

# C2E2: Simulation-Based Verification of Hybrid Systems

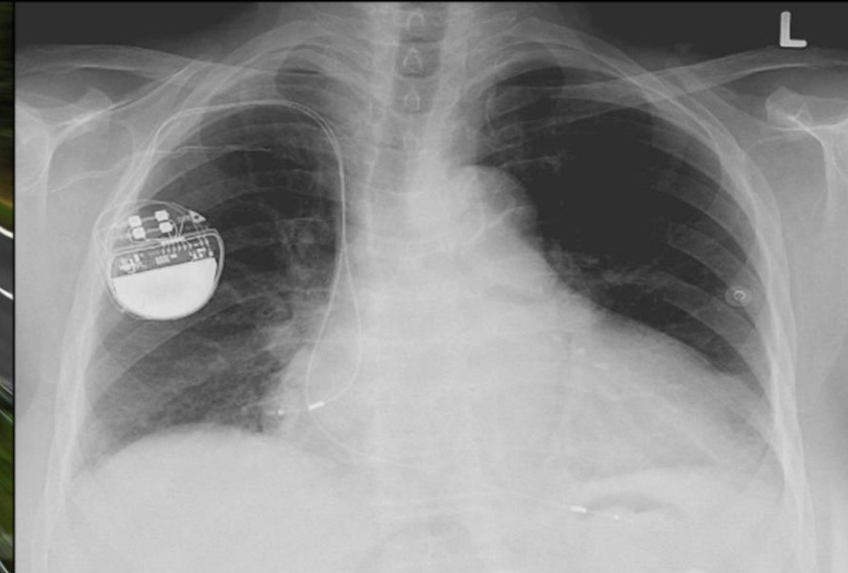
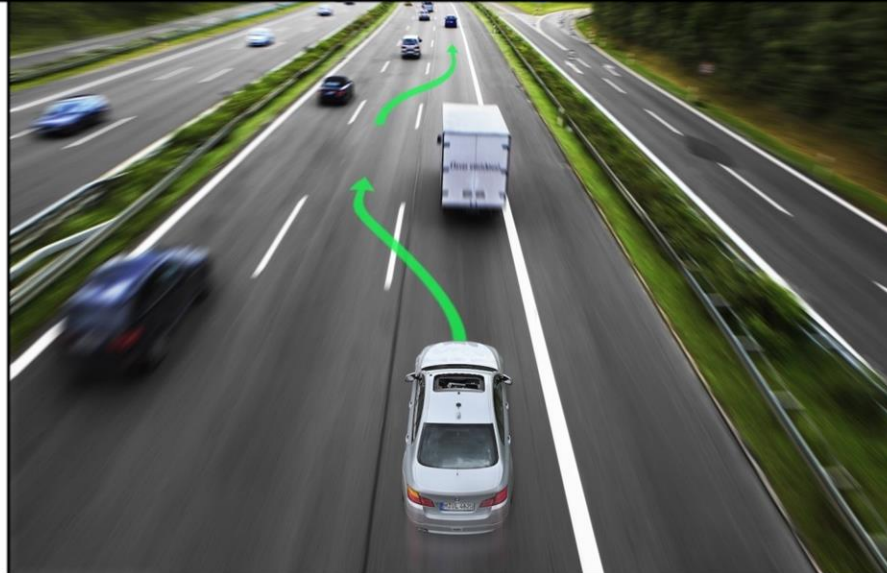
Parasara Sridhar Duggirala, Chuchu Fan, Matthew Potok,  
Bolun Qi, Sayan Mitra, Mahesh Viswanathan



