

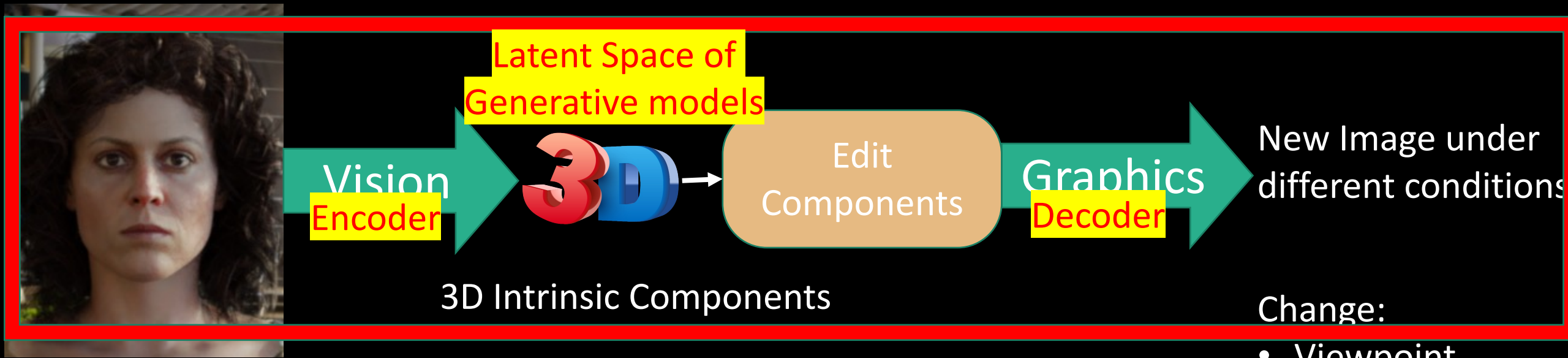
# Lecture 5: Generative Adversarial Networks (GANs)

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**Feel free to share your questions...**

# Next few lectures: Generative models for direct image based rendering.



Current Image

Implicit: Use a Neural Network (Conditional Generative networks) Often, end-to-end.

- Change:
- Viewpoint
  - Lighting
  - Reflectance
  - Background
  - Attributes
  - Many others...

