HyLAA: A Tool for Computing Simulation-Equivalent Reachability for Linear Systems

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Overview

- Computing Simulation-Equivalent Reachability using **Linear Stars**
- Invariant Constraint Trimming / Sucessor Deaggregation
- Hylaa Tool Demonstration

Motivation

- Observation: Numerical simulations are *extremely* useful
  - High-dimension scalability
  - Tunable accuracy
  - Fast
  - Trusted in practice

- But simulation is not perfect:
  - Model fidelity issues
  - Simulation accuracy
  - Point-based analysis (not on continuous trajectories)
  - Insufficient coverage of a system's nondeterminism (initial states / inputs / switching / disturbances)

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Simulation-Equivalent Reachability

- We strive to compute **exactly the set of states that any simulation might reach**, which we call simulation-equivalent reachability
- For continuous systems, this is like discrete-time reachability. For hybrid systems, a false invariant forces a transition (no sophisticated zero-crossing).
- For every state that is reachable, however, there should be a corresponding simulation which can be produced (counter-example generation)

Generalized Star Sets

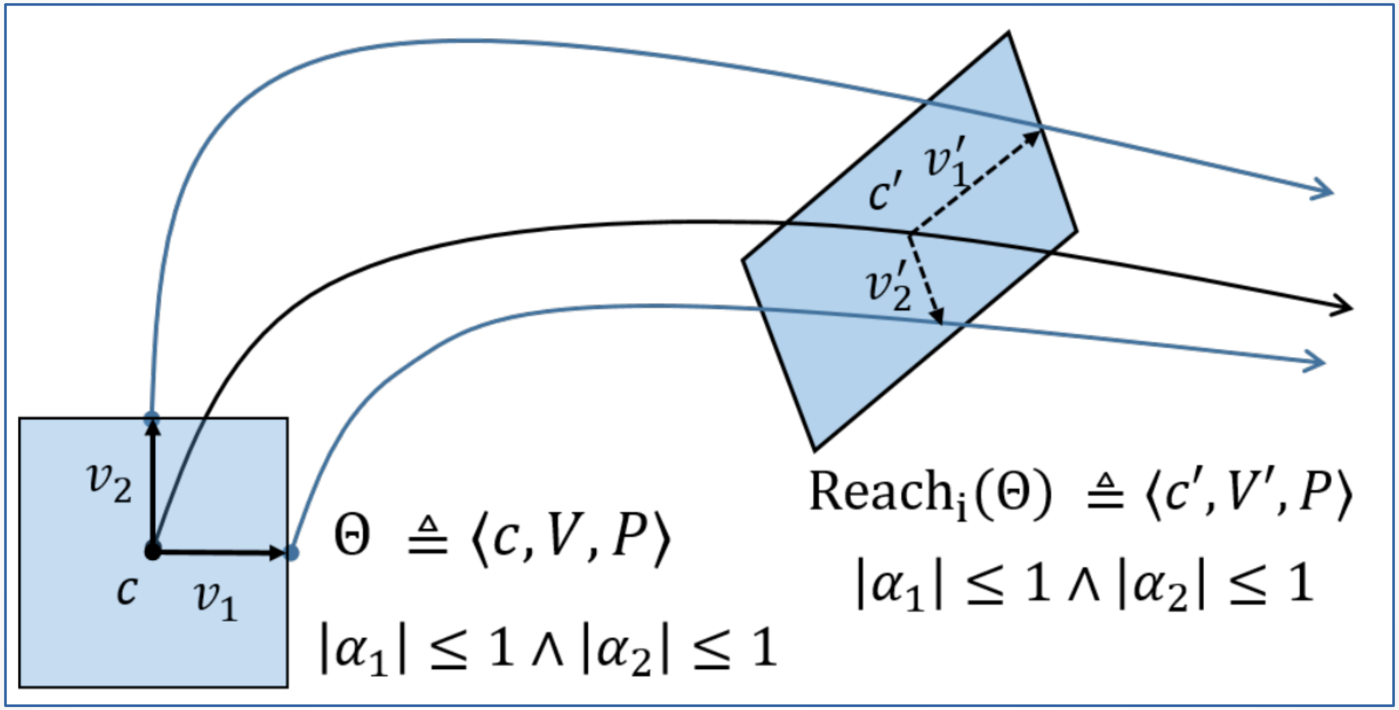
- Hylaa uses a state representation which is a version of a generalized star set.