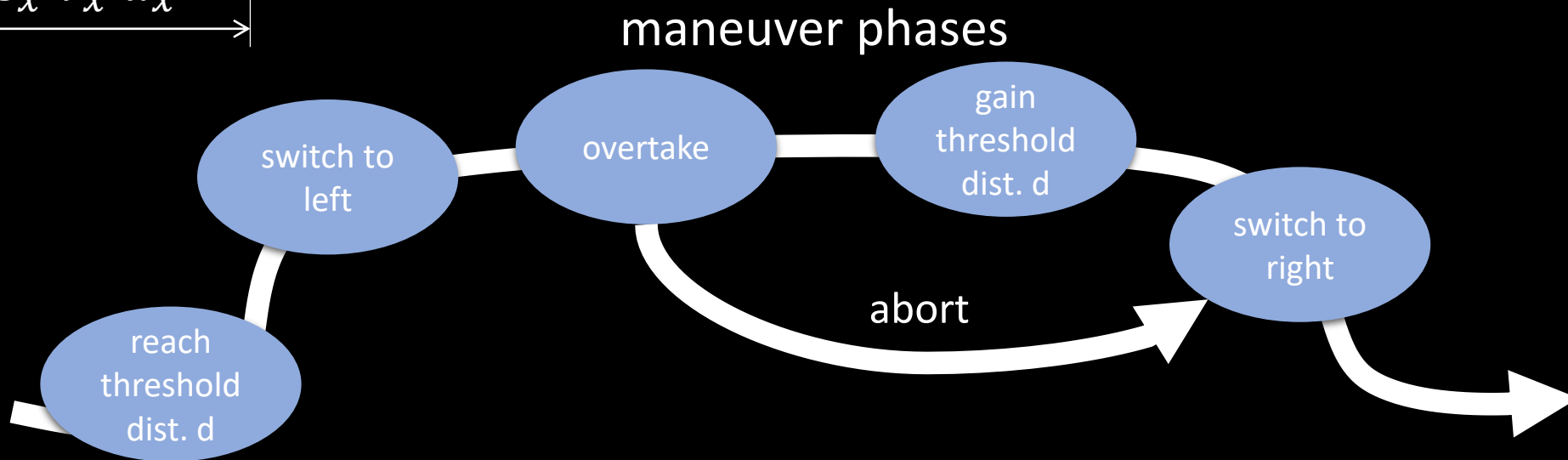
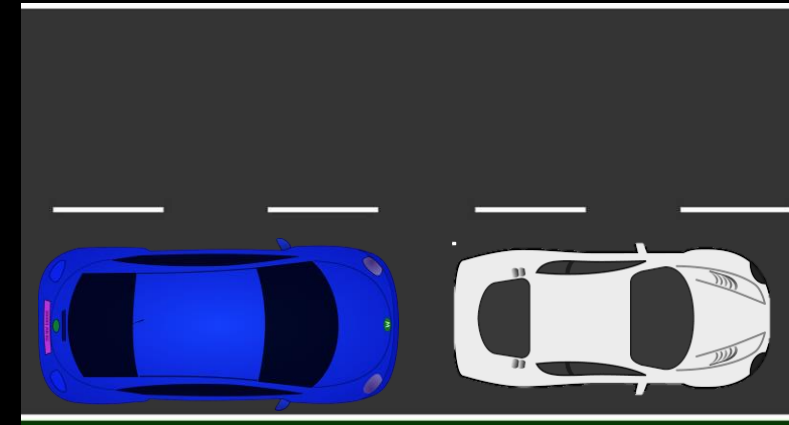
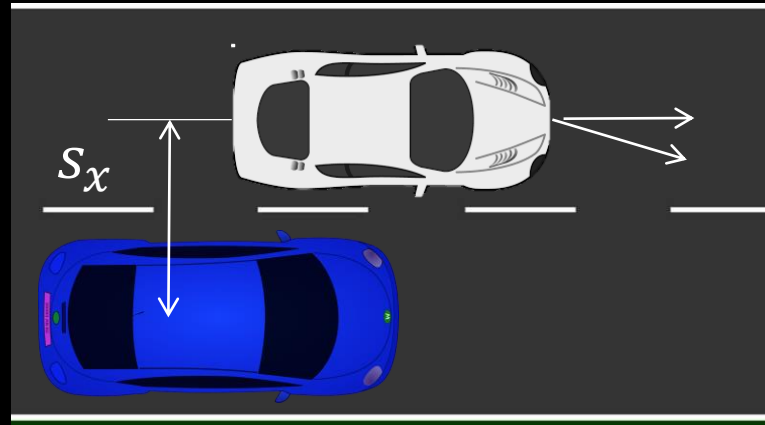
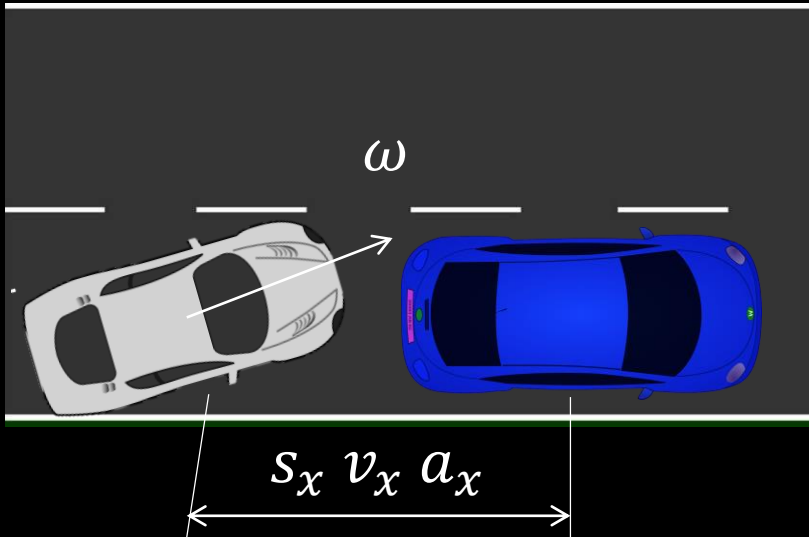
# Outline