# Taxonomy of Generative Models

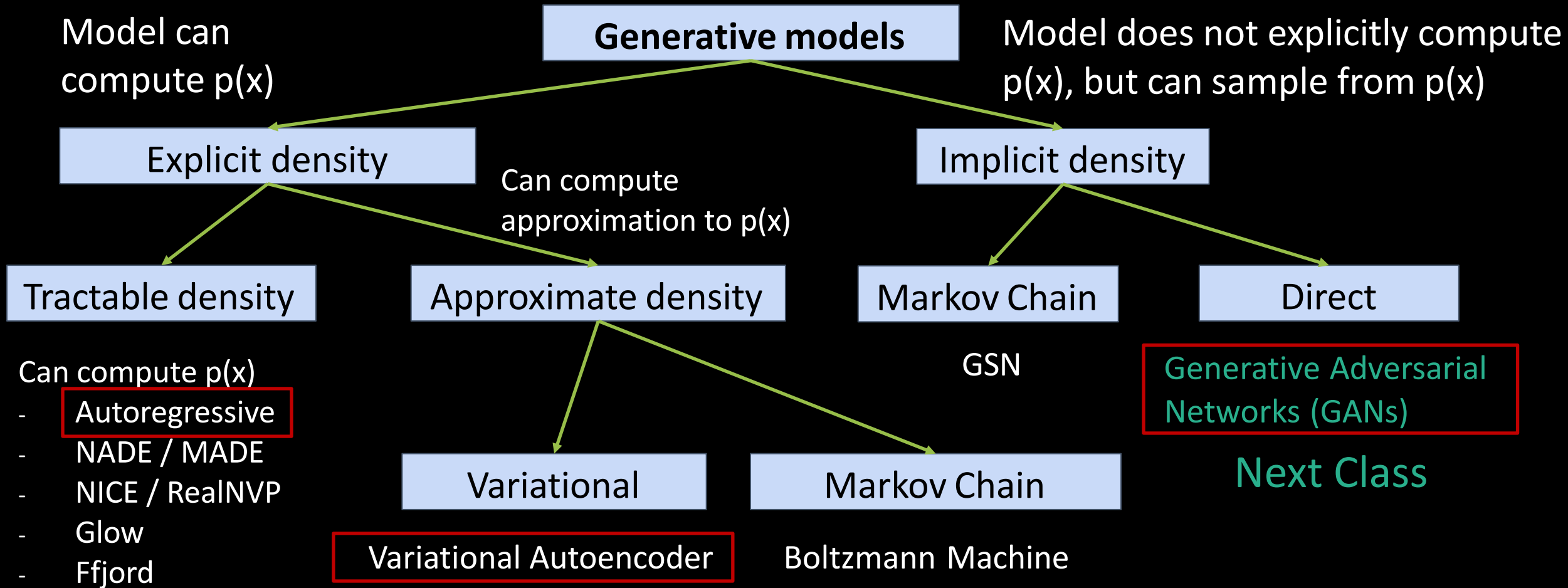
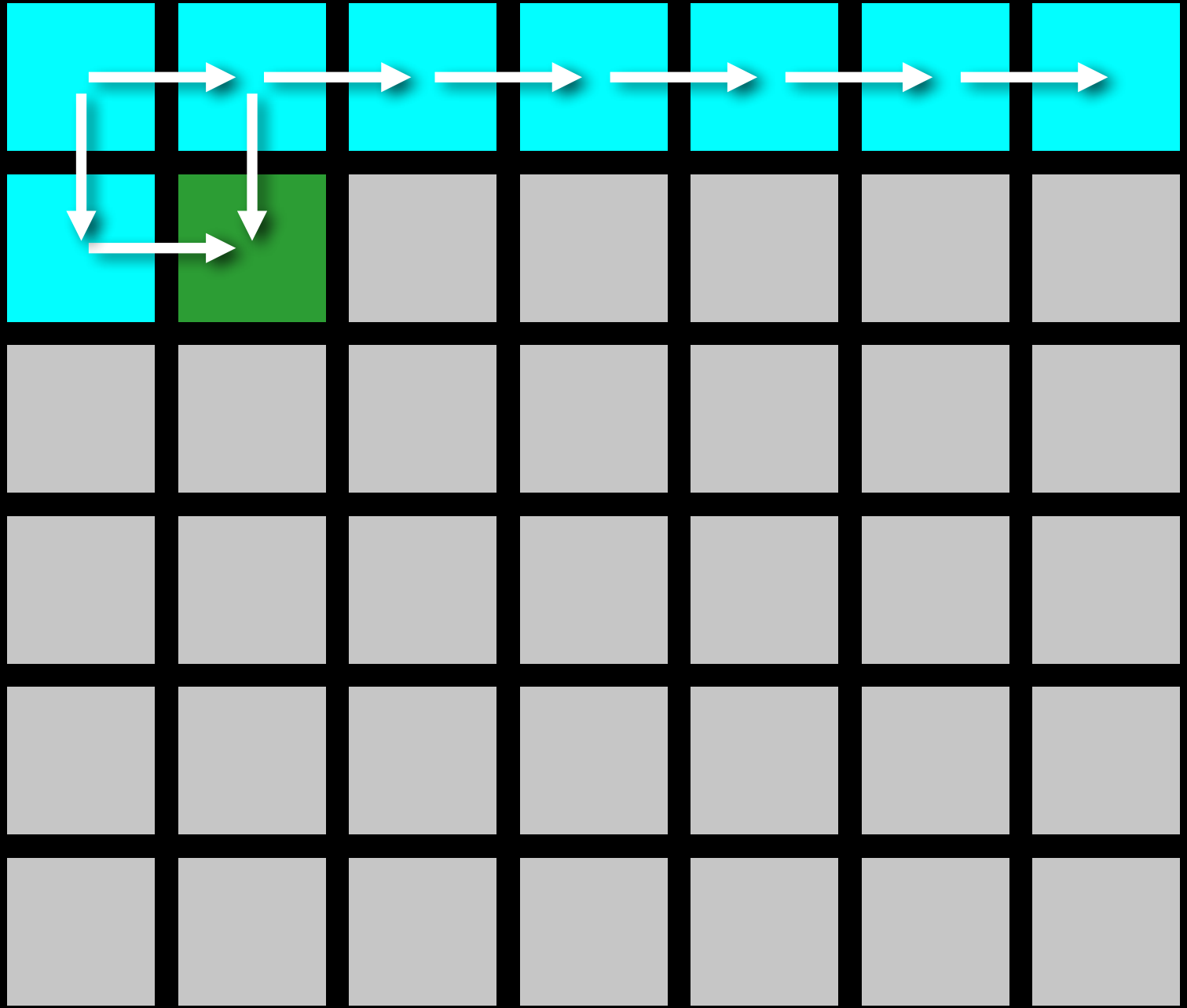