DEFINITION 5. *A* generalized star Θ *is a tuple* $\langle c, V, P\rangle$ *where* $c \in \mathbb{R}^n$ *is called the* center, $V = \{v_1, v_2, \ldots, v_m\}$ *is a set of* m $(\leq n)$ *vectors in* $\mathbb{R}^n$ *called the* basis vectors, *and* $P : \mathbb{R}^n \to \{\top, \bot\}$ *is a predicate. A generalized star* Θ *defines a subset of* $\mathbb{R}^n$ *as follows.*

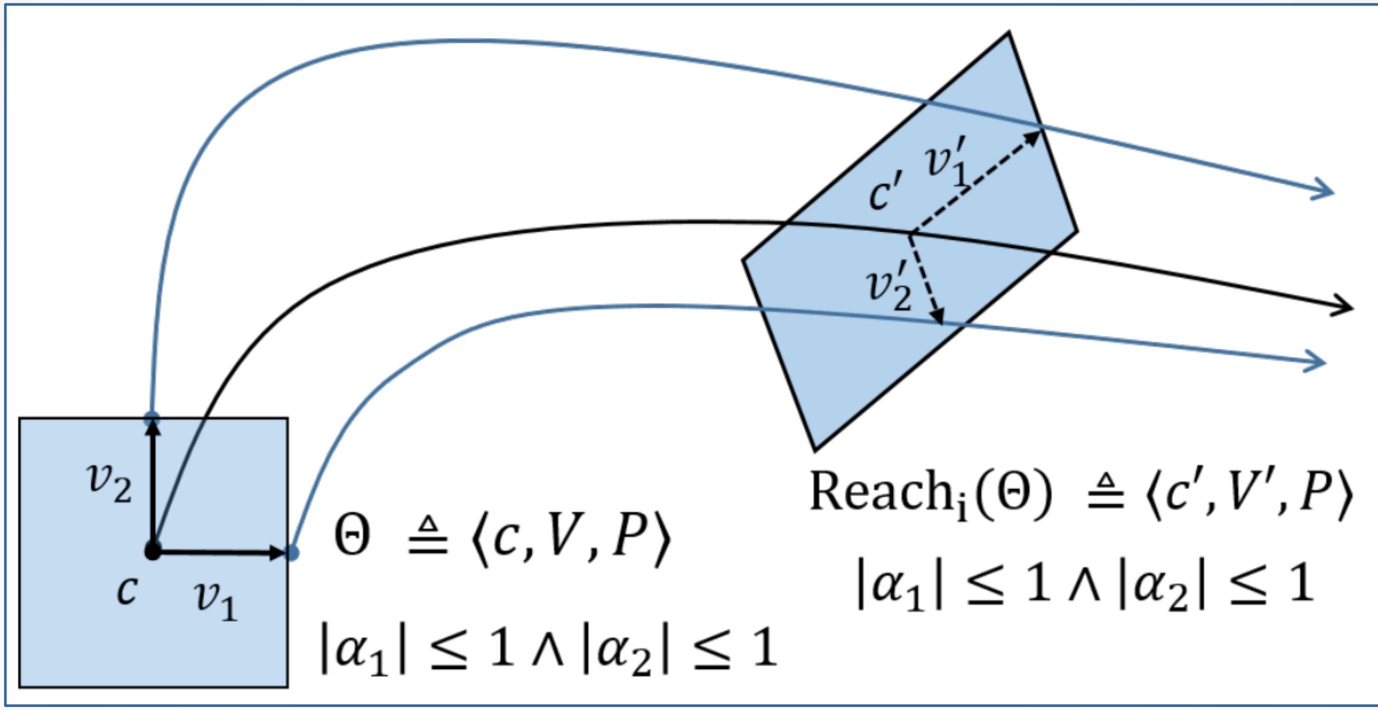
$$[\![\Theta]\!] = \{x \mid \exists \bar{\alpha} = [\alpha_1, \ldots, \alpha_m]^T \text{ such that } x = c + \Sigma_{i=1}^{n} \alpha_i v_i \text{and } P(\bar{\alpha}) = \top\}$$