- CPS Verification – challenges
- C2E2 – simulation based verification technique for CPS verification
- Features of C2E2
- Demo

# Safety verification problems in CPS



# Auto-passing system



# Safety verification problem of ODEs

Consider an **nonlinear ODE** model  $\dot{x} = f(x)$ ,  $x \in \mathbb{R}^n$

Discrete transitions

**Reach**( $\Theta, T$ ): states reachable from initial set  $\Theta \subseteq \mathbb{R}^n$  up to time  $T$

Safety verification problem: given initial set  $\Theta$ , unsafe set  $U$ , time bound  $T$ , decide whether  $\text{Reach}(\Theta, \infty) \cap U = \emptyset$

Safety verification is undecidable in general [Henzinger et al., 95]

Bounded time verification with over-approximation in existing tools:

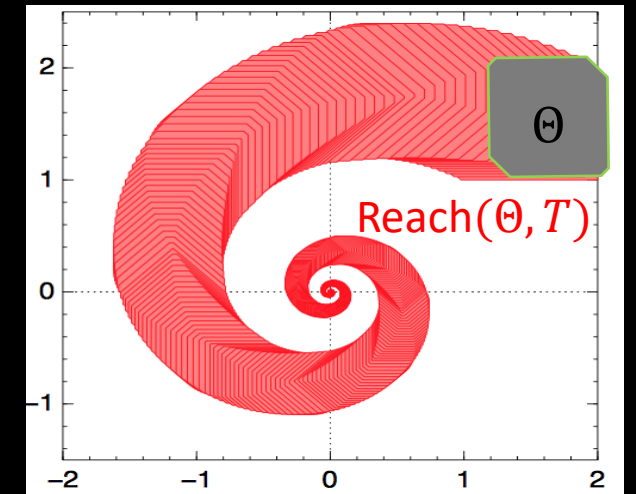
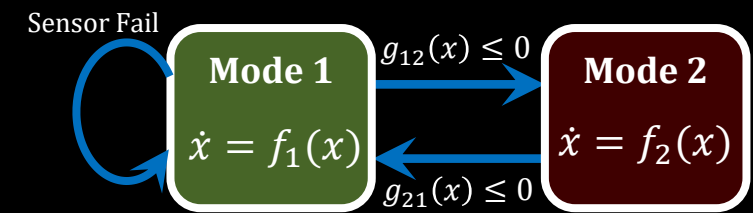
Linear dynamics: PHAVer [Frehse 05], SpaceEx [Frehse 11], d/dt [Asarin 01],

Nonlinear dynamics: Flow\* [Chen 12], etc.