Figure adapted from Ian Goodfellow, Tutorial on Generative Adversarial Networks, 2017.

PixelRNN  
Or  
PixelCNN



# Explicit Density: Autoregressive Models

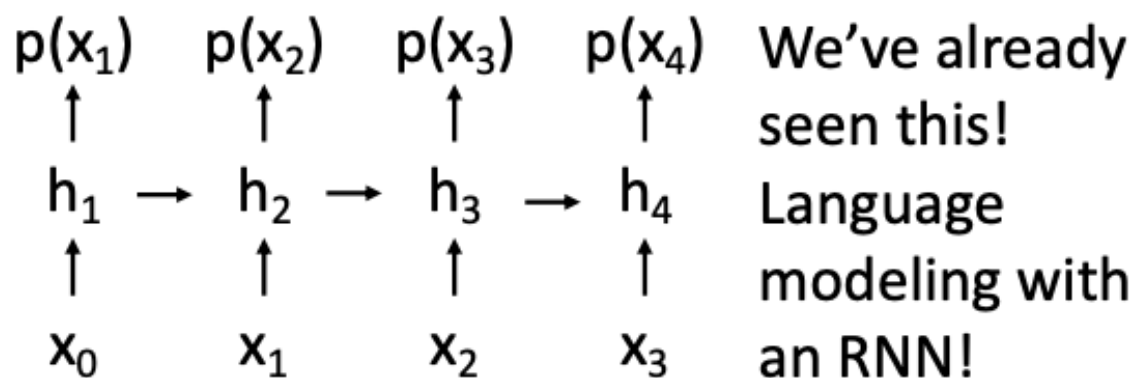
**Goal:** Write down an explicit function for  $p(x) = f(x, W)$

Assume  $x$  consists of multiple subparts:

$$x = (x_1, x_2, x_3, \dots, x_T)$$

Break down probability using the chain rule:

$$\begin{aligned} p(x) &= p(x_1, x_2, x_3, \dots, x_T) \\ &= p(x_1)p(x_2 | x_1)p(x_3 | x_1, x_2) \dots \\ &= \prod_{t=1}^T p(x_t | x_1, \dots, x_{t-1}) \end{aligned}$$