"Parsimonious, Simulation Based Verification of Linear Systems", P. S. Duggirala and M. Viswanathan. International Conference on Computer Aided Verification, (CAV 2016)

Superposition



Point Containment



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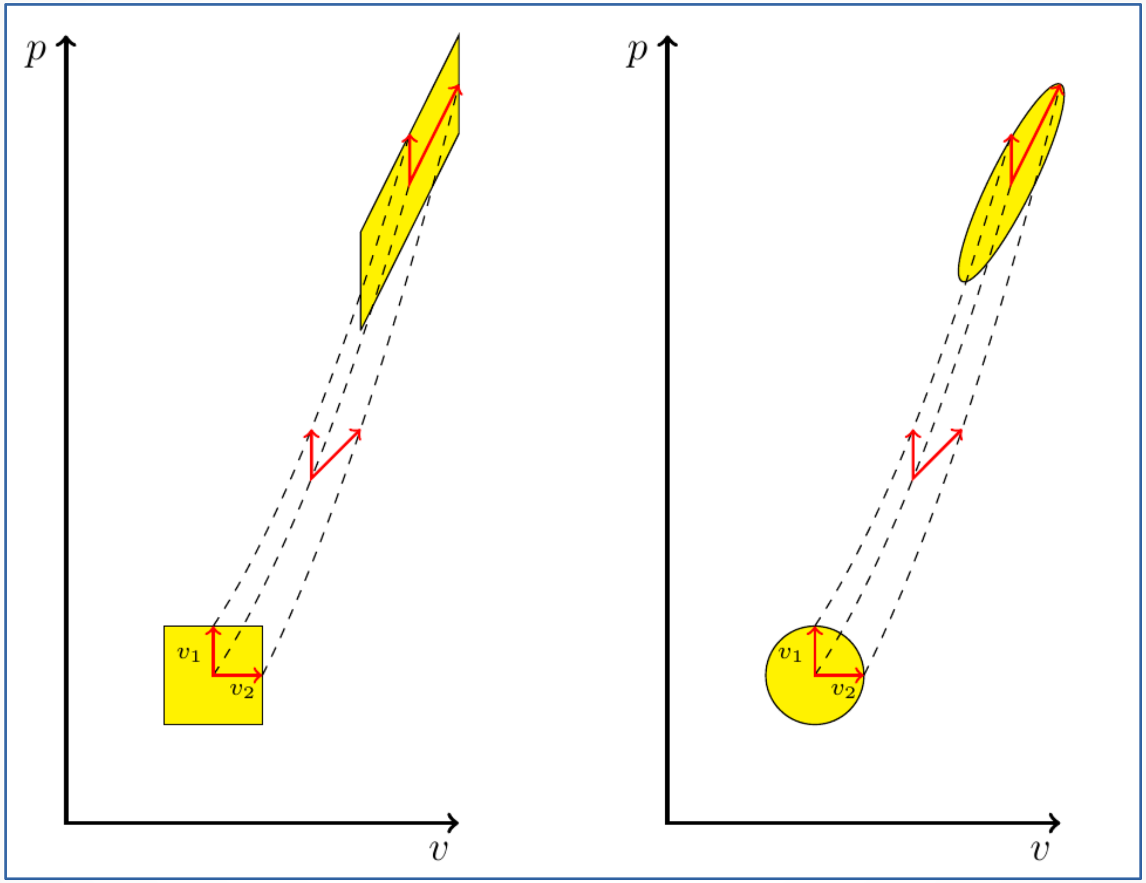

