Lecture 4: Generative Models

Next few lectures: Generative models for direct image based rendering.

Current Image

Vision

3D Intrinsic Components

Edit Components

Graphics

New Image under different conditions

Change:

- Viewpoint
- Lighting
- Reflectance
- Background
- Attributes
- Many others…

Implicit: Use a Neural Network (Conditional Generative networks) Often, end-to-end.

Supervised vs Unsupervised Learning

Supervised Learning

Data: (x, y)

x is data, y is label

Goal: Learn a *function* to map x -> y

Examples: Classification, regression, object detection, semantic segmentation, image captioning, etc.

Classification

Cat

Supervised vs Unsupervised Learning

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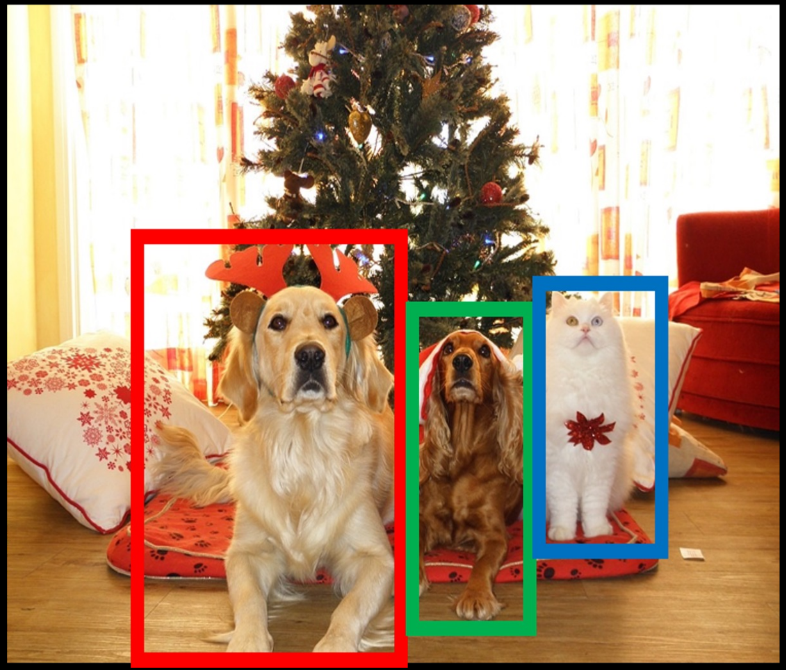
Object Detection



DOG, DOG, CAT

Supervised vs Unsupervised Learning

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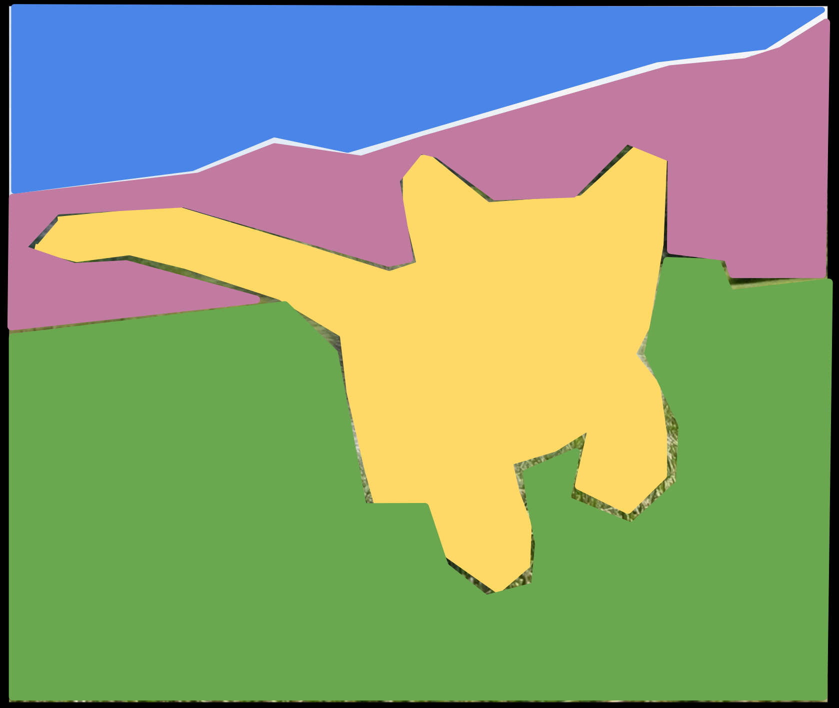
Examples: Classification, regression, object detection, semantic segmentation, image captioning, etc.

Semantic Segmentation



GRASS, CAT, TREE, SKY

Supervised vs Unsupervised Learning

Supervised Learning

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Goal: Learn a *function* to map x -> y

Examples: Classification, regression, object detection, semantic segmentation, image captioning, etc.

Image captioning

A cat sitting on a suitcase on the floor

Supervised vs Unsupervised Learning

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Unsupervised Learning

Data: x

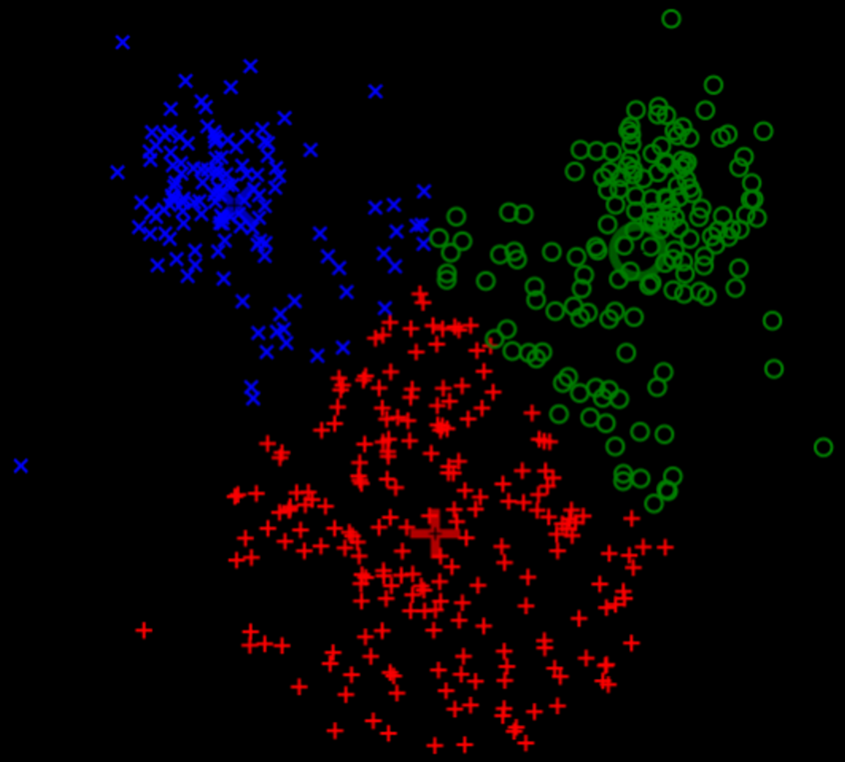
Just data, no labels!

Goal: Learn some underlying hidden *structure* of the data

Examples: Clustering, dimensionality reduction, feature learning, density estimation, etc.

Supervised vs Unsupervised Learning

Clustering
(e.g. K-Means)



Unsupervised Learning

Data: x

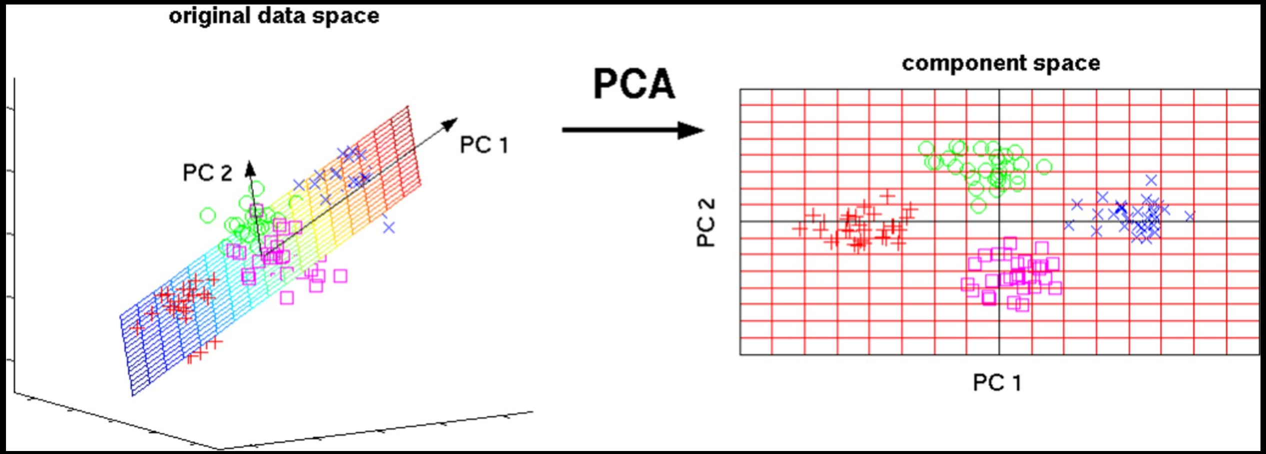
Just data, no labels!

Goal: Learn some underlying hidden *structure* of the data

Examples: Clustering, dimensionality reduction, feature learning, density estimation, etc.

Supervised vs Unsupervised Learning

Dimensionality Reduction
(e.g. Principal Components Analysis)



3D → 2D

Unsupervised Learning

Data: x

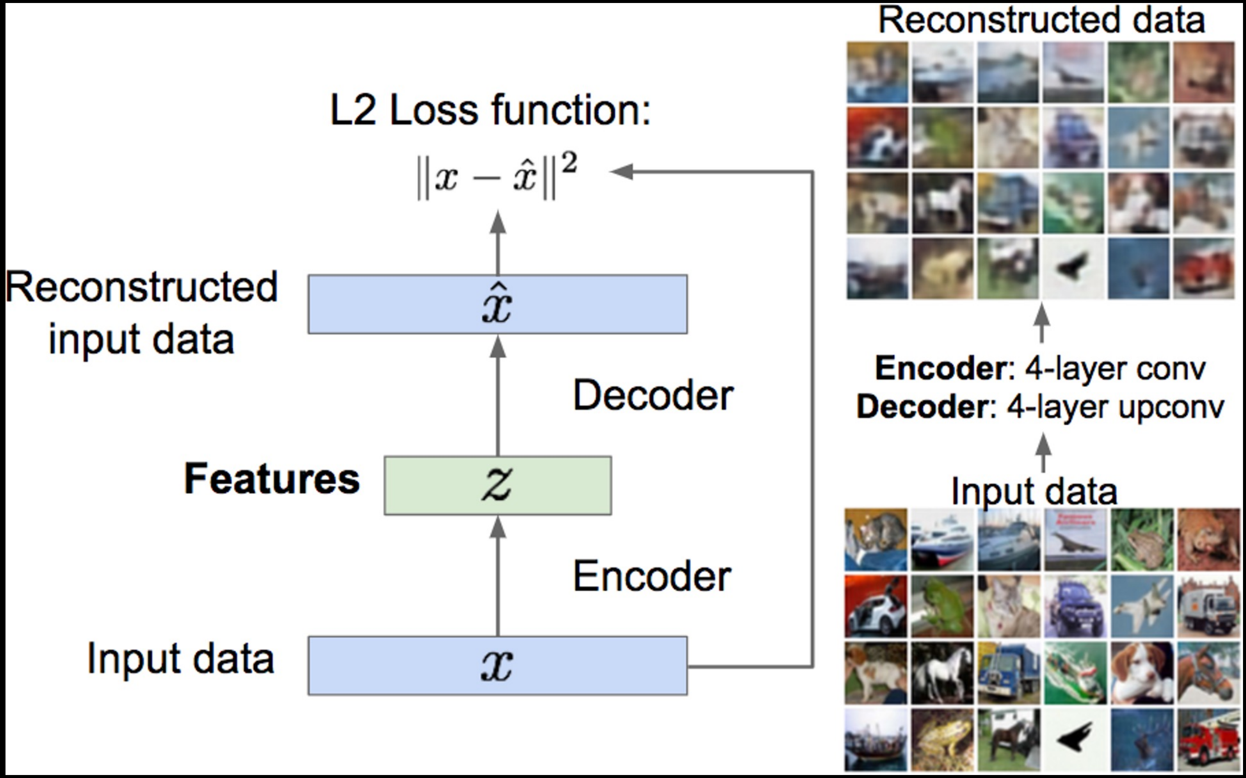
Just data, no labels!

Goal: Learn some underlying hidden *structure* of the data

Examples: Clustering, dimensionality reduction, feature learning, density estimation, etc.

Supervised vs Unsupervised Learning

Feature Learning
(e.g. autoencoders)



Unsupervised Learning

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Supervised vs Unsupervised Learning

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Discriminative vs Generative Models

Discriminative Model:
Learn a probability distribution p(y|x)

Generative Model:
Learn a probability distribution p(x)

Conditional Generative Model: Learn p(x|y)

Data: x

Label: y

Cat

Discriminative vs Generative Models

Discriminative Model:
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Conditional Generative Model: Learn $p(x|y)$

Data: x

Label: y

Cat

Probability Recap:

Density Function
$p(x)$ assigns a positive number to each possible x; higher numbers mean x is more likely

Density functions are **normalized**:

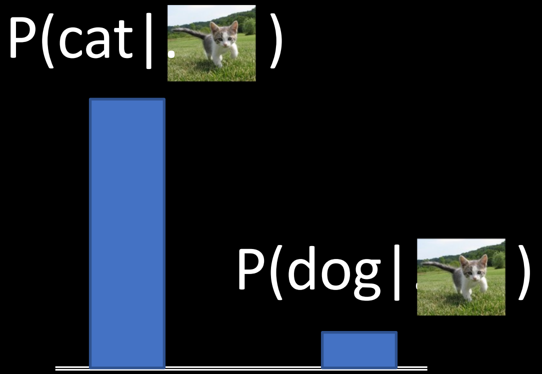
$$\int_X p(x)dx = 1$$

Different values of x **compete** for density

Discriminative vs Generative Models

Discriminative Model: Learn a probability distribution p(y|x)

Data: x

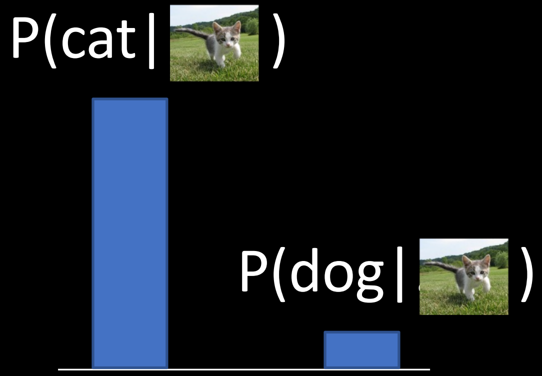


Generative Model: Learn a probability distribution p(x)

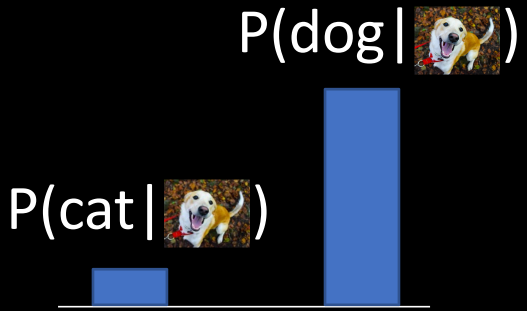
Conditional Generative Model: Learn p(x|y)