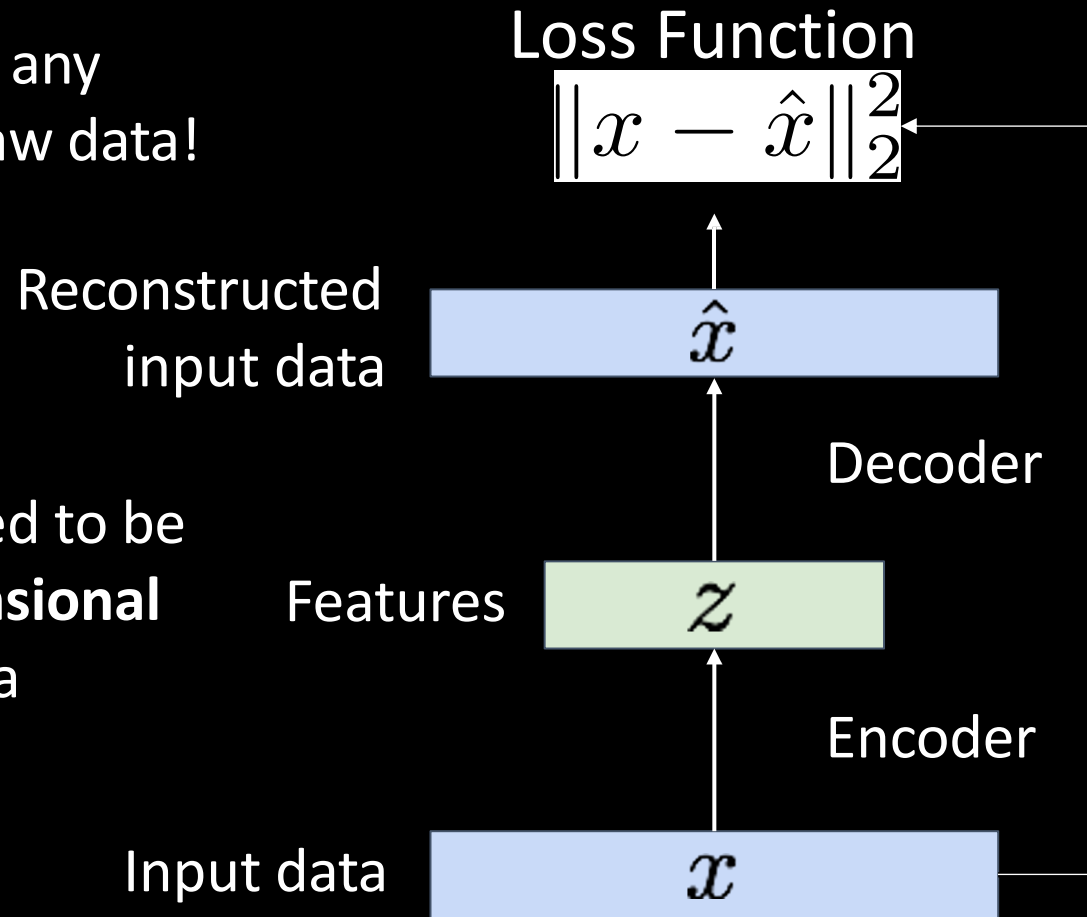
Probability of the next subpart given all the previous subparts

# (Regular, non-variational) Autoencoders

**Loss:** L2 distance between input and reconstructed data.

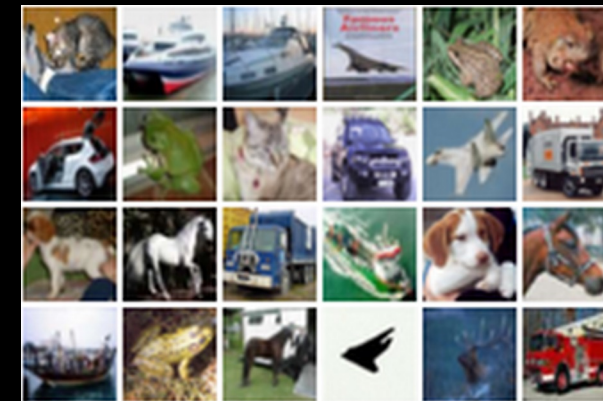
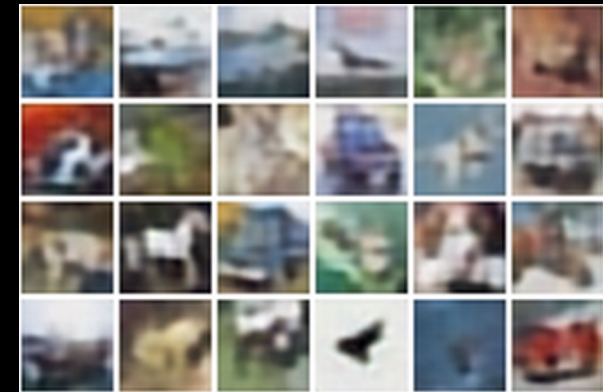
Does not use any labels! Just raw data!