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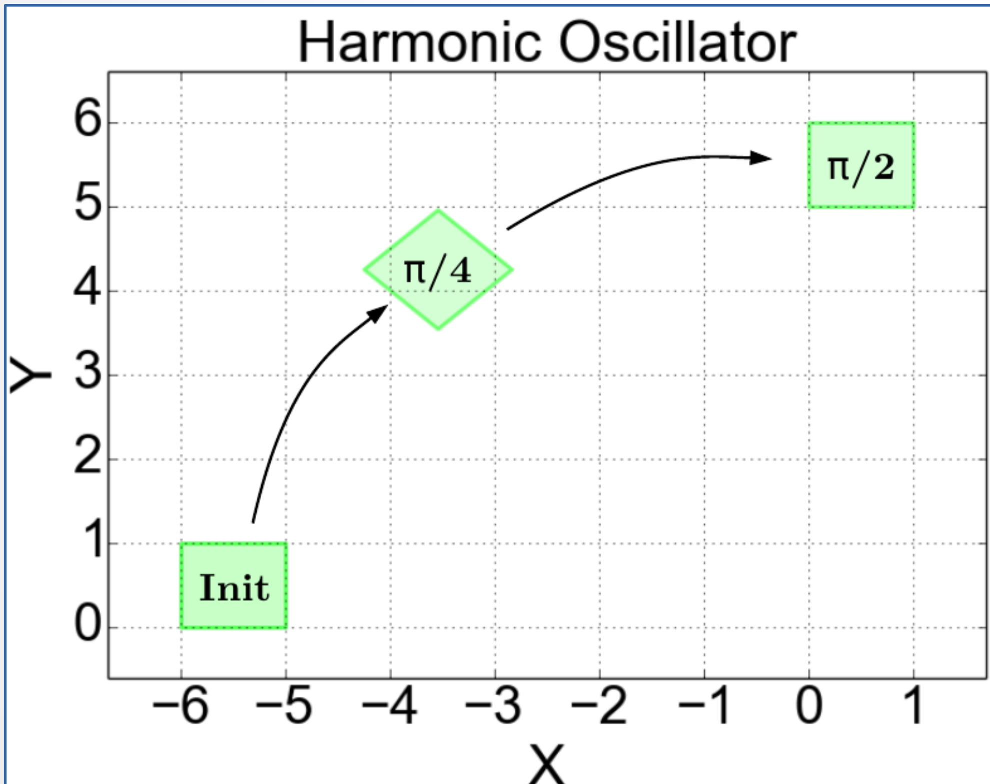
Harmonic Oscillator Example

Dynamics: x' = y, y' = -x

Initial condition: $x(0) \in [-6, -5]$, $y(0) \in [0, 1]$

At time π/4 - basis vector #1: **(1, 0) → (0.707, -0.707)**
basis vector #2: **(0, 1) → (0.707, 0.707)**



Basis Matrix at π/4

$$\begin{pmatrix} -1 & 0 & 0.707 & 0.707 \\ 0 & -1 & -0.707 & 0.707 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ \alpha_1 \\ \alpha_2 \end{pmatrix} \begin{matrix} = \\ = \\ \leq \\ \leq \\ \leq \\ \leq \end{matrix} \begin{pmatrix} 0 \\ 0 \\ 6 \\ -5 \\ 0 \\ 1 \end{pmatrix}$$