Discriminative vs Generative Models

Discriminative Model: Learn a probability distribution $p(y|x)$



Generative Model: Learn a probability distribution $p(x)$



Conditional Generative Model: Learn $p(x|y)$

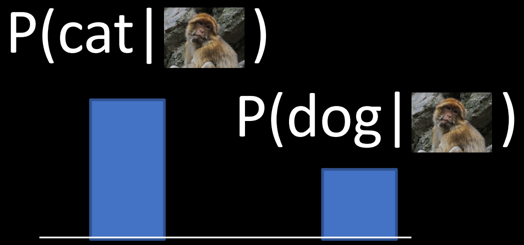
Discriminative model: the possible labels for each input "compete" for probability mass. But no competition between **images**

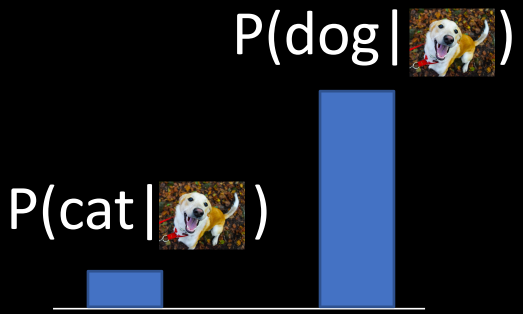
Discriminative vs Generative Models

Discriminative Model: Learn a probability distribution p(y|x)

Generative Model: Learn a probability distribution p(x)

Conditional Generative Model: Learn p(x|y)

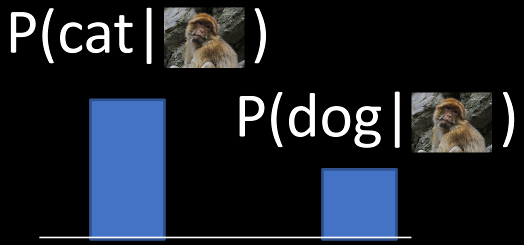




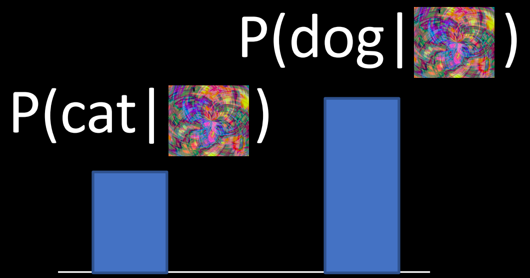
Discriminative model: No way for the model to handle unreasonable inputs; it must give label distributions for all images

Discriminative vs Generative Models

Discriminative Model: Learn a probability distribution p(y|x)



Generative Model: Learn a probability distribution p(x)



Conditional Generative Model: Learn p(x|y)

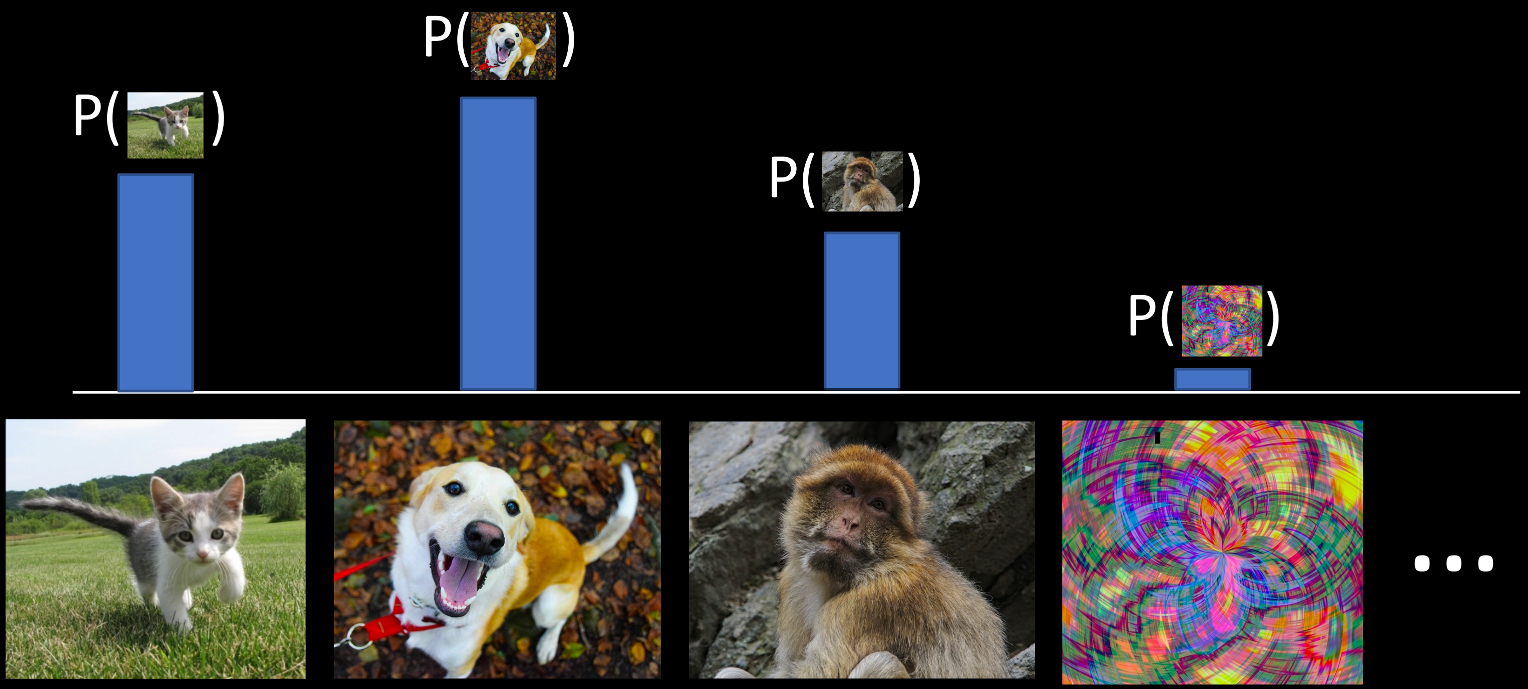
Discriminative model: No way for the model to handle unreasonable inputs; it must give label distributions for all images

Discriminative vs Generative Models

Discriminative Model: Learn a probability distribution $p(y|x)$

Generative Model: Learn a probability distribution $p(x)$

Conditional Generative Model: Learn $p(x|y)$



Generative model: All possible images compete with each other for probability mass

Requires deep image understanding! Is a dog more likely to sit or stand? How about 3-legged dog vs 3-armed monkey?

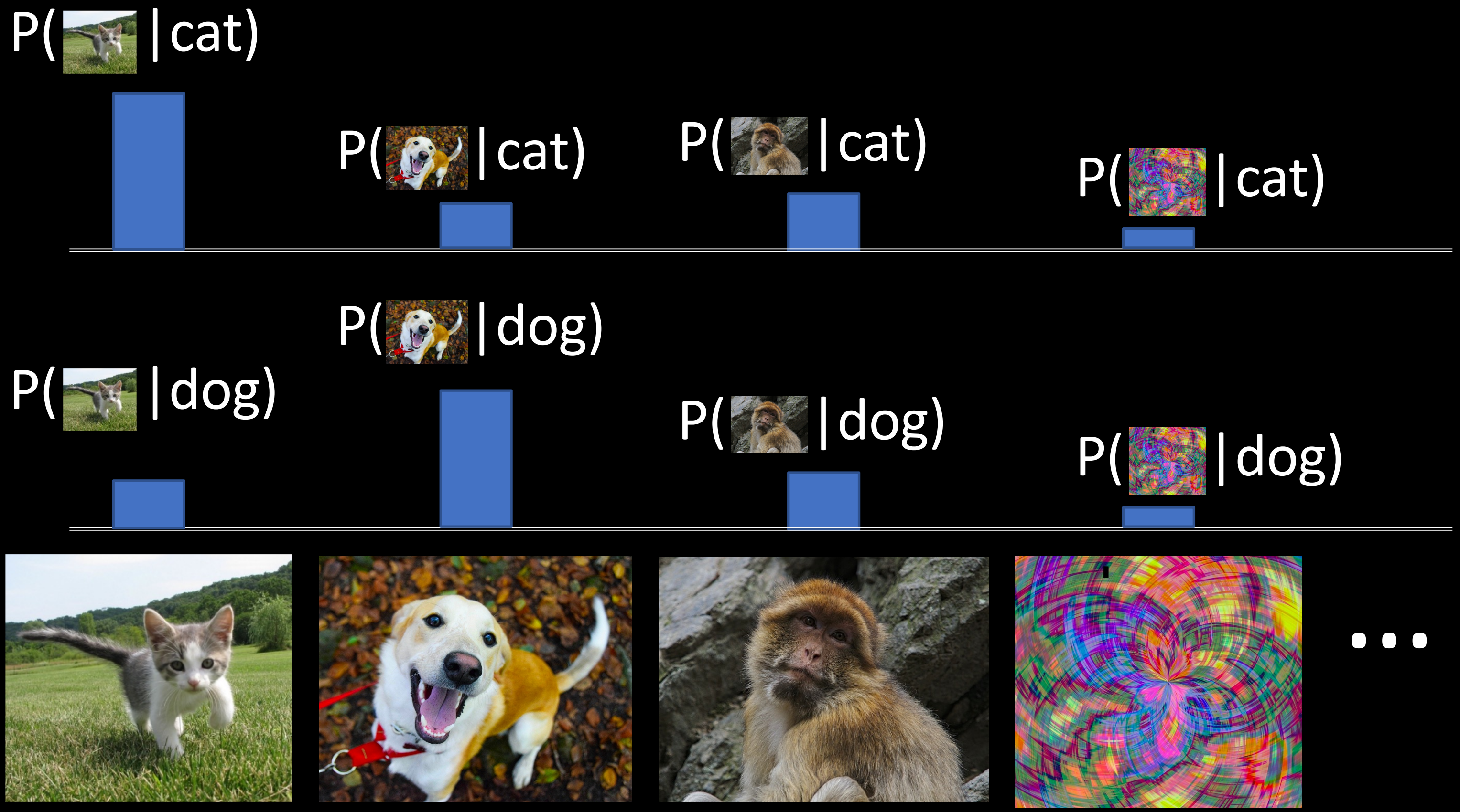
Model can “reject” unreasonable inputs by assigning them small values

Discriminative vs Generative Models

Discriminative Model: Learn a probability distribution p(y|x)

Generative Model: Learn a probability distribution p(x)

Conditional Generative Model: Learn p(x|y)



Conditional Generative Model: Each possible label induces a competition among all images

Discriminative vs Generative Models

Discriminative Model: Learn a probability distribution p(y|x)

Generative Model: Learn a probability distribution p(x)

Conditional Generative Model: Learn p(x|y)

Recall **Bayes' Rule:**

$$P(x \mid y) = \frac{P(y \mid x)}{P(y)} P(x)$$

Discriminative vs Generative Models

Discriminative Model:
Learn a probability distribution p(y|x)

Generative Model:
Learn a probability distribution p(x)

Conditional Generative Model: Learn p(x|y)

Recall **Bayes' Rule:**

$$\underbrace{\boxed{P(x \mid y)}}_{\text{Conditional Generative Model}} = \frac{\overbrace{\boxed{P(y \mid x)}}^{\text{Discriminative Model}}}{\underbrace{\boxed{P(y)}}_{\text{Prior over labels}}} \overbrace{\boxed{P(x)}}^{\text{(Unconditional) Generative Model}}$$

We can build a conditional generative model from other components!

What can we do with a discriminative model?

- **Discriminative Model:** Learn a probability distribution p(y|x) → Assign labels to data; Feature learning (with labels)

- **Generative Model**: Learn a probability distribution p(x)

- **Conditional Generative Model:** Learn p(x|y)

What can we do with a generative model?

- **Discriminative Model:** Learn a probability distribution $p(y|x)$ → Assign labels to data; Feature learning (with labels)

- **Generative Model**: Learn a probability distribution $p(x)$ → Detect outliers; Feature learning (without labels); Sample to **generate** new data

- **Conditional Generative Model:** Learn $p(x|y)$

What can we do with a generative model?

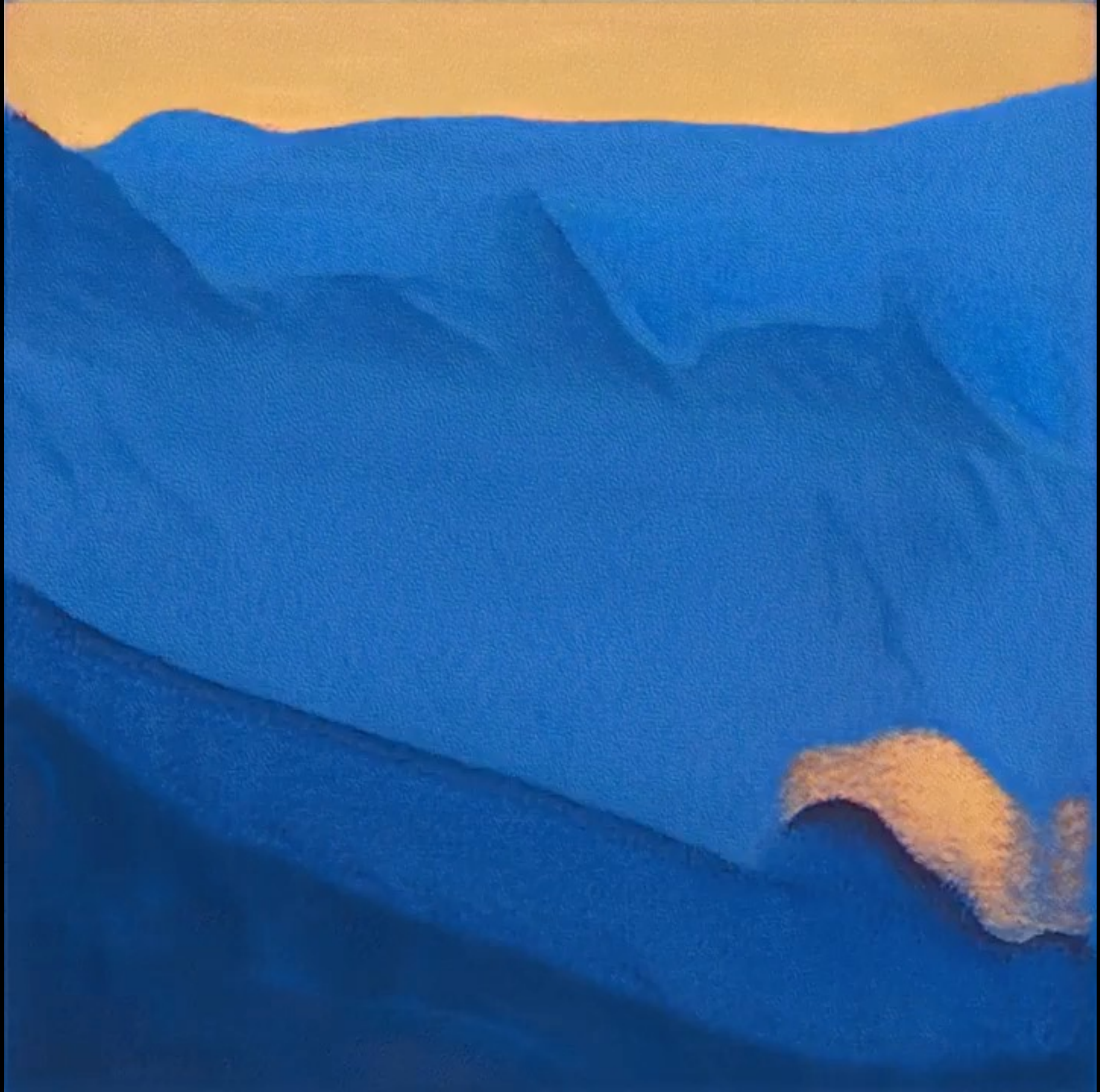
- **Discriminative Model:** Learn a probability distribution $p(y|x)$ → Assign labels to data; Feature learning (with labels)

- **Generative Model**: Learn a probability distribution $p(x)$ → Detect outliers; Feature learning (without labels); Sample to **generate** new data

- **Conditional Generative Model:** Learn $p(x|y)$ → Assign labels, while rejecting outliers!; Generate new data conditioned on input labels

Introduction to Generative Models (Conditional and Unconditional)

What cool things can we do with it?