Features need to be **lower dimensional** than the data



Decoder:  
4 tconv layers  
Encoder:  
4 conv layers

Reconstructed data



Input Data

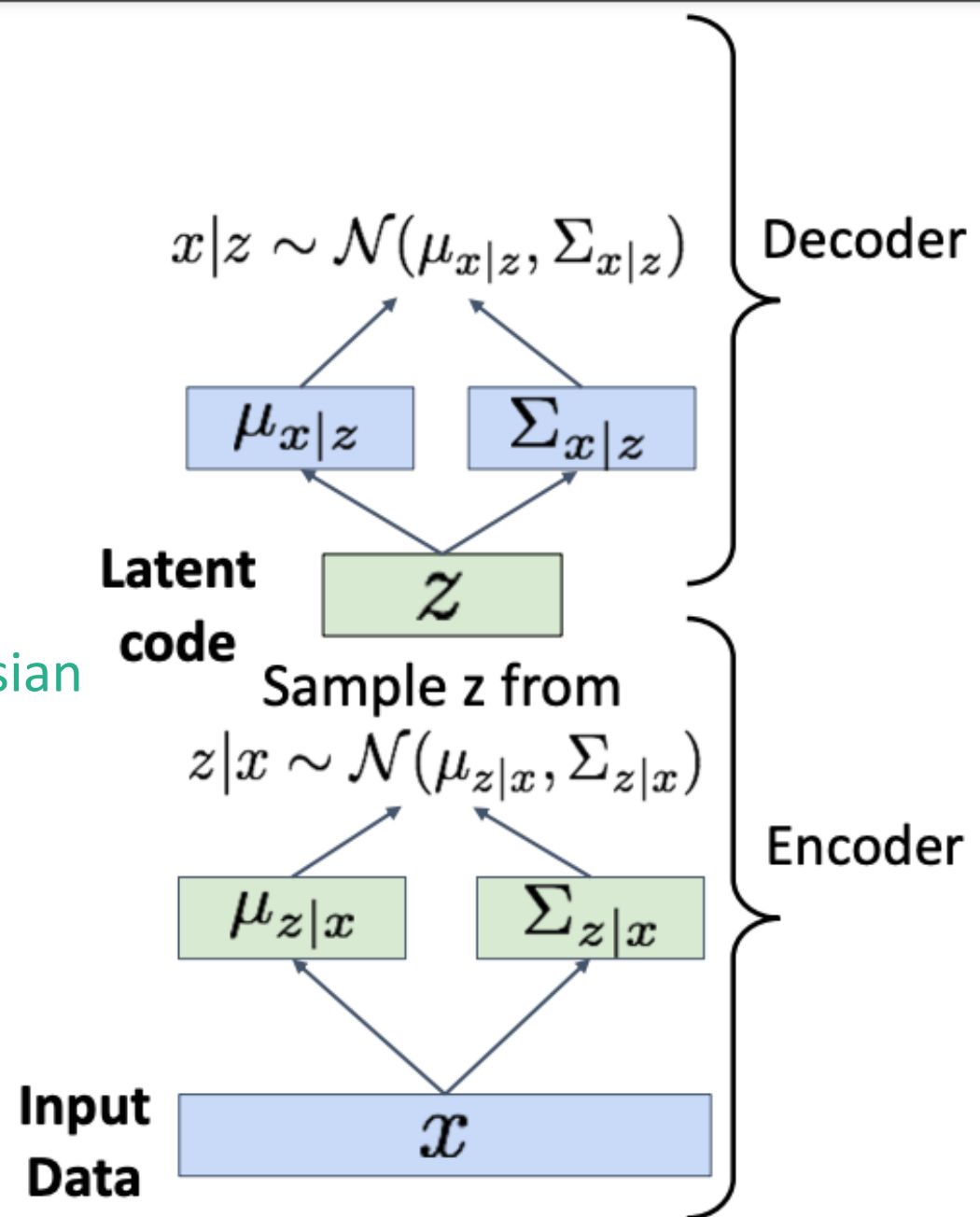
**Not probabilistic: No way to sample new data from learned model**