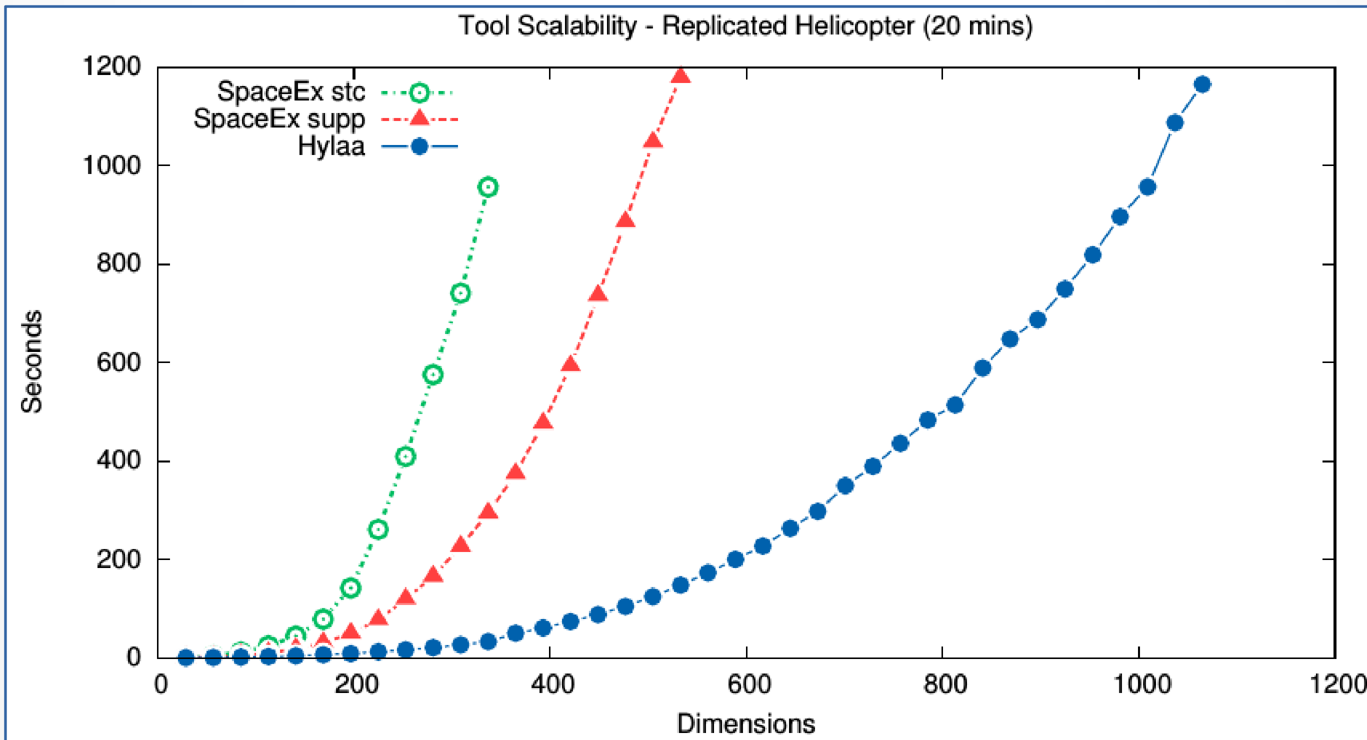
LP solver assigns:
(x, y) – state at time t
(α_1, α_2) – initial state

Continuous Post Scalability

- In 2 dimensions, we need to do 3 simulations (one for each basis vector, and one for the center)
- In **N**-dimensions, we need **N+1** simulations
- Two main computations:
  - Run n+1 simulations
  - Solve a linear program
- Both seem scalable... how scalable is the method?