Credits: Neural Synesthesia

PLAY ABOUT METHODS LEARN PRESS CONTACT BOOK CALLING BS

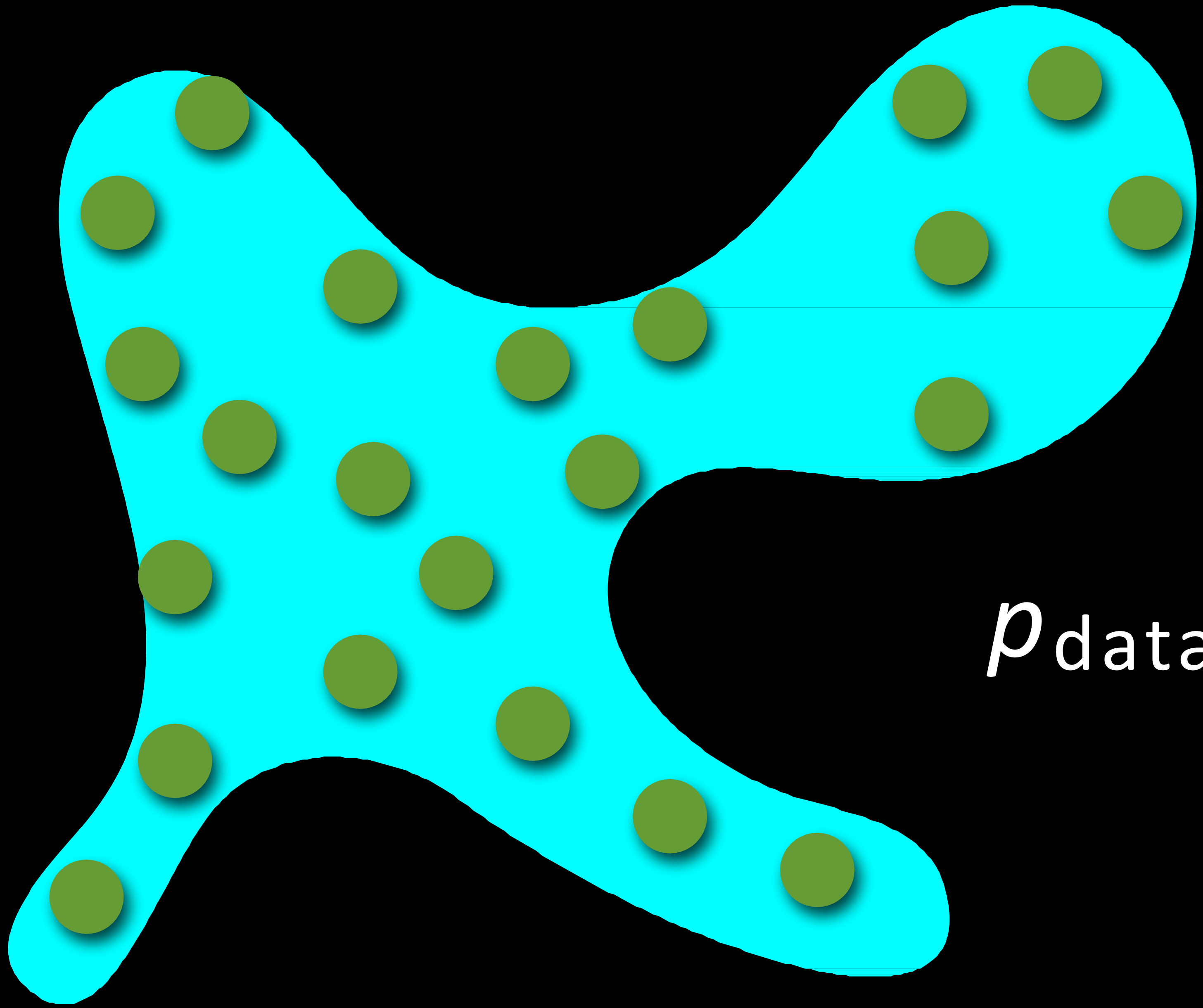
Click on the person who is real.

https://www.whichfaceisreal.com/index.php

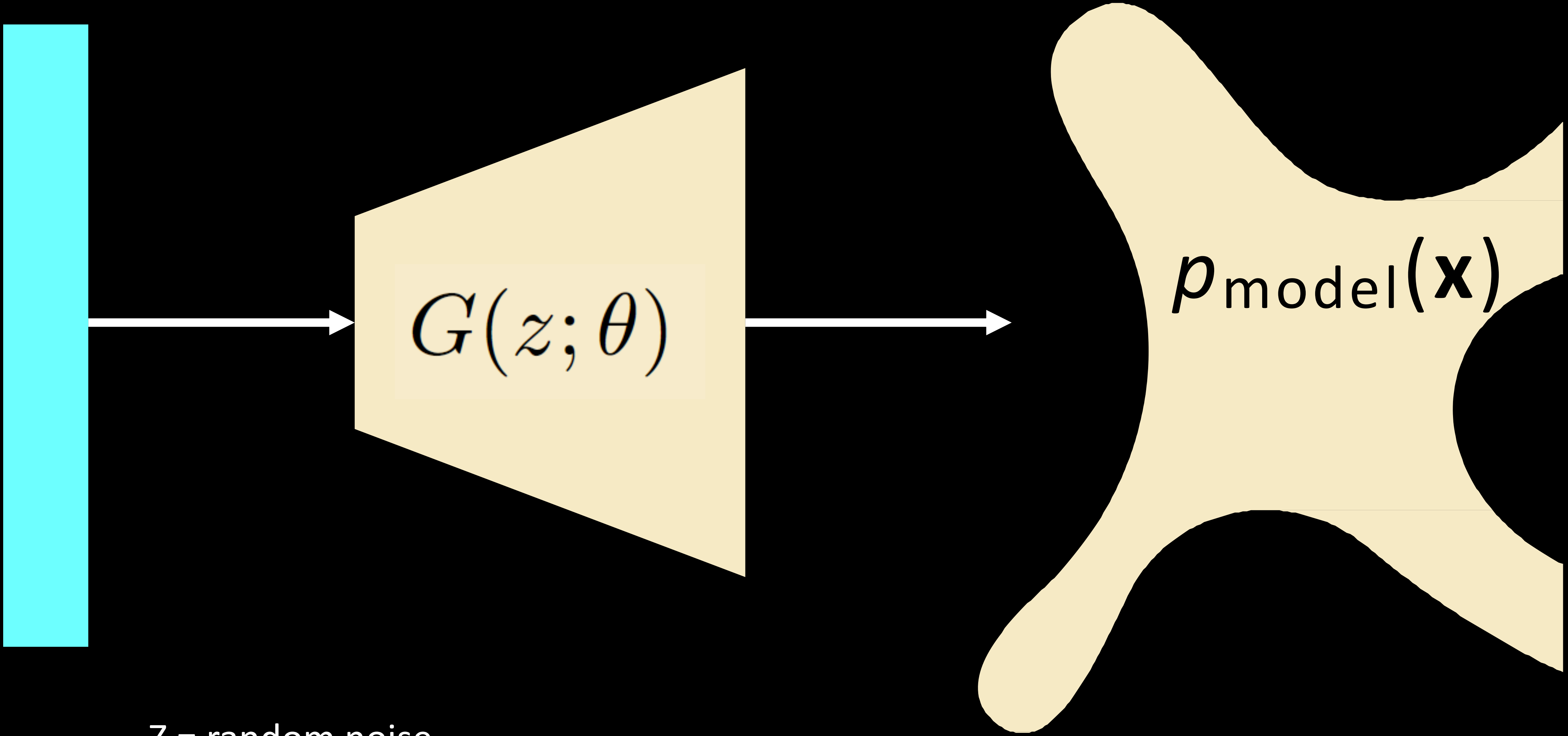
$$p_{\text{data}}(\mathbf{x})$$

$$p_{\mathrm{data}}(\mathbf{x}) \approx p_{\mathrm{model}}(\mathbf{x})$$

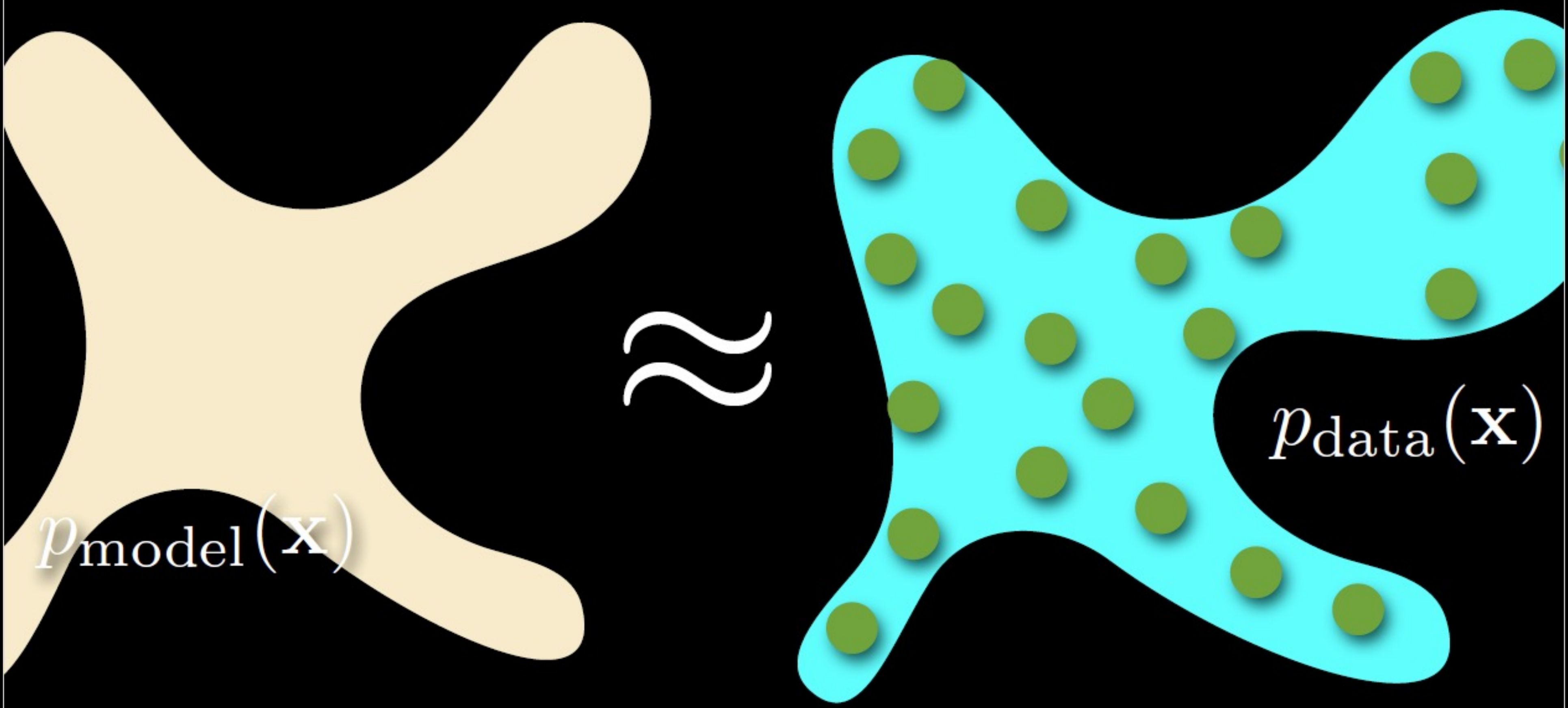

TNT



$p_{\text{data}}(\mathbf{x})$



Z = random noise
(samples from latent space)



$p_{\text{model}}(\mathbf{x})$
$\approx$
$p_{\text{data}}(\mathbf{x})$

Taxonomy of Generative Models

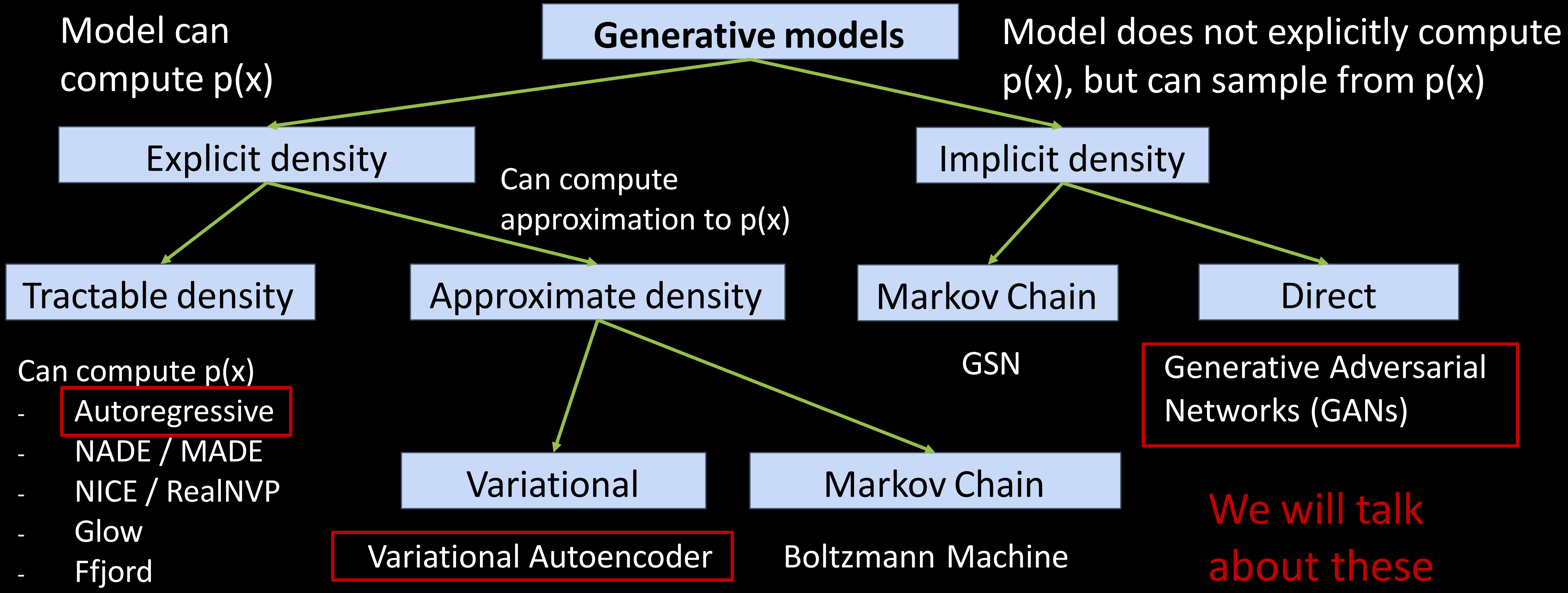


Figure adapted from Ian Goodfellow, Tutorial on Generative Adversarial Networks, 2017.

