory	Page *a*	•	•	•	•	-	-	-	-	-	*F*	•
	Page *b*	-	-	-	-	*F*	•	•	•	•	-	-
	Page *c*	-	*F*	•	•	•	•	•	•	•	•	•
	Page *d*	•	•	•	•	•	•	•	•	•	-	*F*
	Page *e*	•	•	•	•	-	-	*F*	•	•	•	•
Faults			•			•		•			•	•
$t_{cur} - t_{last}$			1			3		2			3	1

Load Control: Fundamental Trade-off

- High multiprogramming level
  - $MPL_{max} = \dfrac{\textit{number of page frames}}{\textit{minimum number of frames required for a process to execute}}$
- Low paging overhead
  - MPL_{min} = 1 process
- Issues
  - What criterion should be used to determine when to increase or decrease the *MPL*?
  - Which task should be swapped out if the *MPL* must be reduced?

Load Control Done Wrong



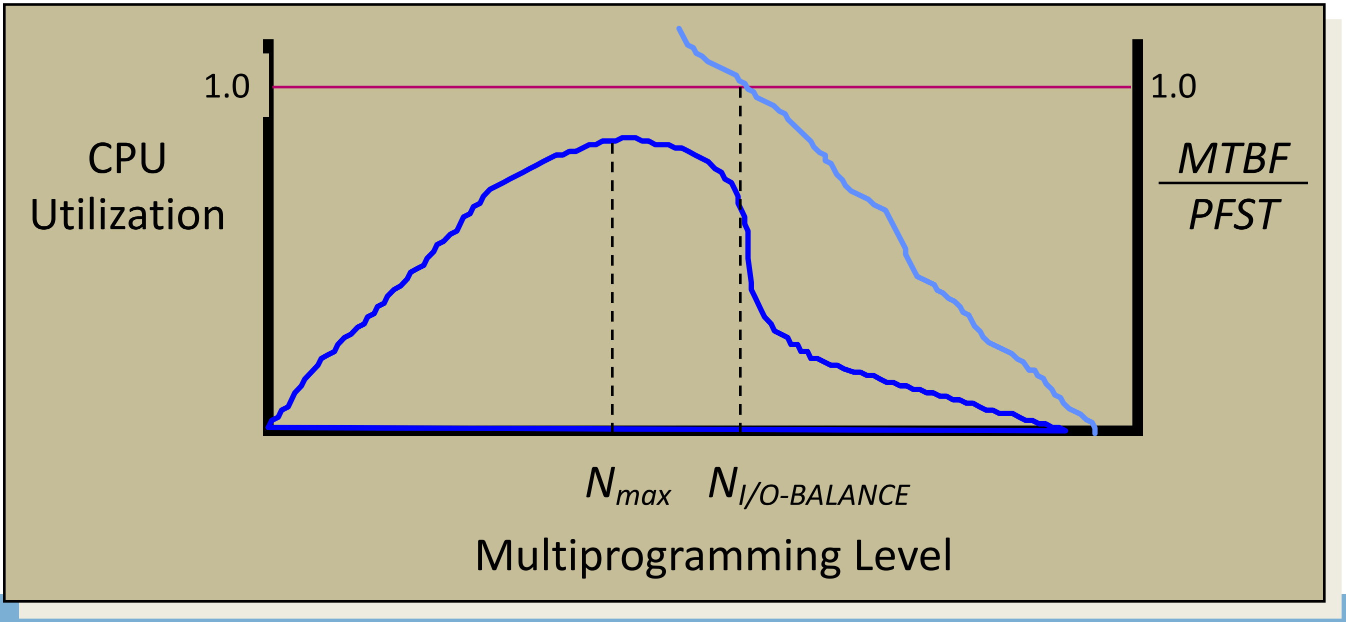
i.e., based on CPU utilization

- Assume memory is nearly full
- A chain of page faults occur
  - A queue of processes forms at the paging device
- CPU utilization falls
- Operating system increases *MPL*
  - New processes fault, taking memory away from existing processes
- CPU utilization goes to 0, the OS increases the *MPL* further...