Scalability Comparison

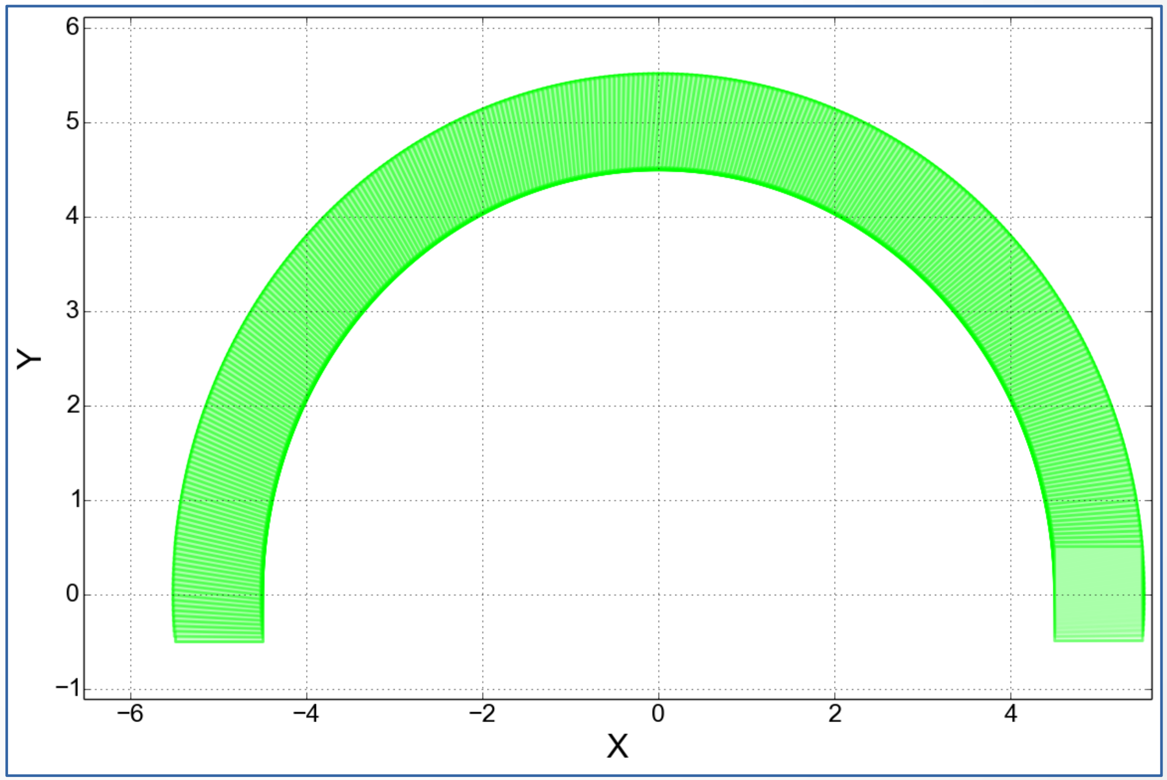
- Comparison of Hylaa vs SpaceEx
  - Replicated Helicopter (28 dims each)



(b) 20 min limit

Mode Invariant Error

- The "standard" reachability algorithm:
  - Continuous Post until invariant is false
  - Trim to invariant
  - Discrete Post
  - (repeat)