Autoregressive models: PixelRNN & PixelCNN

Explicit Density Estimation

Learn weights W

Goal: Write down an explicit function for $p(x) = f(x, \boxed{W})$

Given dataset $x^{(1)}, x^{(2)}, \ldots x^{(N)}$, train the model by solving:

$$W^* = \arg\max_{W} \prod_i p(x^{(i)})$$

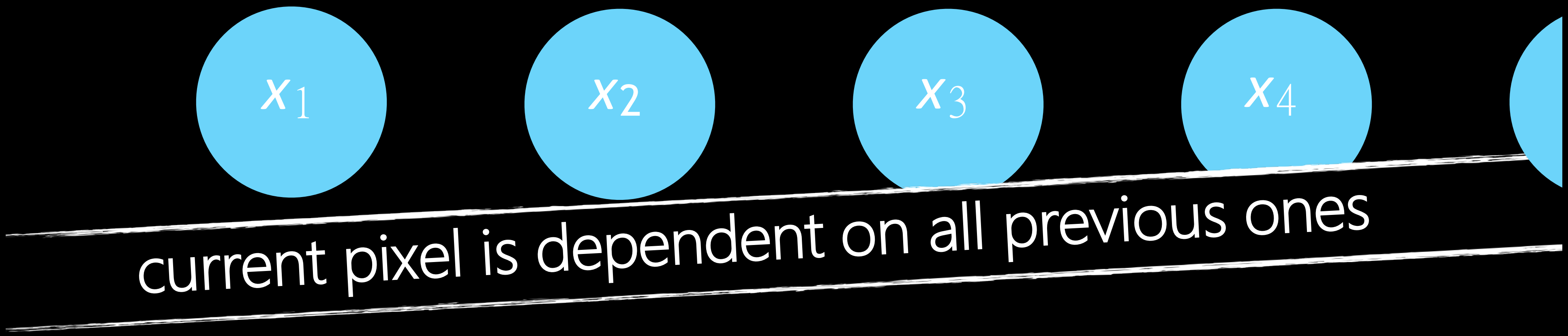
Maximize probability of training data
(Maximum likelihood estimation)

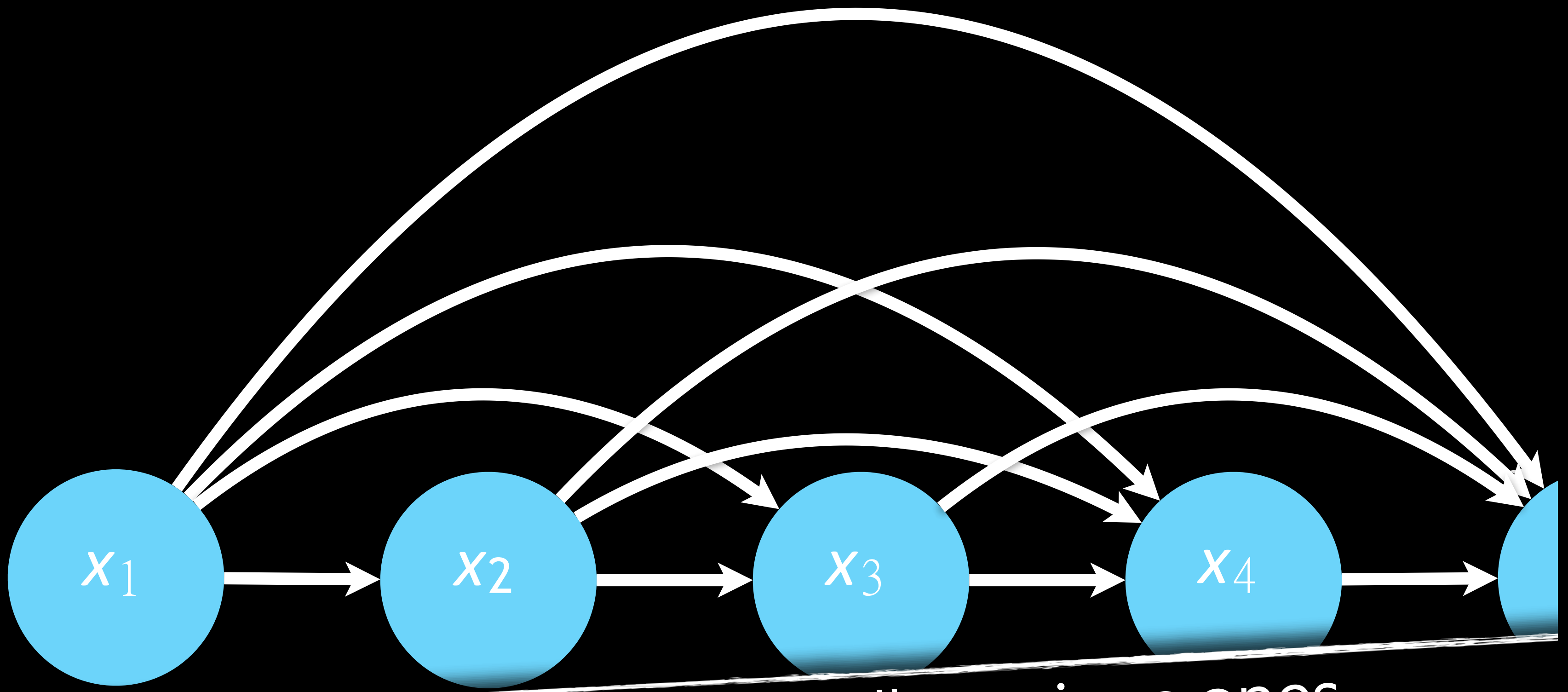
$$= \arg\max_{W} \sum_i \log p(x^{(i)})$$

Log trick to exchange product for sum

$$= \arg\max_{W} \sum_i \log f(x^{(i)}, W)$$

This will be our loss function!
Train with gradient descent


x_1
x_2
x_3
x_4
current pixel is dependent on all previous ones



current pixel is dependent on all previous ones

$$p_{\mathrm{model}}(\mathbf{x}) = \prod_{i=1}^{N} p(x_i \mid x_1, \ldots, x_{i-1})$$

use chain rule to decompose

$$p_{\mathrm{model}}(\mathbf{x}) = \prod_{i=1}^{N} p(x_i | x_1, \ldots, x_{i-1})$$

$$p_{\mathrm{model}}(\mathbf{x}) = \prod^{N} p(x_i | x_1, \ldots, x_{i-1})$$

probability of ith pixel given all previous pixels

$$p_{\text{model}}(\mathbf{x}) = \prod^{N} p(x_i|x_1, \ldots, x_{i-1})$$

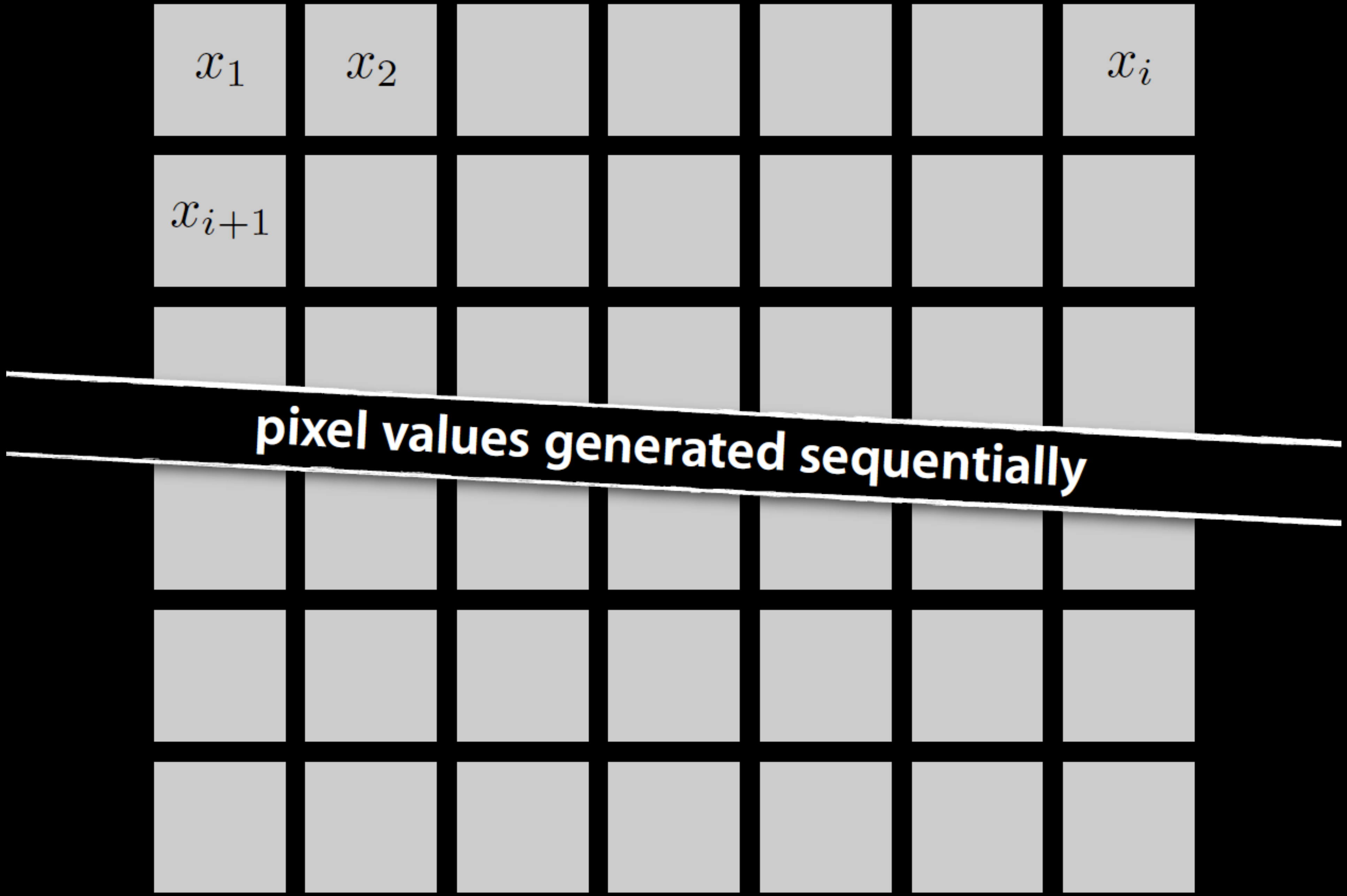
express distribution over pixel values as neural network

$$p_{\text{model}}(\mathbf{x}) = \prod_{i=1}^{N} p(x_i | x_1, \ldots, x_{i-1})$$

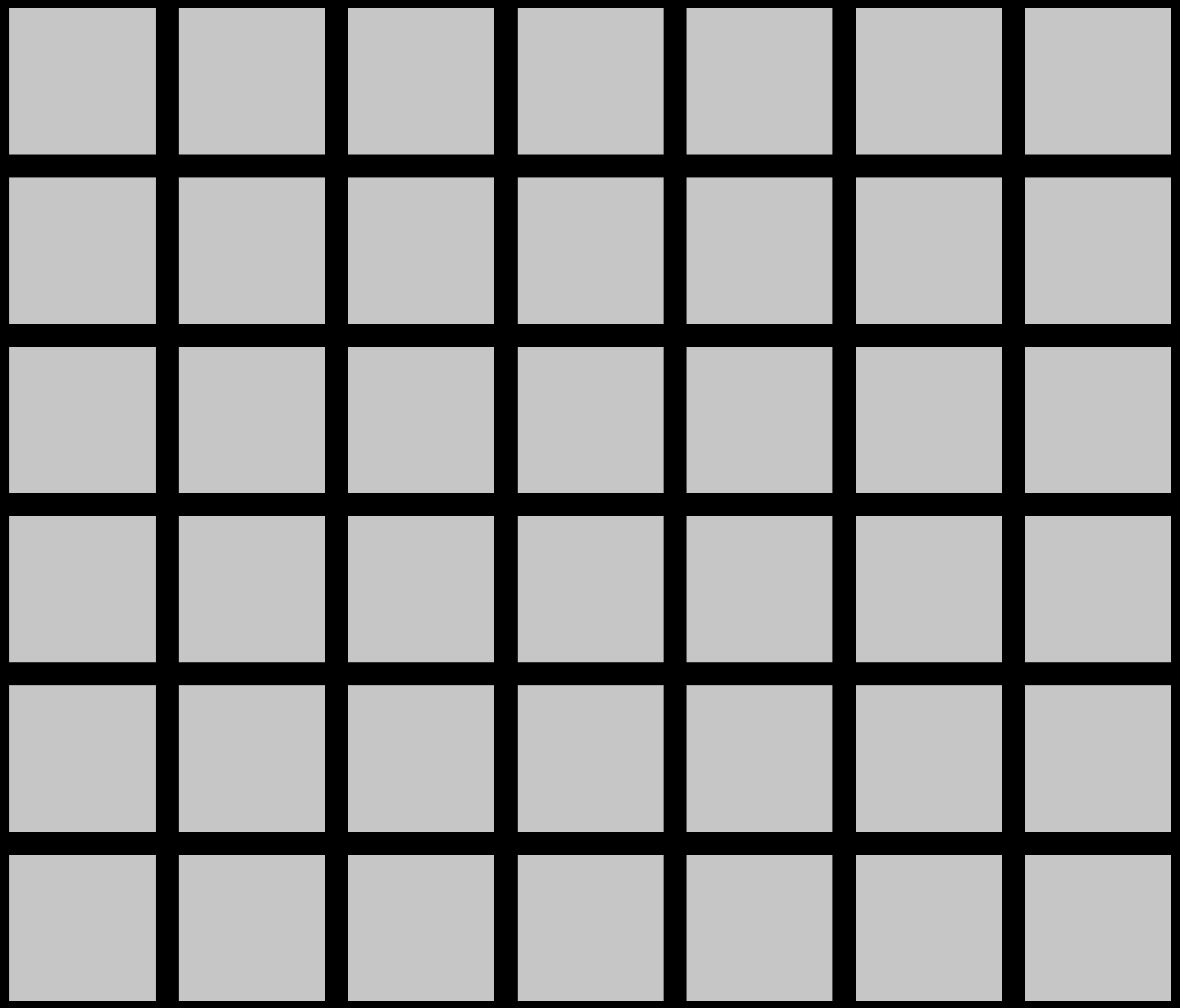
maximize likelihood of training data with respect to network weights