C2E2: bounded time verification for nonlinear hybrid systems

Simulation-driven approach

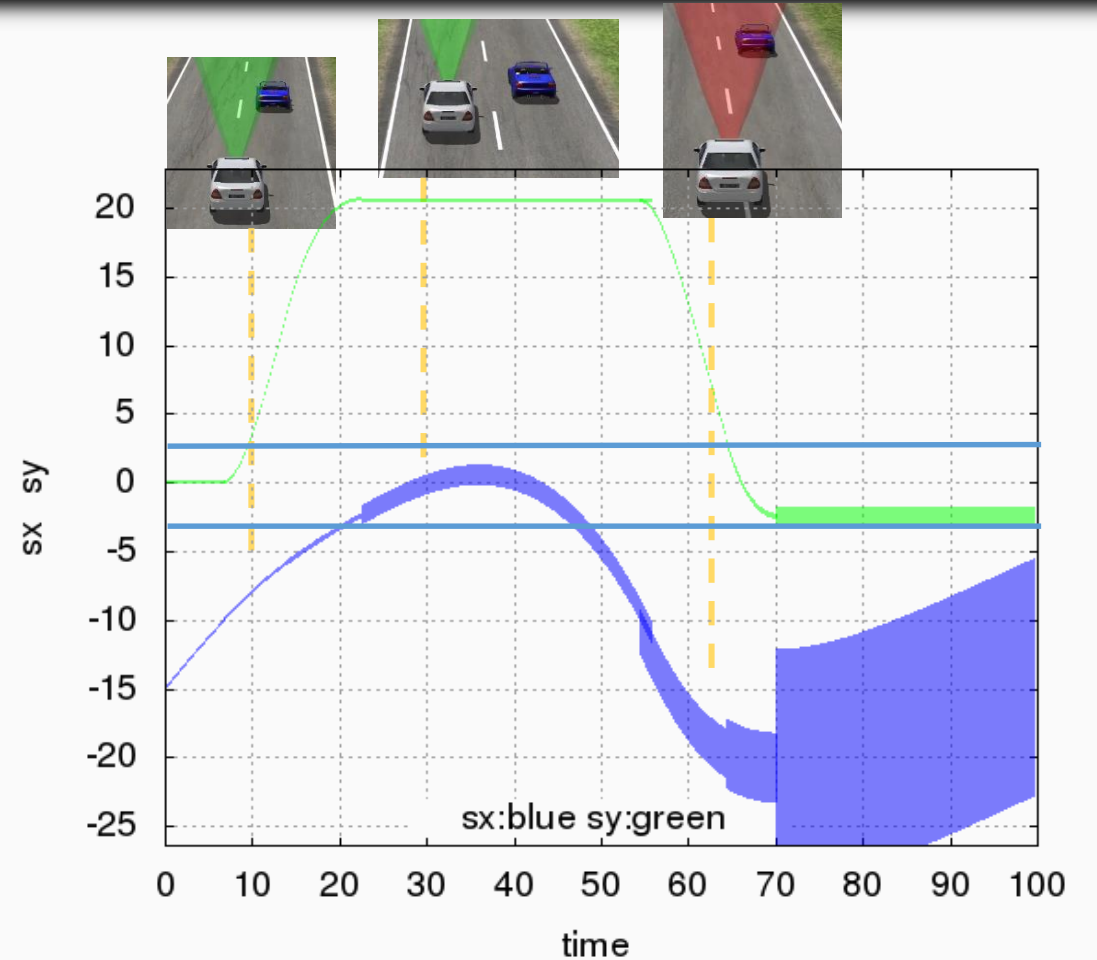