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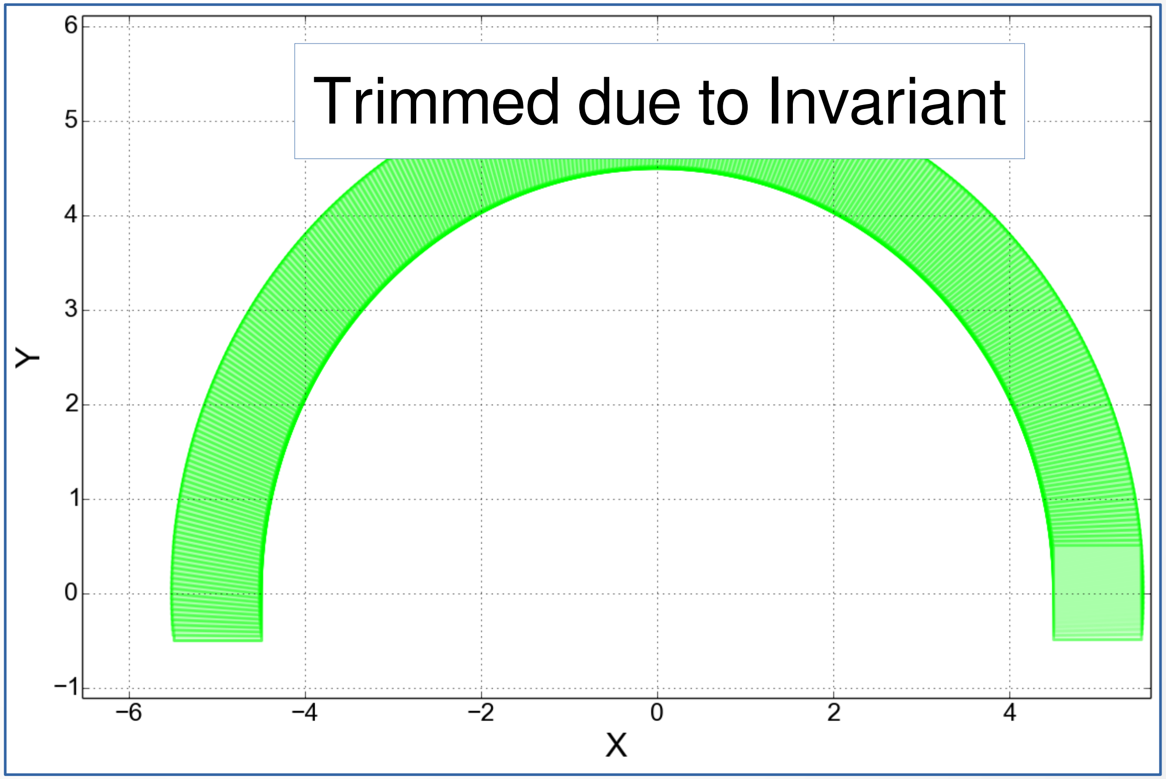
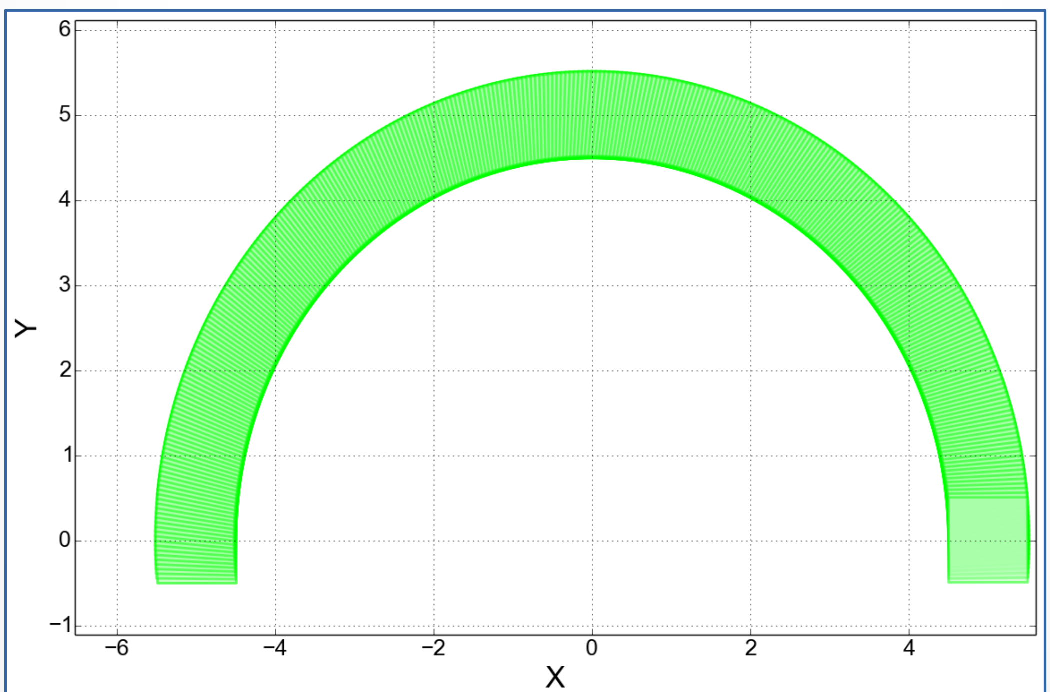