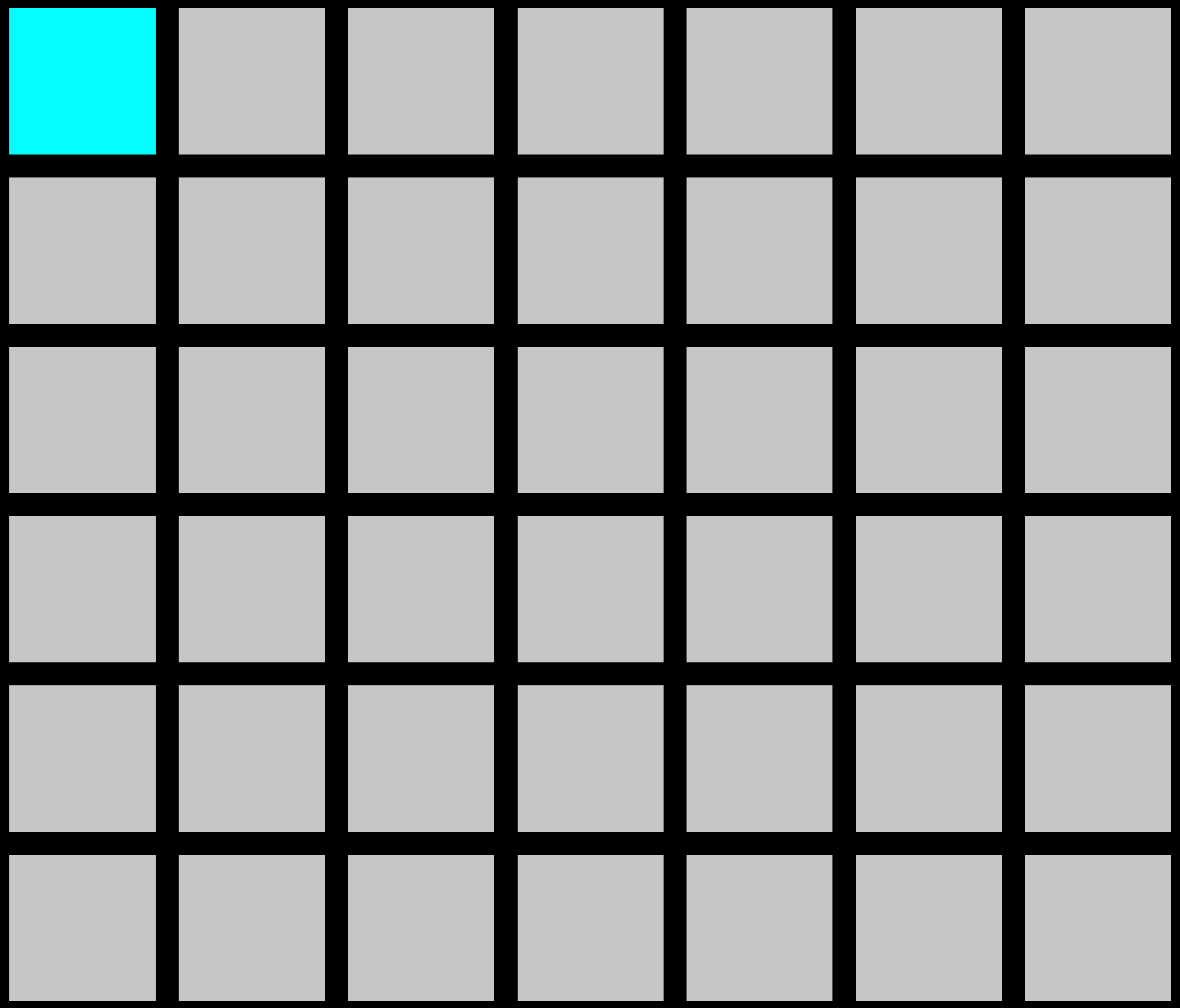
Maximum Likelihood estimate: $$arg \max_{\theta} \; log f_{model}(x;\theta)$$

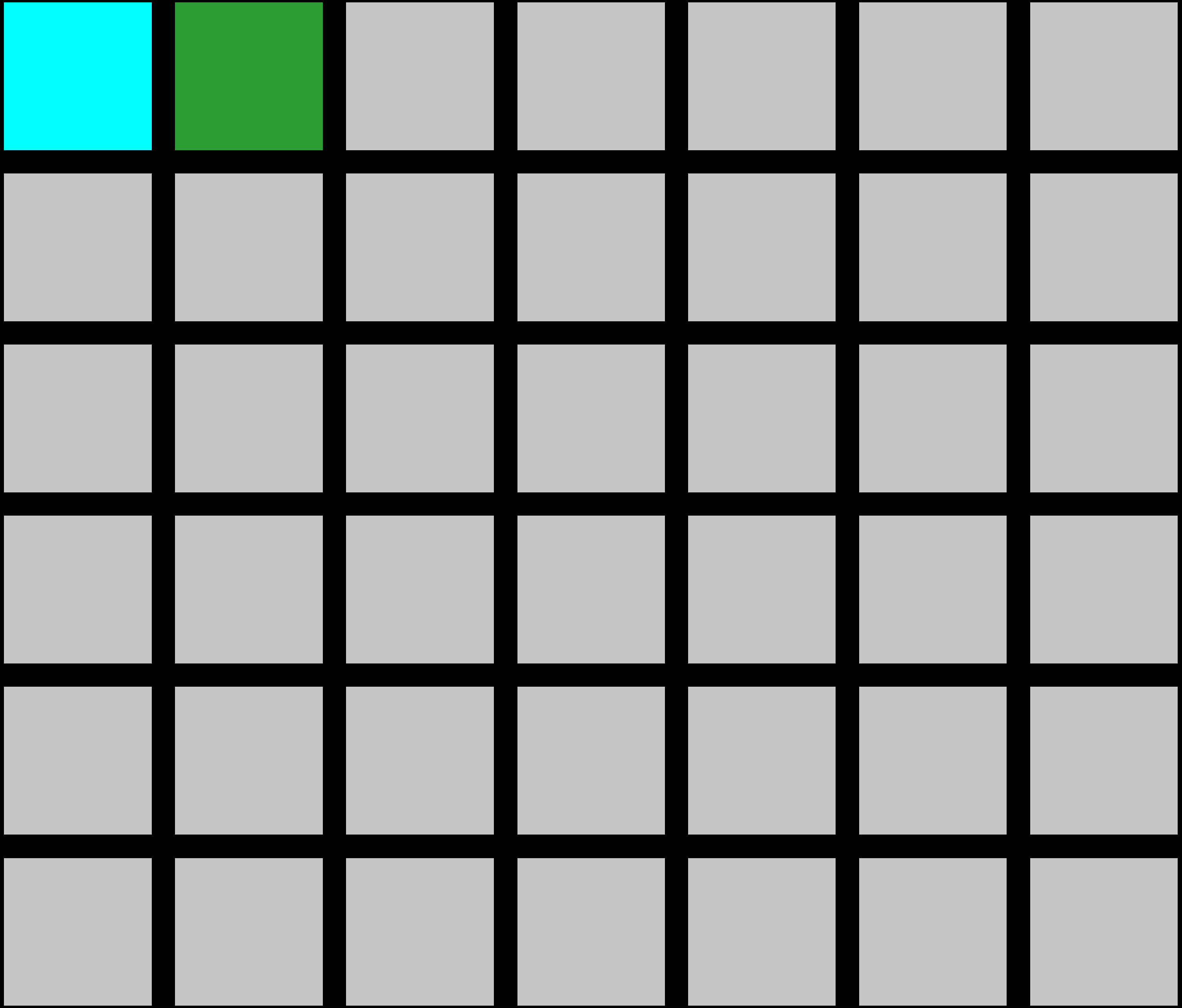
- Maximize this estimator w.r.t. neural network parameter $\boldsymbol{\theta}$.
- In practice we minimize negative of the above estimator as our loss function.