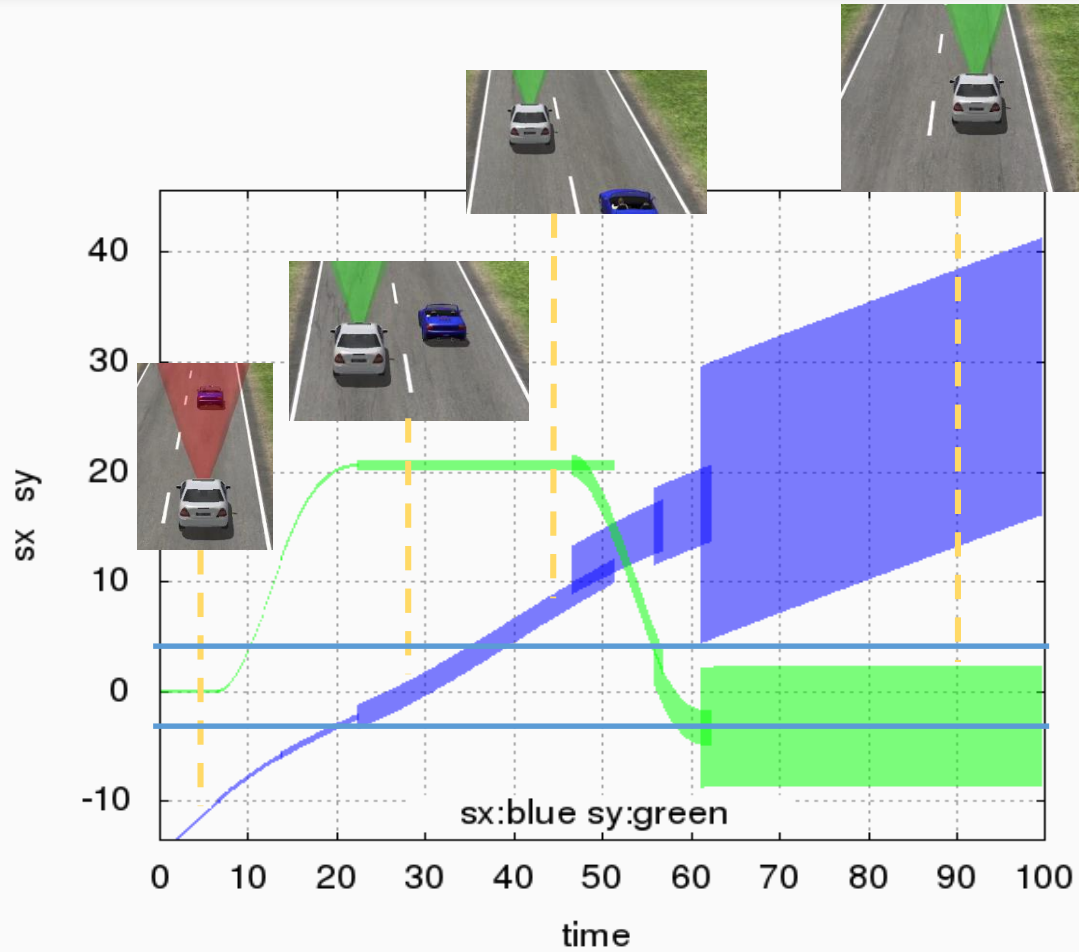
Provides soundness and relative completeness guarantees