(Hylaa Demo)

invariant_trim.py

Aggregation Error

- Upon taking a discrete transition, successors are aggregated
- Desired operation is set **union**, but often only **convex-hull** is possible



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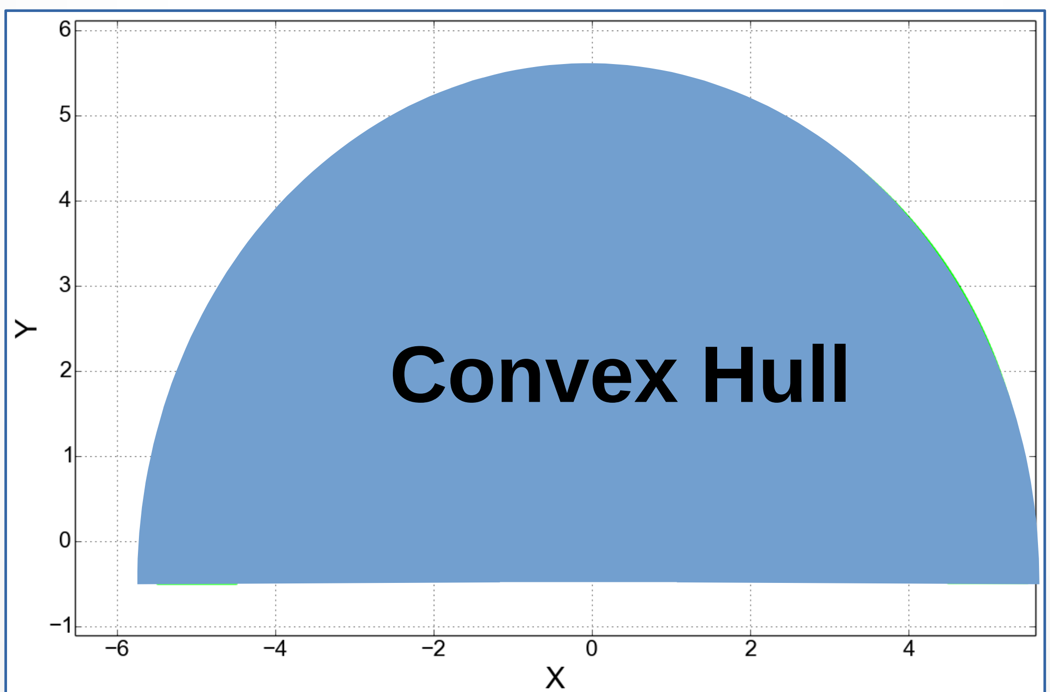

Deaggregation

- To eliminate this error, we still perform aggregation, but then **deaggregate** (split) upon reaching a subsequent guard

- Example:
  - Steps 10 to 20 have a guard enabled, and get aggregated into a single set [10, 20]
  - In the successor mode, continuous post for 1.5 seconds before another guard is reached
  - Split into two sets, [10, 14] and [15, 20]
  - Continue with each of those two sets, skipping the first 1.5 seconds

Deaggregation and Simulation-Equivalence

- With deaggregation, only states with concrete simulations can pass through guards
- Unsafe states are defined as entire modes
- Therefore, unsafe states are reachable only if a concrete simulation exists
  - *Simulation-equivalent safety*

Conclusion

Hylaa is a new tool that computes *simulation-equivalent reachability.*

The Hylaa tool code, repeatability scripts, an interactive demo, and videos are all available online:

stanleybak.com/hylaa

Our ARCH2017 paper used Hylaa to verify linear systems with over 10000 dimensions*!

* “Direct Verification of Linear Systems with over 10000 Dimensions”,
S. Bak and P. S. Duggirala, Applied Verification for Continuous and Hybrid Systems (ARCH 2017)