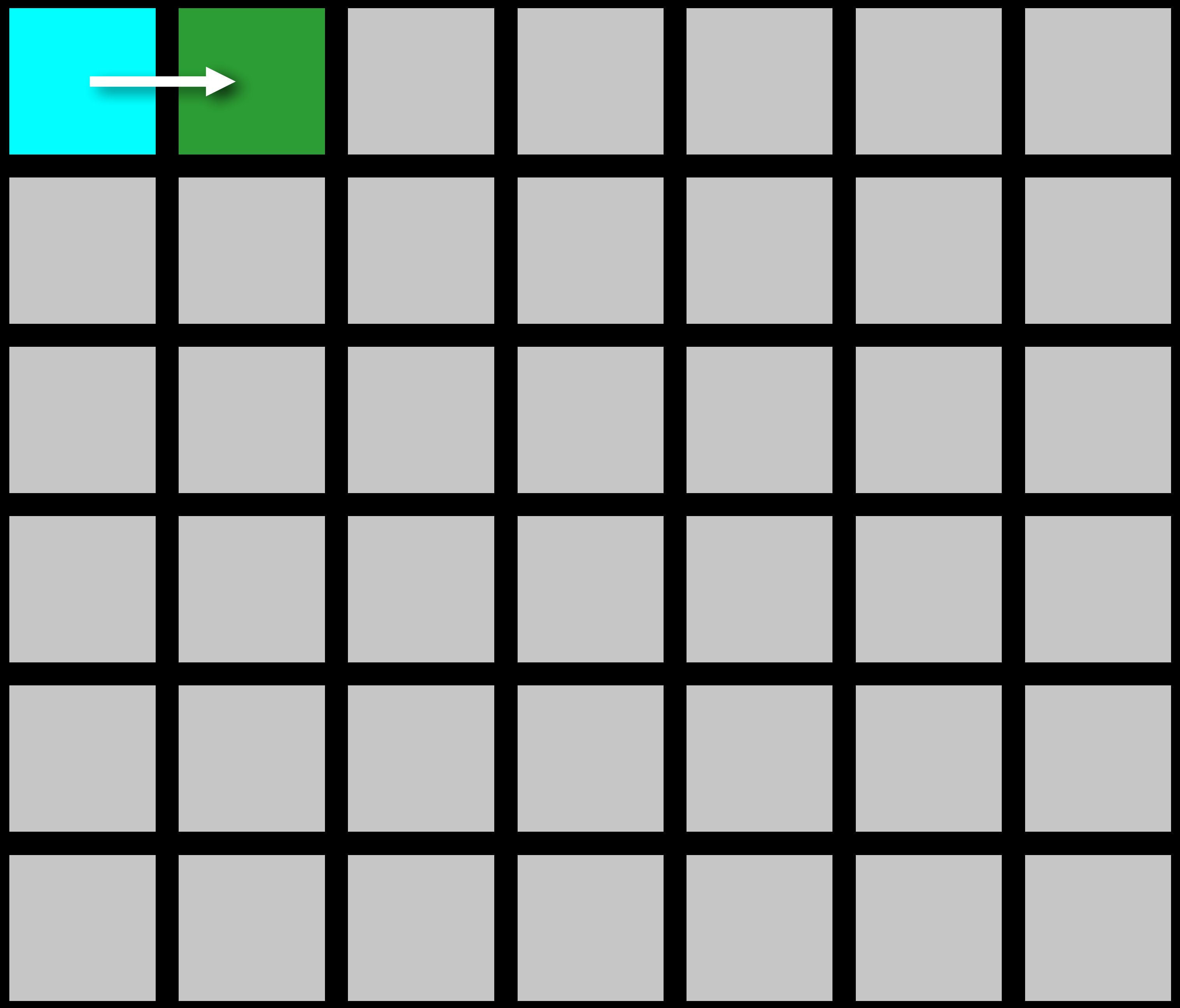

x_1
x_2
x_i
x_{i+1}
pixel values generated sequentially

PixelRNN

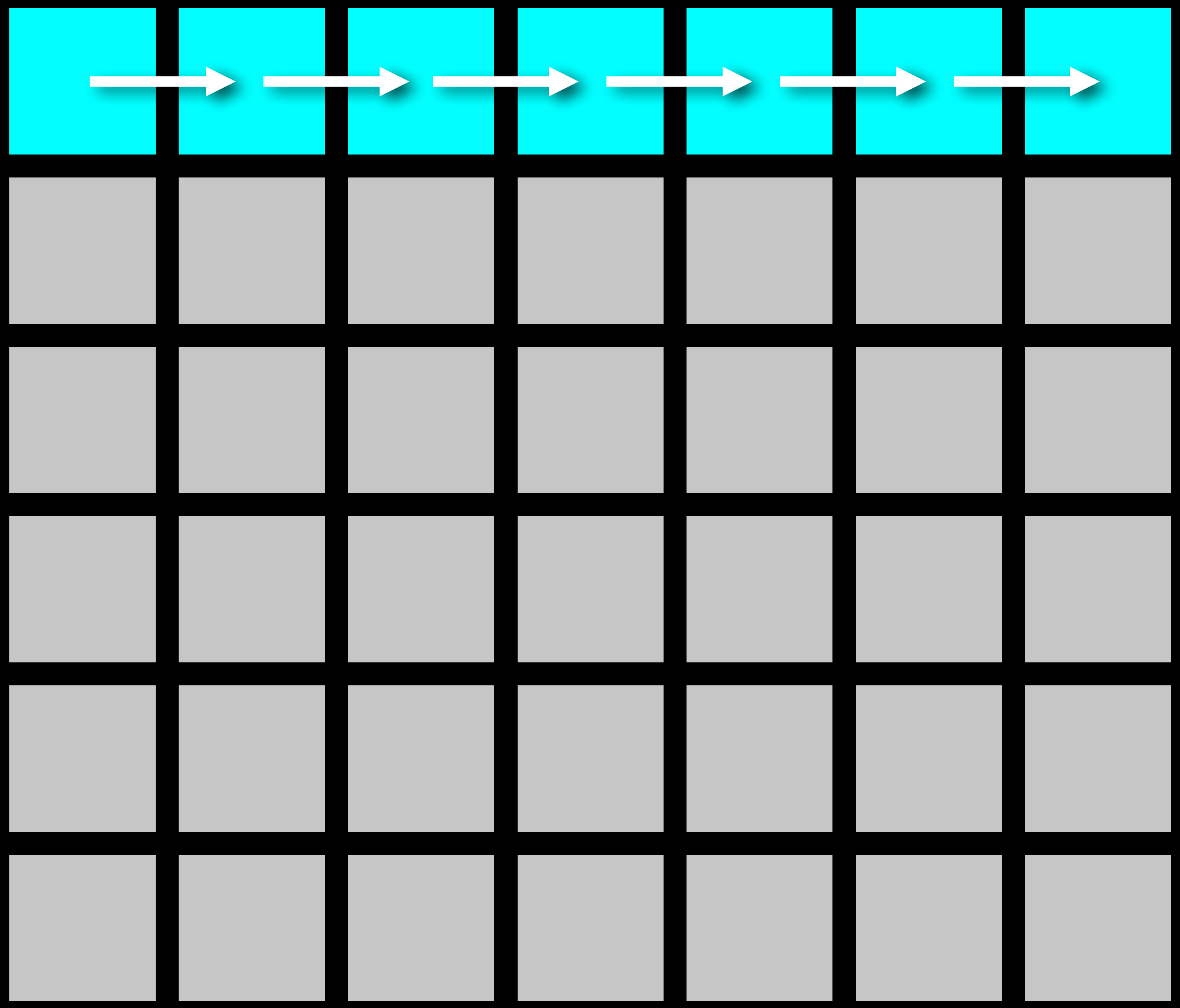


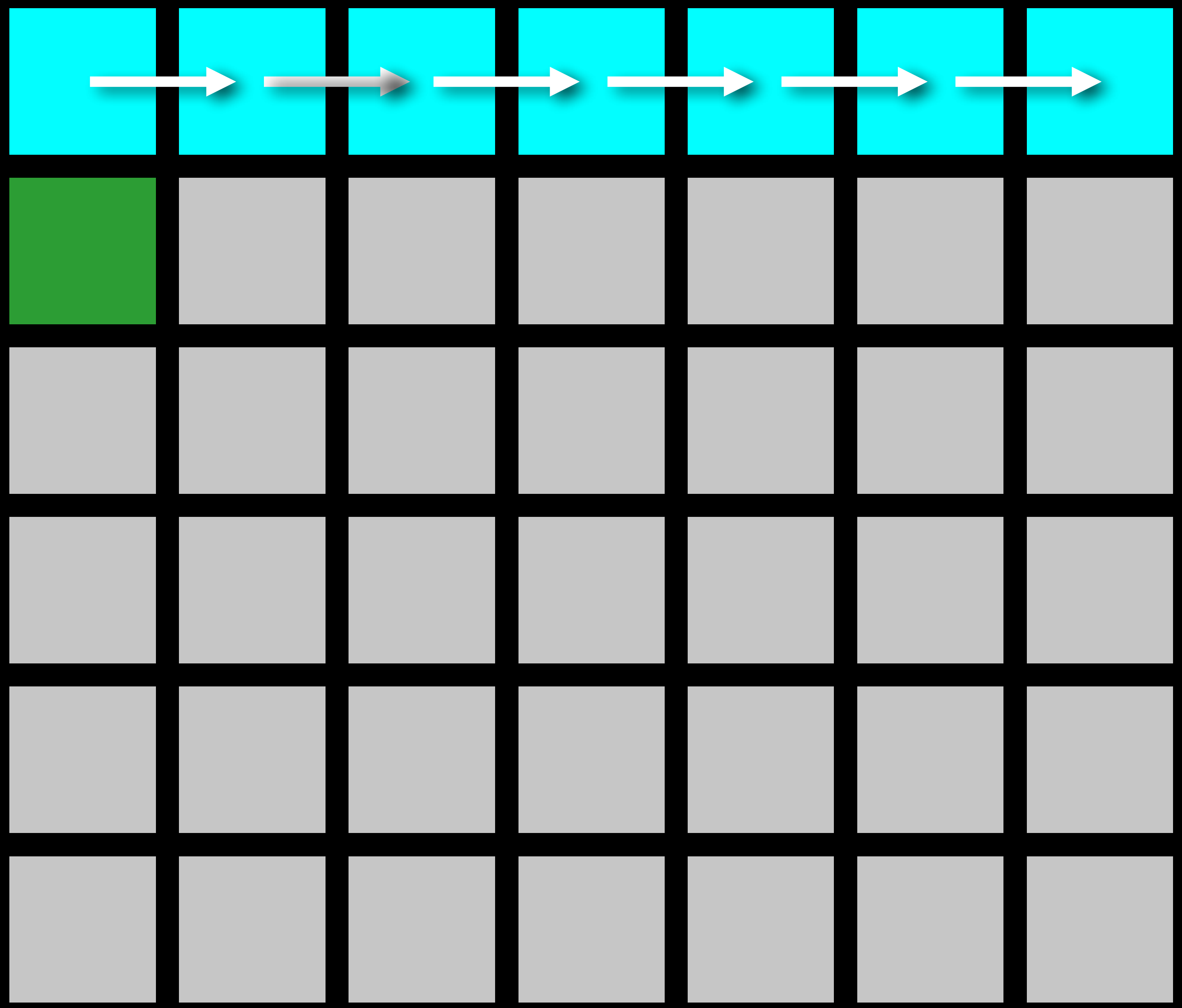


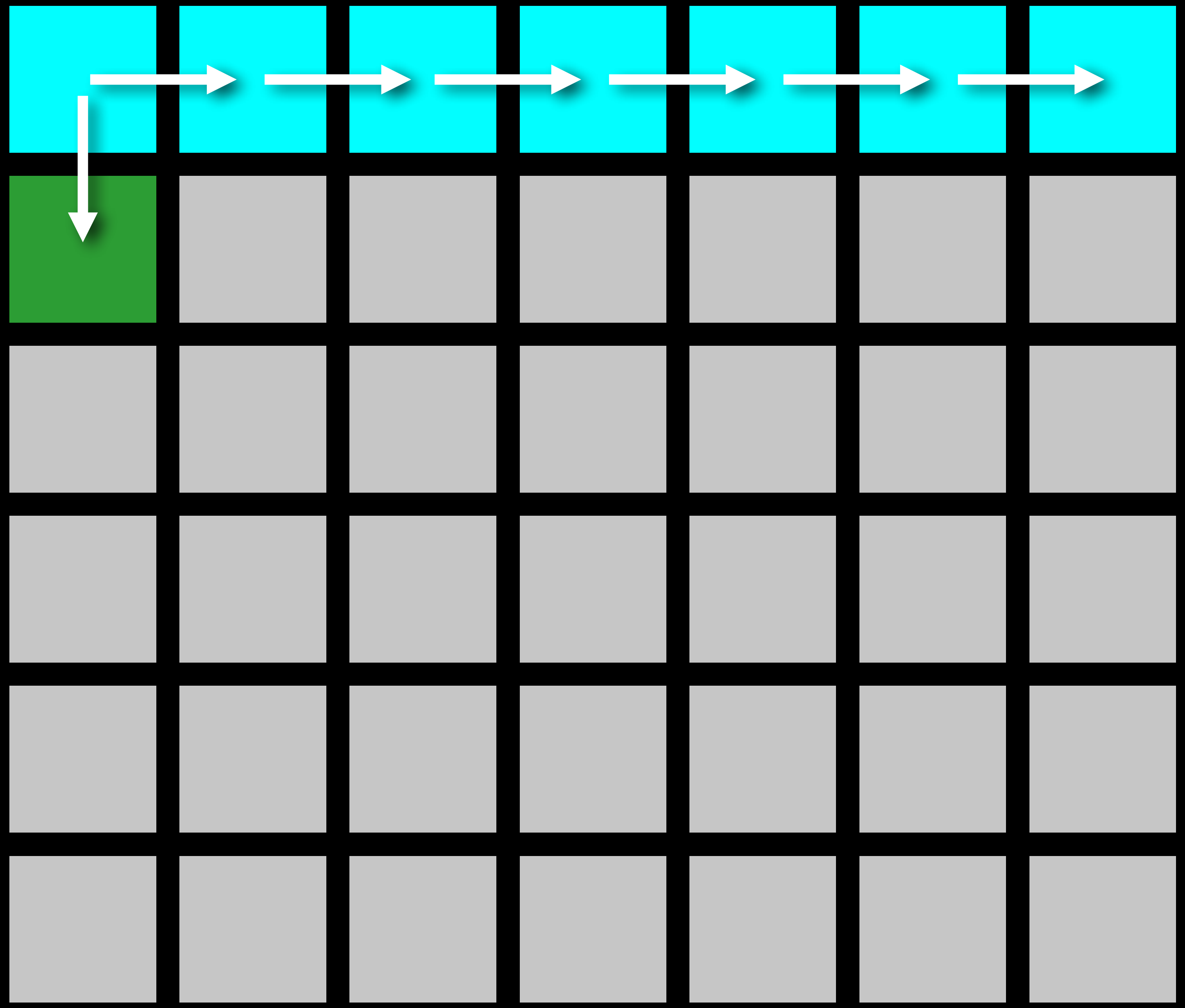


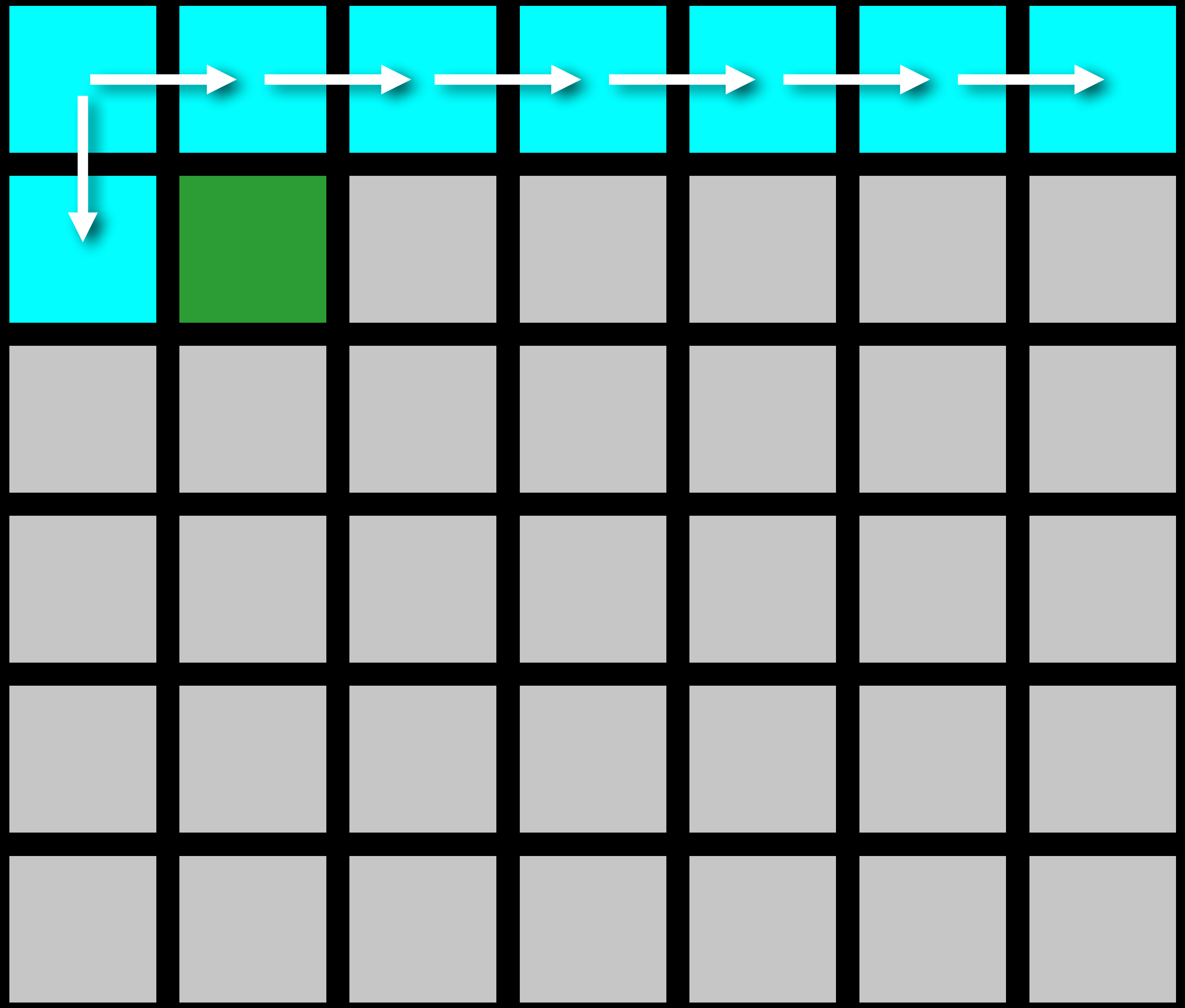


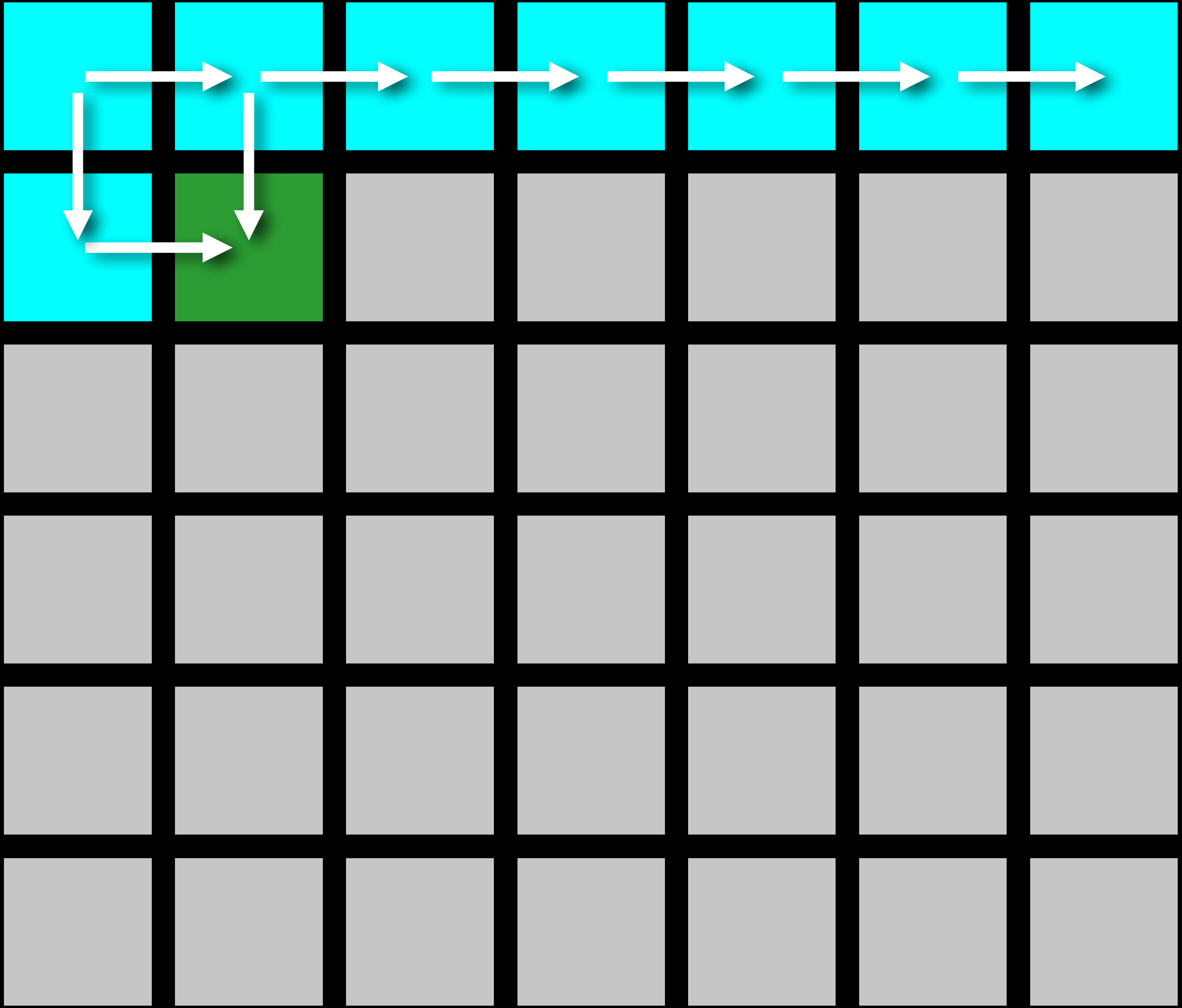

dependency on previous pixels modelled using an RNN



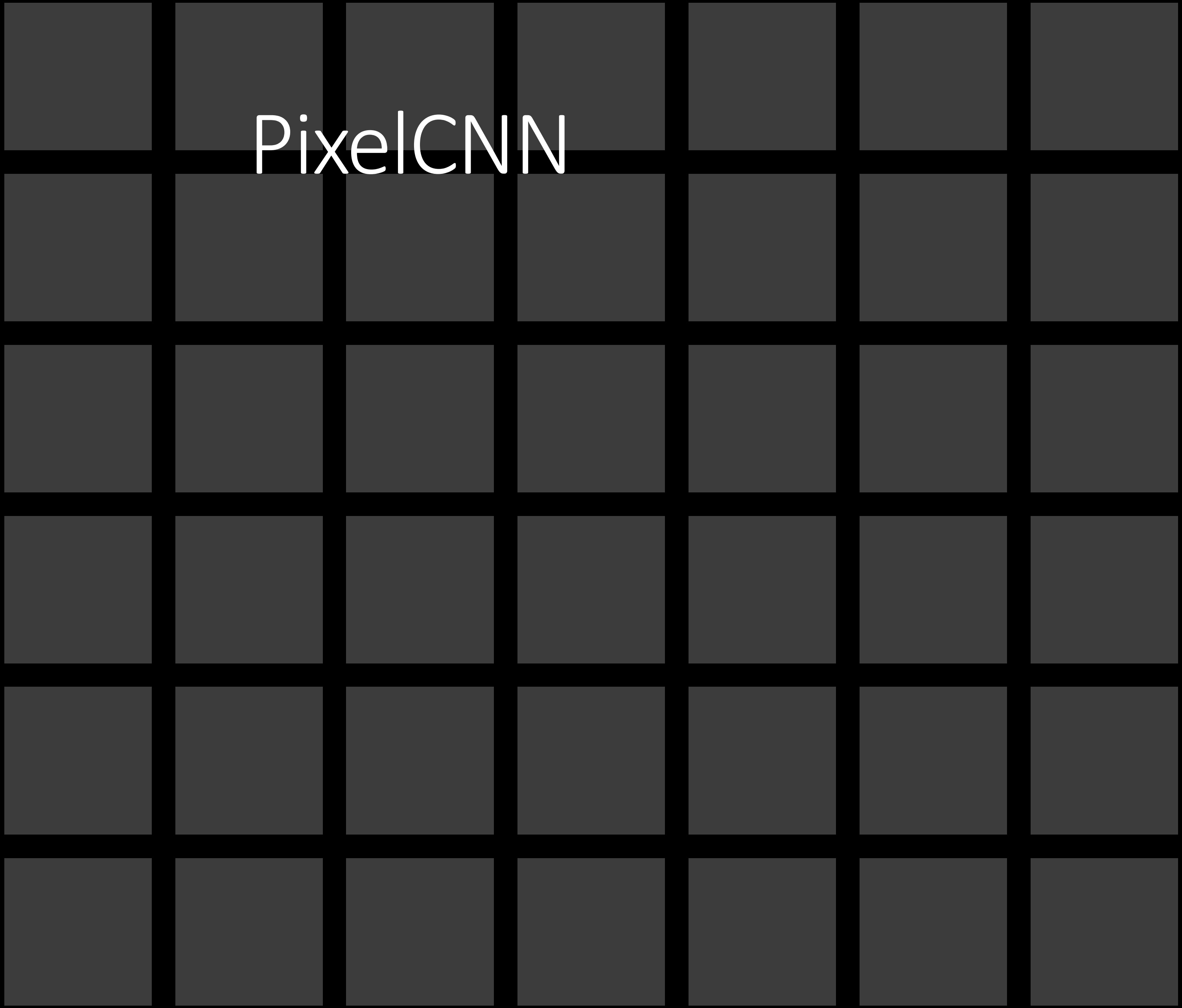


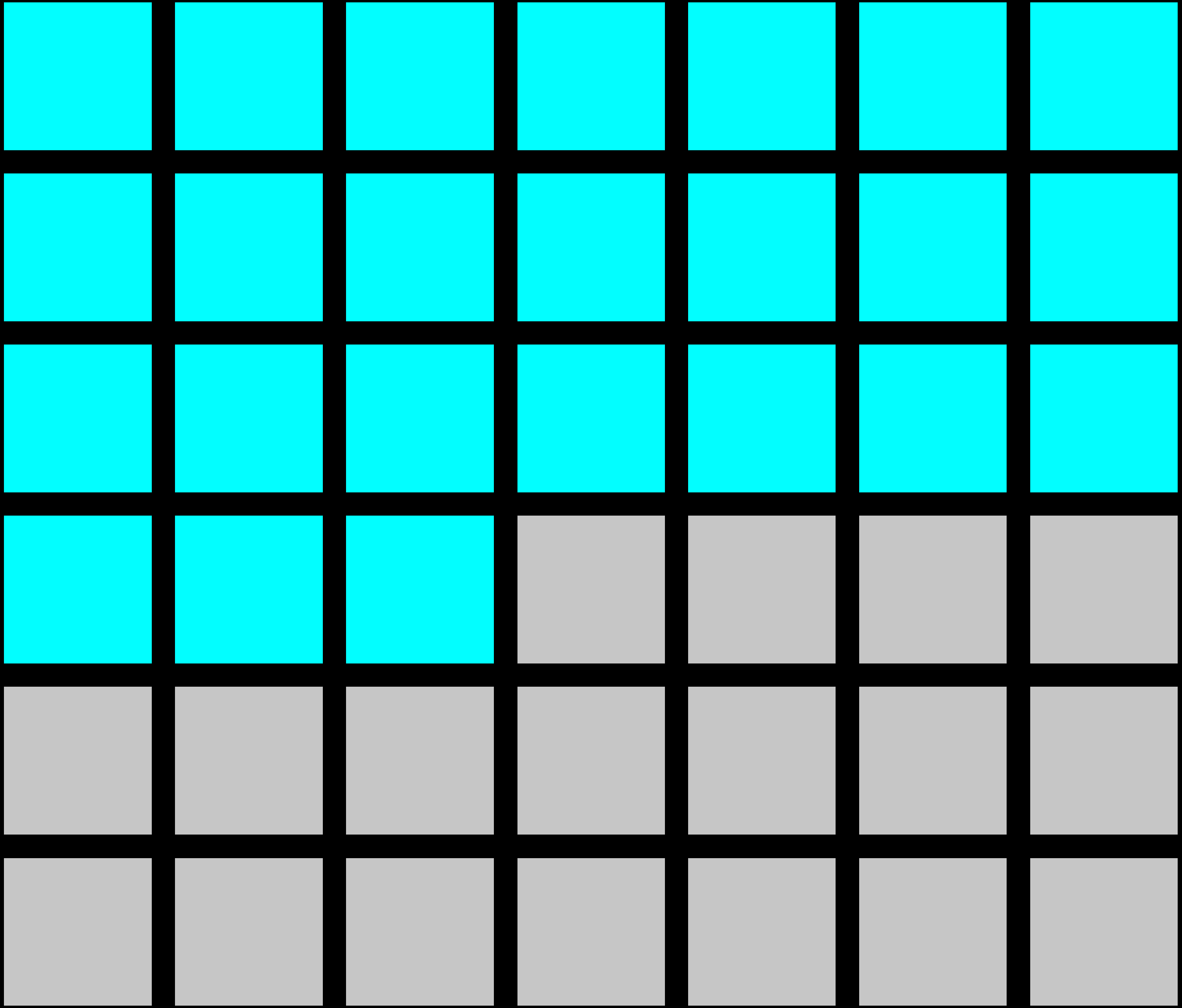


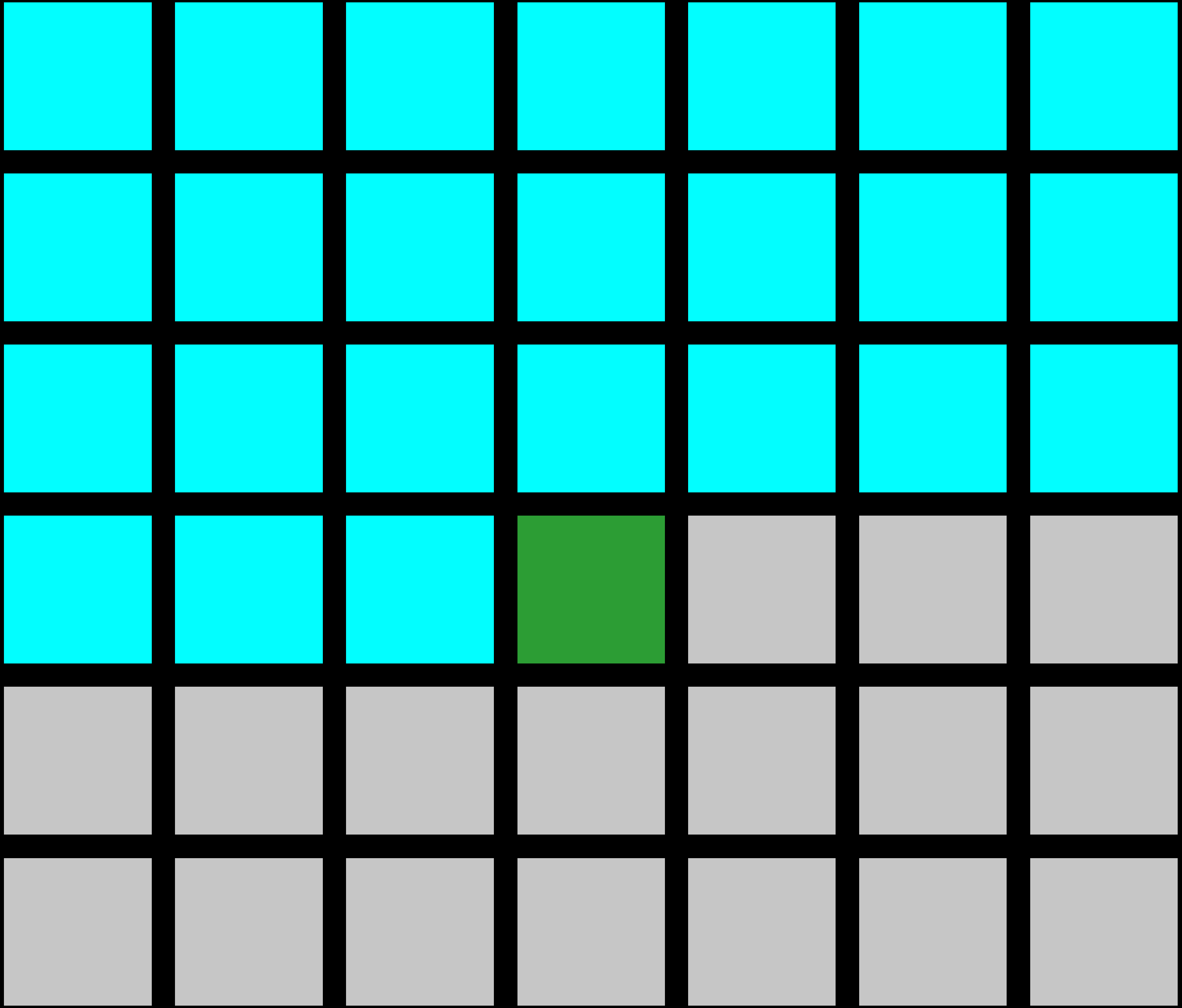


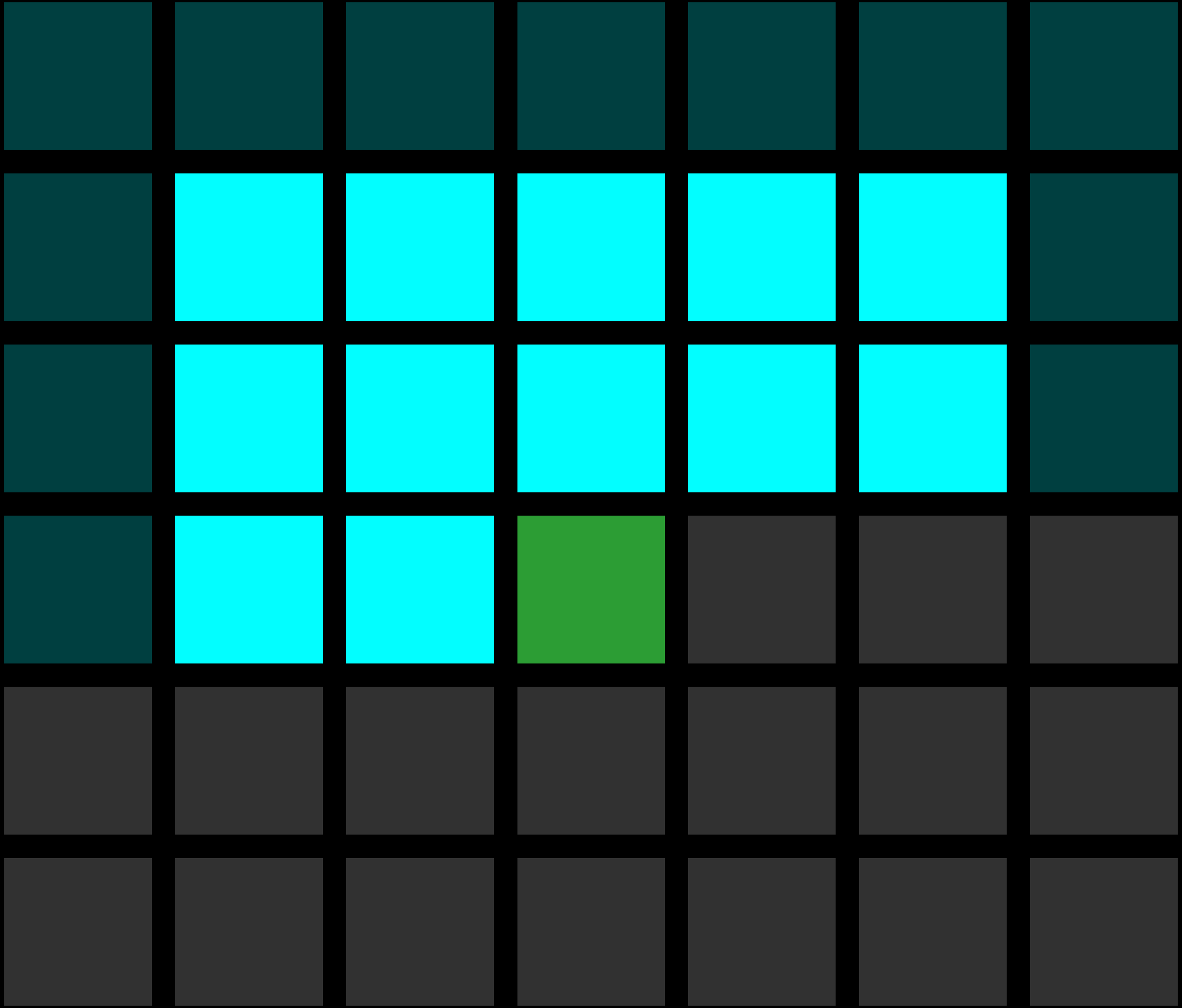


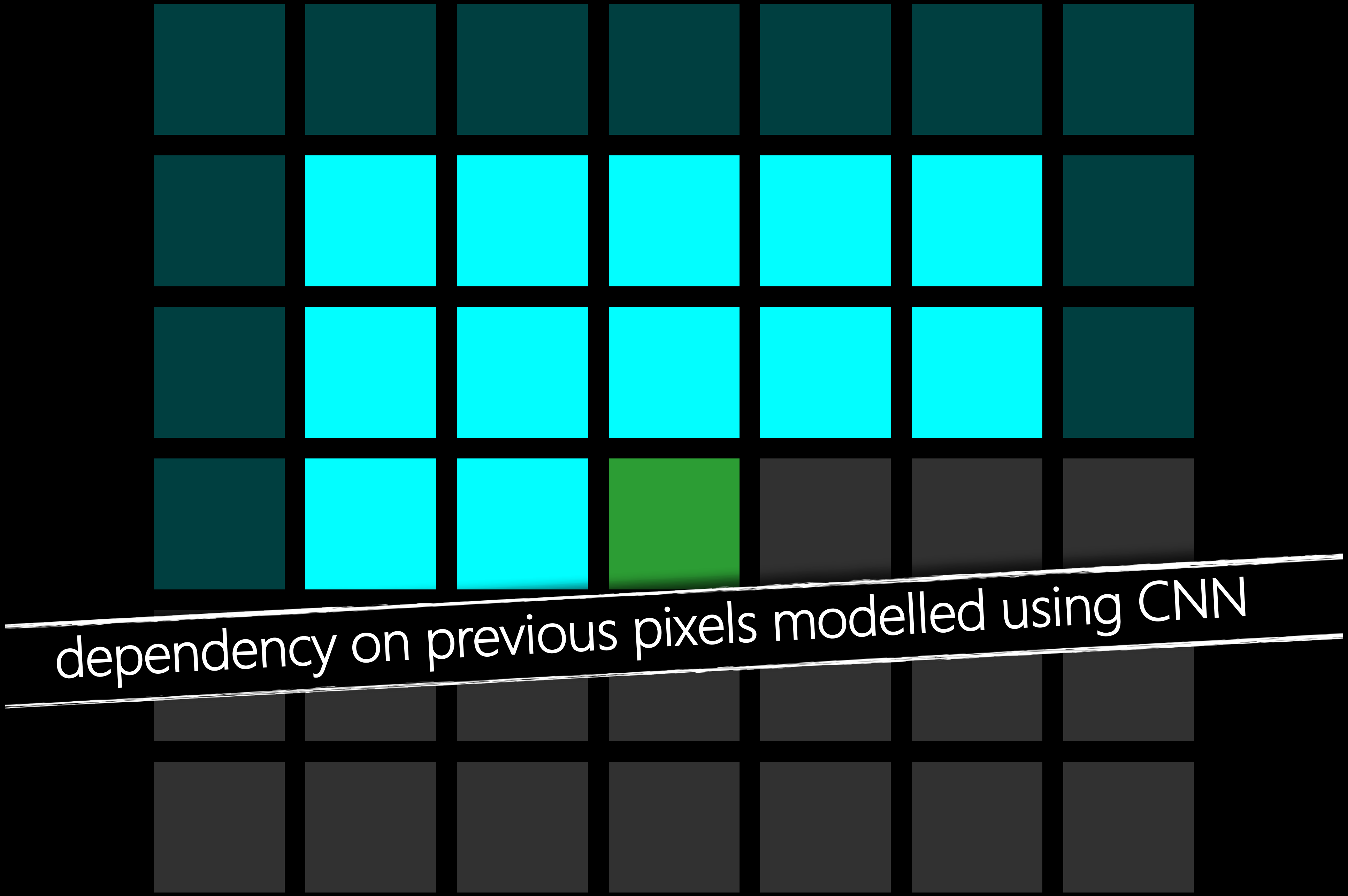
PixelCNN

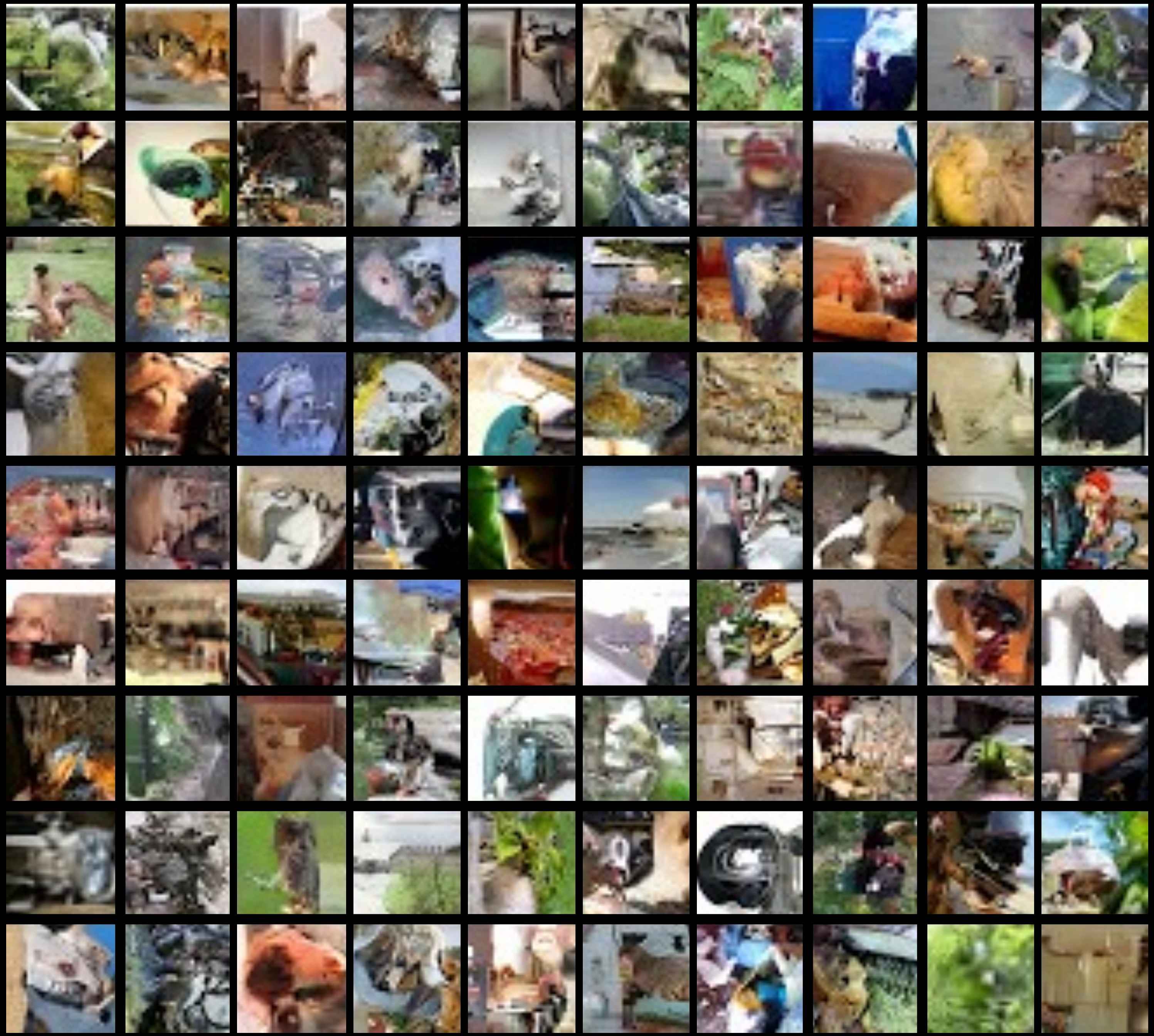


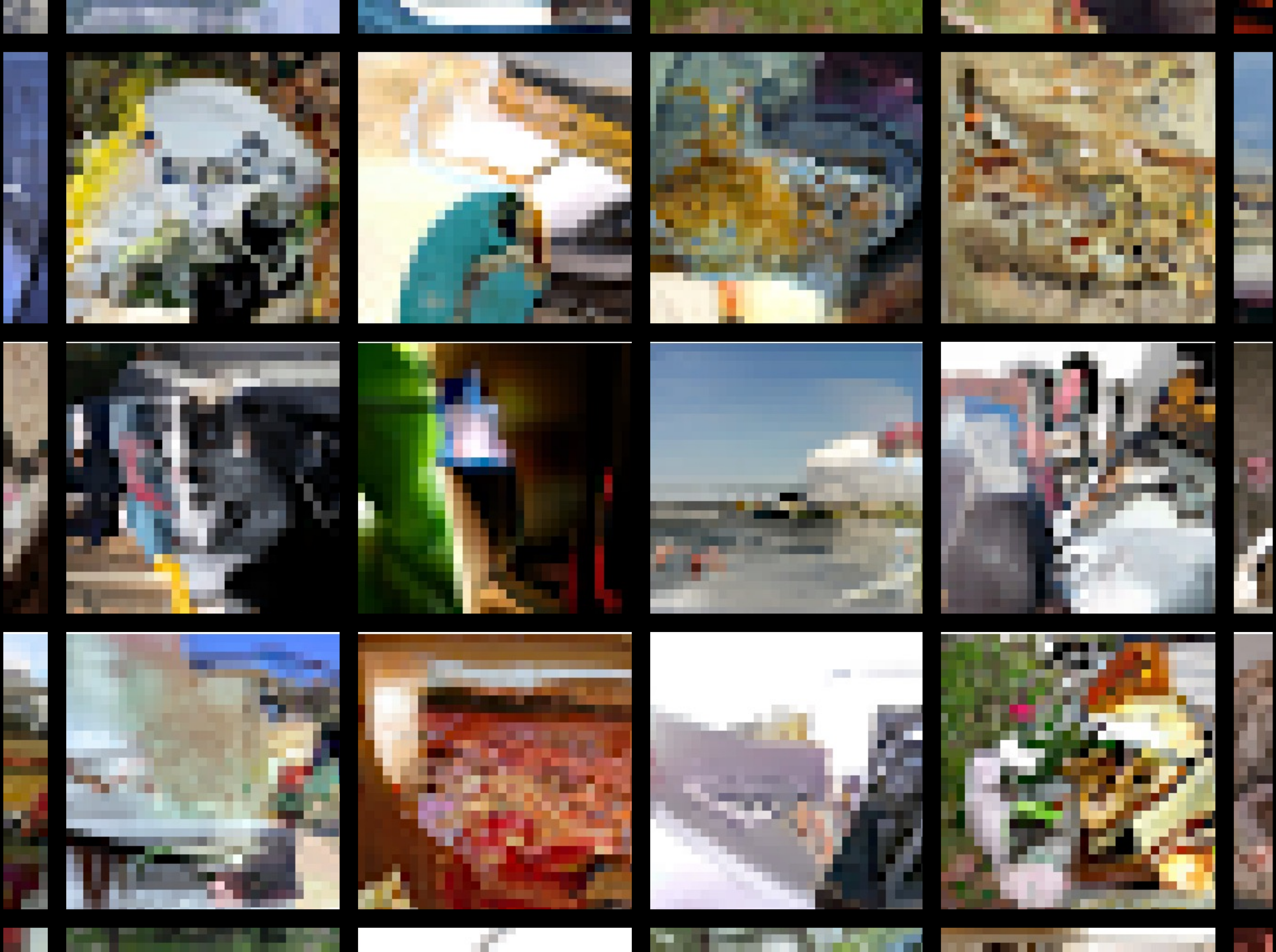




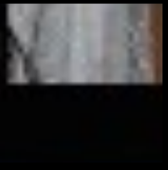


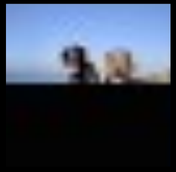

dependency on previous pixels modelled using CNN





occluded