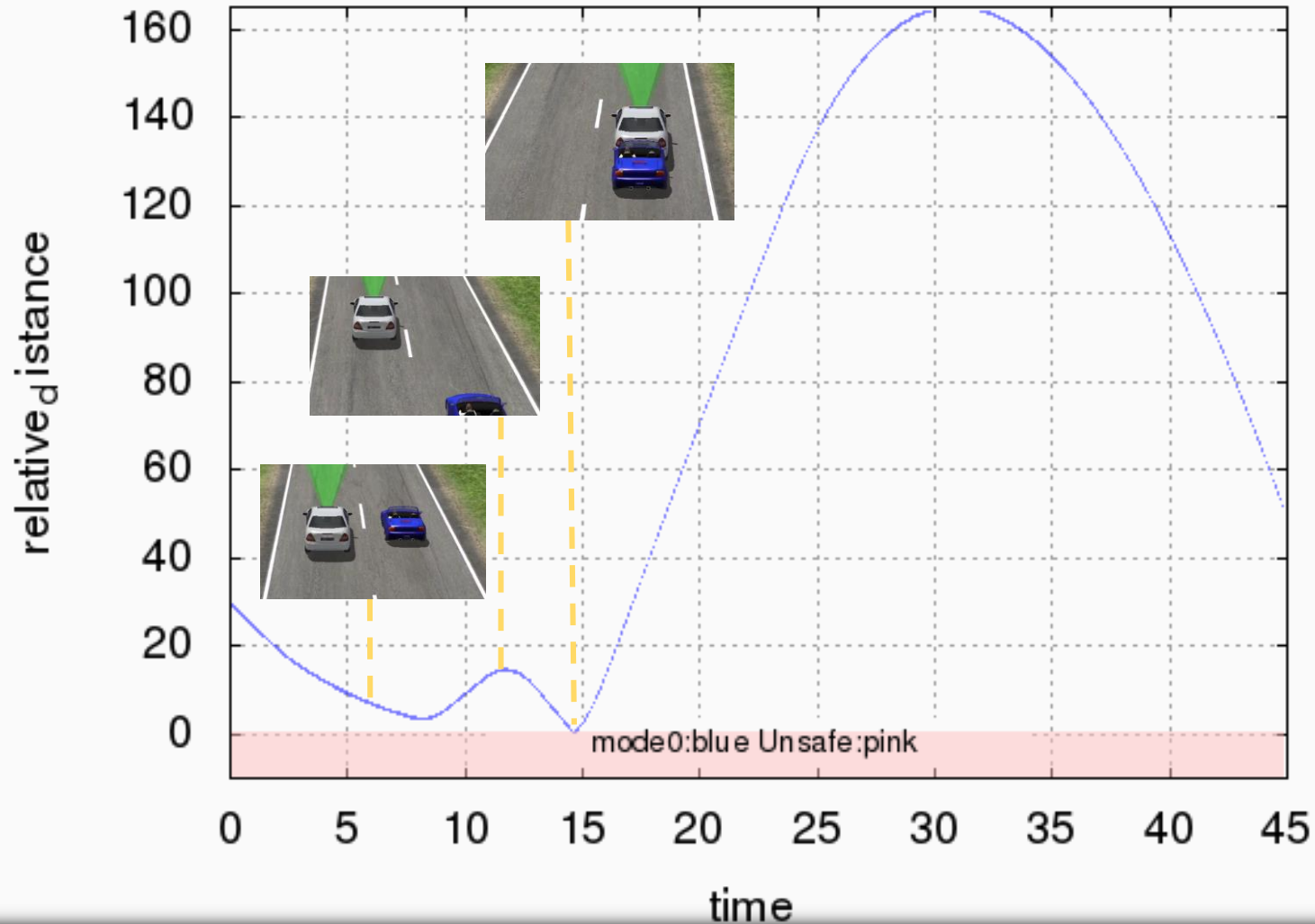
# Automatic simulation-driven strategy

- Given start  and unsafe 
- Compute finite cover of initial set
- Simulate from the center  $x_0$  of each cover
- **Bloat** simulation so that bloated tube contains trajectories from the cover
- Union = over-approximation of reach set
- Check intersection/containment with  $U$  and refine

# Verification of auto-passing system



# Auto-passing system – counter-example





# New features in C2E2

## Usability improvement

- Automatic reachability with piece-wise on-the-fly discrepancy algorithm

## Efficiency improvement

- Automatic detection and handling of different classes dynamics
  - Global discrepancy function for linear dynamics  $\dot{x} = Ax$
  - On-the-fly discrepancy for nonlinear dynamics  $\dot{x} = f(x)$
  - Special handling of constant dynamics  $\dot{x} = k$

## New testing scripts and a command line interface

# Demo

1. Website, downloading, and installation instructions.
2. C2E2 usability features.
3. Verification, results, and visualizations.
  - Cardiac cell
  - Autonomous vehicle passing
  - Powertrain control system
  - Robotic arms
4. Reachable sets, other data.

# Conclusion

Simulation-driven verification can be used for safety analysis of CPS

Automatic reachability analysis

Provides soundness and relative completeness

C2E2: our invariant verification tool for hybrid systems is able to solve some hard problems--try it

Check out more examples at the C2E2 webpage

<https://publish.illinois.edu/c2e2-tool/>

**C2E2 verification tool**  
analyze Simulink and hybrid system models

Home About Documents Download Examples Powertrain Challenge

## Main

**Compare Execute Check Engine (C2E2)** is a tool for verifying bounded-time invariant properties of hybrid system and Stateflow models. It supports nonlinear dynamics. Email [c2e2help@gmail.com](mailto:c2e2help@gmail.com) for support.

# Questions?

Send an email to [psd@uconn.edu](mailto:psd@uconn.edu), [cfan10@illinois.edu](mailto:cfan10@illinois.edu) or [c2e2help@gmail.com](mailto:c2e2help@gmail.com)