# Variational Autoencoders

Train by maximizing the  
**variational lower bound**

$$E_{z \sim q_{\phi}(z|x)} [\log p_{\theta}(x|z)] - D_{KL}(q_{\phi}(z|x), p(z))$$

1. Run input data through **encoder** to get a distribution over latent codes *Try to make z gaussian*
2. **Encoder output should match the prior p(z)!**
3. Sample code z from encoder output
4. Run sampled code through **decoder** to get a distribution over data samples
5. **Original input data should be likely under the distribution output from (4)!**





# Few Math recap: What is Expectation?

Definition:

$$E[X] = \sum_x xp_X(x)$$

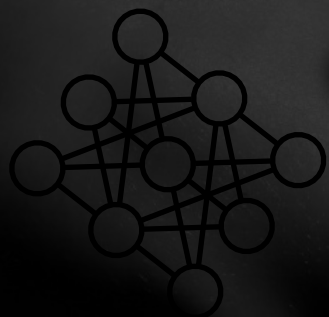
$$E_{z \sim q_\phi(z|x)} [\log p_\theta(x|z)]$$

- Sample  $z_i$  from the latent space (gaussian).
- Pass  $z_i$  through decoder to reconstruct image  $x'_i$
- Loss can be computed as binary cross-entropy loss between the real images and the generated images =  $\sum \{x'_i * \log(x_i)\}$   
(Some implementation also use regular MSE loss).



# GAN

**Generative  
Adversarial Network**





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# Generative Adversarial Nets

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**Ian J. Goodfellow, Jean Pouget-Abadie\*, Mehdi Mirza, Bing Xu, David Warde-Farley,  
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Montréal, QC H3C 3J7

## Abstract

We propose a new framework for estimating generative models via an adversarial process, in which we simultaneously train two models: a generative model  $G$  that captures the data distribution, and a discriminative model  $D$  that estimates

# Generative Adversarial Networks


**Setup:** Assume we have data  $x_i$  drawn from distribution  $p_{\text{data}}(x)$ .

Want to sample from  $p_{\text{data}}$ .

**Idea:** Introduce a latent variable  $z$  with simple prior  $p(z)$ .

Sample  $z \sim p(z)$  and pass to a **Generator Network**  $x = G(z)$

Then  $x$  is a sample from the **Generator distribution**  $p_G$ . Want  $p_G = p_{\text{data}}$ !



Generator

Network takes a random input  
and produces a sample from the  
data distribution as output


**image**



$z \leftarrow$  random noise




Similar as decoder for VAE.



Discriminator

Network classifies input as “real” or “fake”





Discriminator

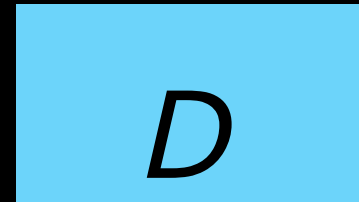
Network classifies input as “real” or “fake”