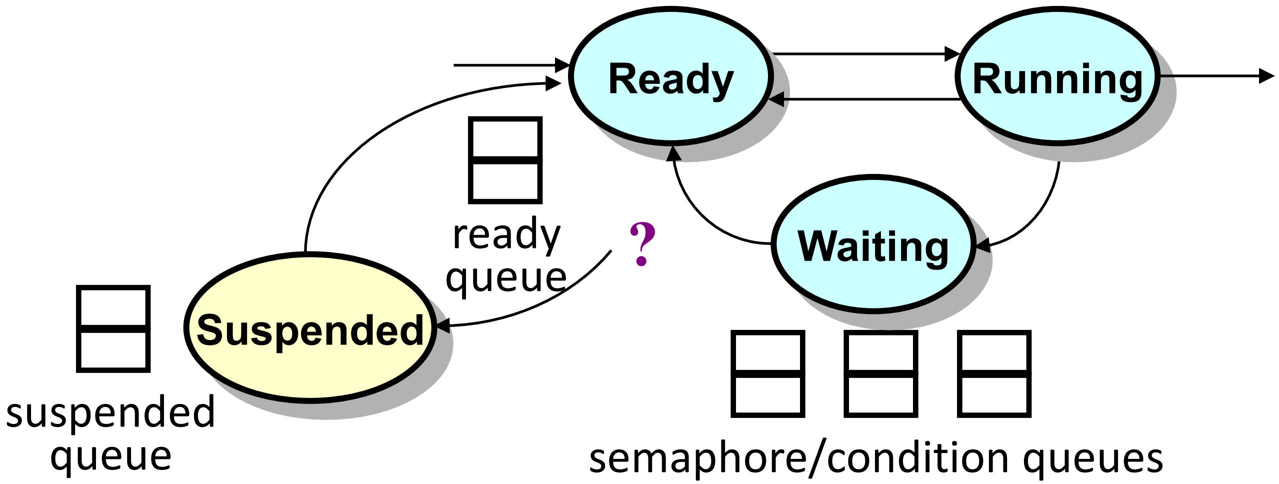
System is *thrashing* — spending all of its time paging

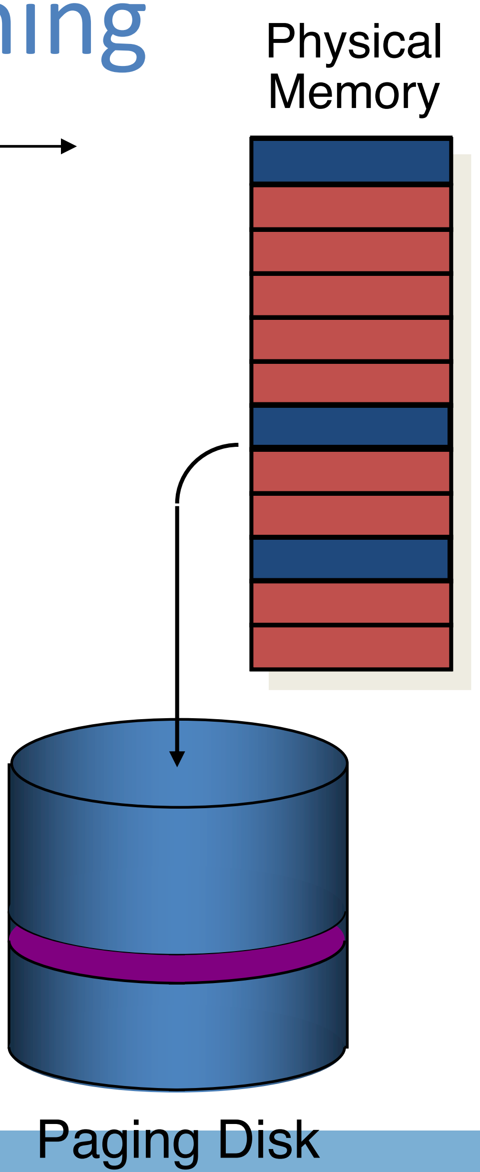
Load Control and Thrashing

- Thrashing can be ameliorated by *local* page replacement
- Better criteria for load control: Adjust MPL so that:
  - *mean time between page faults (MTBF) = page fault service time (PFST)*
  - $\Sigma\, WS_i$ *= size of memory*



Load Control and Thrashing





- When the multiprogramming level should be decreased, which process should be swapped out?
  - Lowest priority process?
  - Smallest process?
  - Largest process?
  - Oldest process?
  - Faulting process?