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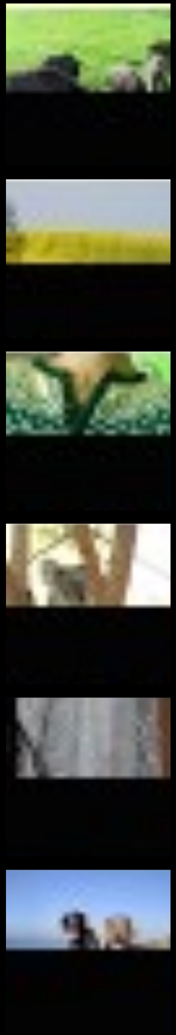
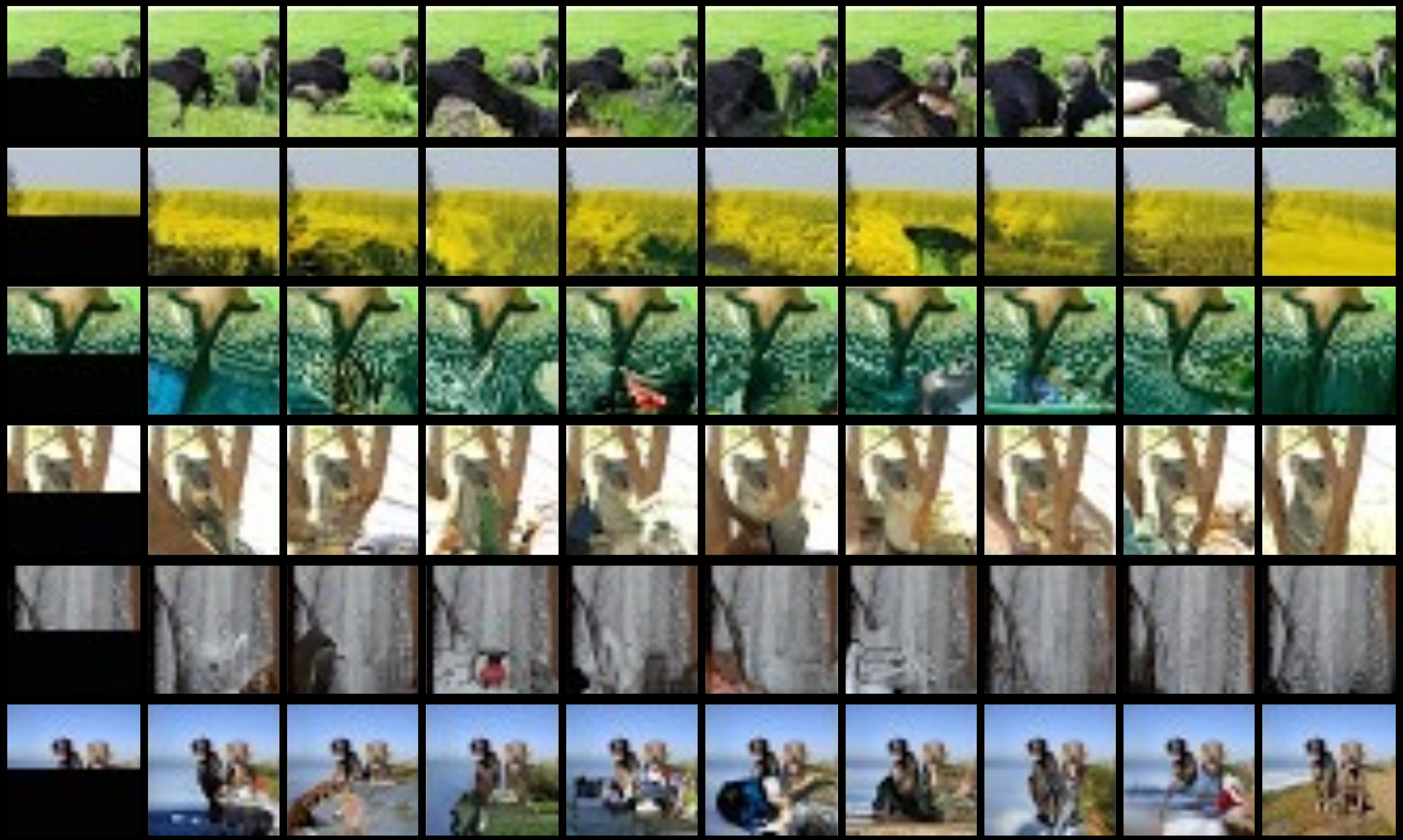


original

occluded completions original



sequential pixel generation is SLOW

pixel generation order matters

Explicit Density Estimation

Learn weights W

Goal: Write down an explicit function for $p(x) = f(x, \boxed{W})$

Given dataset $x^{(1)}, x^{(2)}, \dots x^{(N)}$, train the model by solving:

$$W^* = \arg\max_{W} \prod_i p(x^{(i)})$$

Maximize probability of training data
(Maximum likelihood estimation)

$$= \arg\max_{W} \sum_i \log p(x^{(i)})$$

Log trick to exchange product for sum

$$= \arg\max_{W} \sum_i \log f(x^{(i)}, W)$$

This will be our loss function!
Train with gradient descent

Explicit Density: Autoregressive Models

Goal: Write down an explicit function for $p(x) = f(x, W)$

Assume x consists of multiple subparts:

$$x = (x_1, x_2, x_3, \ldots, x_T\ )$$

Break down probability using the chain rule:

$$\begin{aligned} p(x) &= p(x_1, x_2, x_3, \ldots, x_T\ ) \\ &= p(x_1)p(x_2 \mid x_1)p(x_3 \mid x_1, x_2) \ldots \\ &= \prod_{t=1}^{T} p(x_t \mid x_1, \ldots, x_{t-1}) \end{aligned}$$

Probability of the next subpart given all the previous subparts

$$\begin{tikzcd} p(x_1) & p(x_2) & p(x_3) & p(x_4) \\ h_1 \arrow[u] \arrow[r] & h_2 \arrow[u] \arrow[r] & h_3 \arrow[u] \arrow[r] & h_4 \arrow[u] \\ x_0 \arrow[u] & x_1 \arrow[u] & x_2 \arrow[u] & x_3 \arrow[u] \end{tikzcd}$$

We've already seen this! Language modeling with an RNN!

Slide Credits

- EECS 6322 Deep Learning for Computer Vision, Kosta Derpanis (York University)
- EECS 498 Deep Learning for Computer Vision, Justin Johnson (U. Michigan)
- Many amazing research papers!

Extra Reading:

https://towardsdatascience.com/auto-regressive-generative-models-pixelrnn-pixelcnn-32d192911173