---

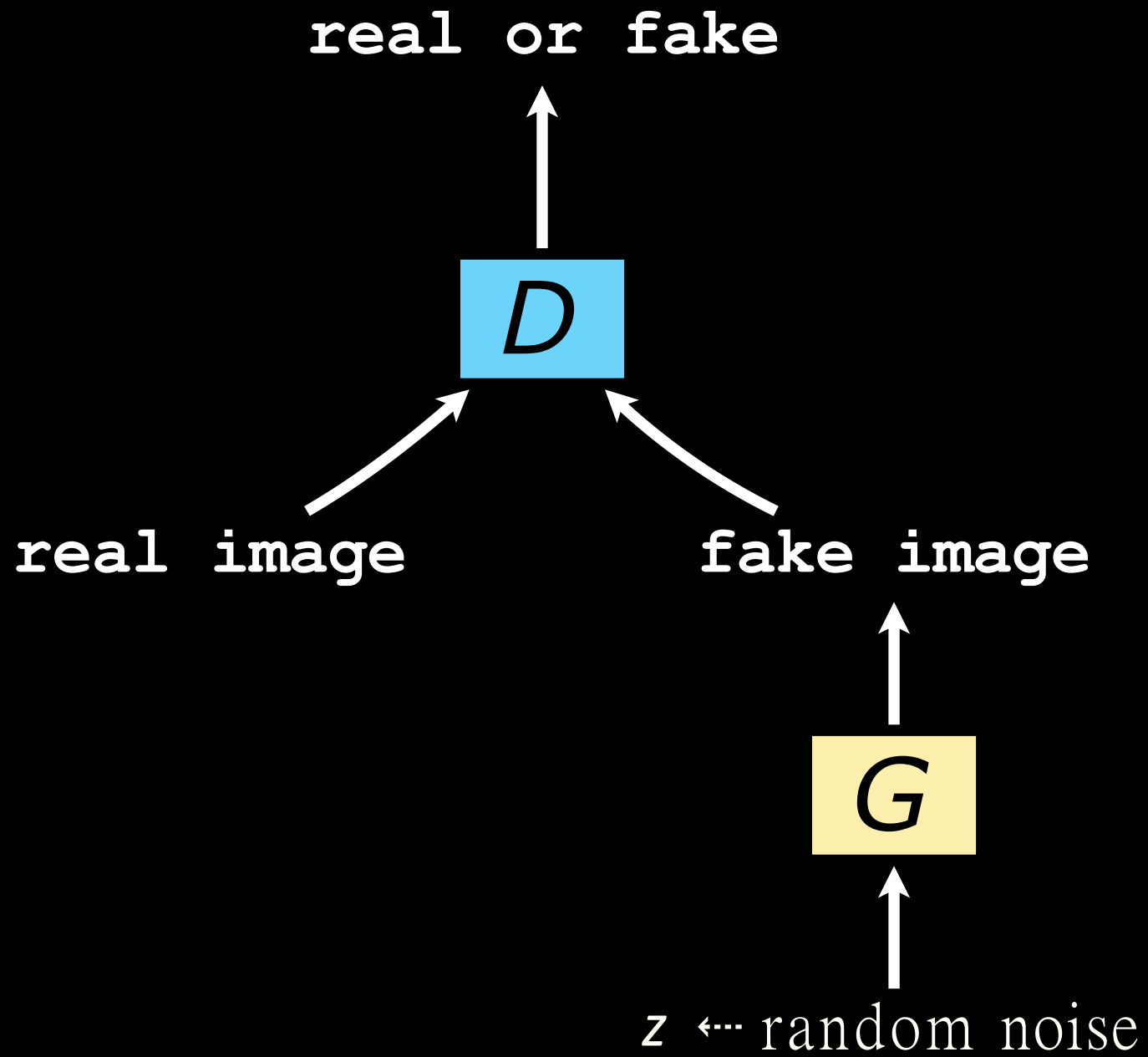
“fake” inputs come from the generator

---

real or fake



image



minimax  
objective

$$\min_{\theta_g} \max_{\theta_d} \left[ \mathbb{E}_{x \sim p_{\text{data}}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

minimax  
objective

$$\mathbb{E}_{x \sim p_{\text{data}}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z)))$$

$\min_{\theta_g} \max_{\theta_d}$

minimax  
objective

$$\mathbb{E}_{x \sim p_{\text{data}}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z)))$$

**cross entropy loss between real and fake images**

minimax  
objective

$$\mathbb{E}_{x \sim p_{\text{data}}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z)))$$

min max  
 $\theta_g \quad \theta_d$

**loss for real images**

minimax  
objective

$$\mathbb{E}_{x \sim p_{\text{data}}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z)))$$

**loss for fake images**



minimax  
objective

$$\min_{\theta_g} \max_{\theta_d} [\mathbb{E}_{x \sim p_{\text{data}}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z)))]$$

**discriminator wants to maximize objective**

minimax  
objective

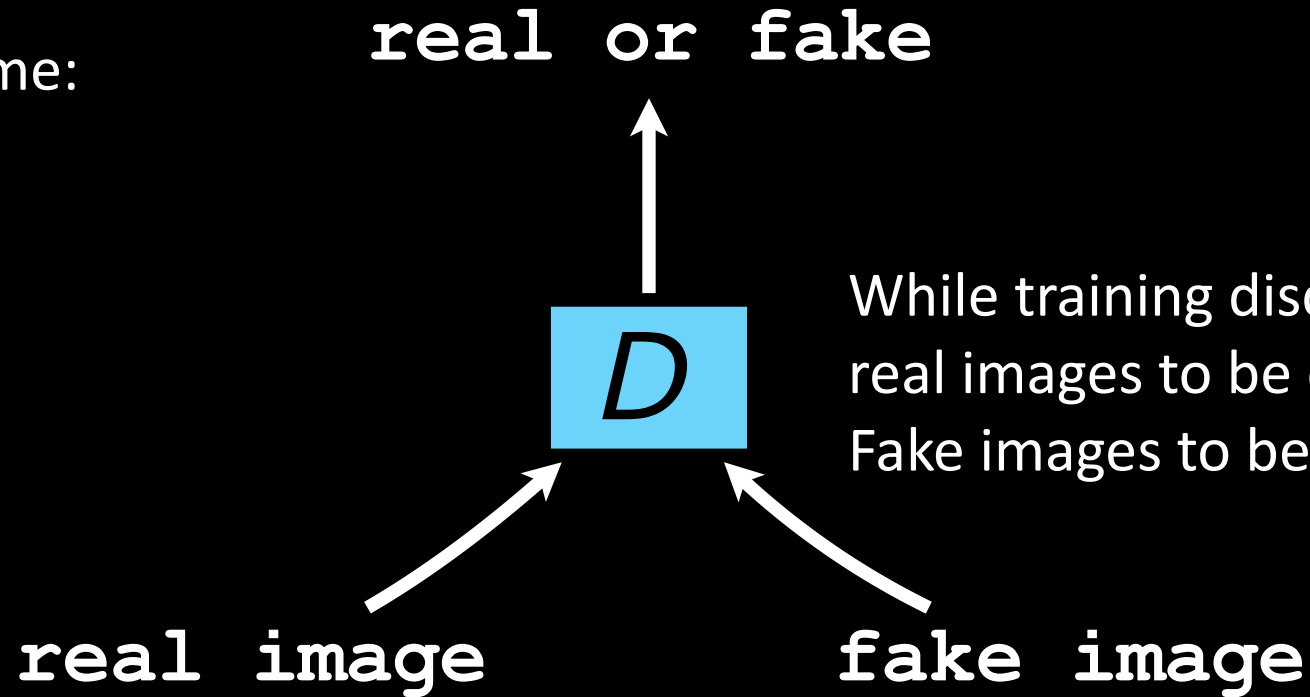
$$\min_{\theta_g} \max_{\theta_d} \left[ \mathbb{E}_{x \sim p_{\text{data}}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

**generator wants to minimize objective**

In practice we assume:

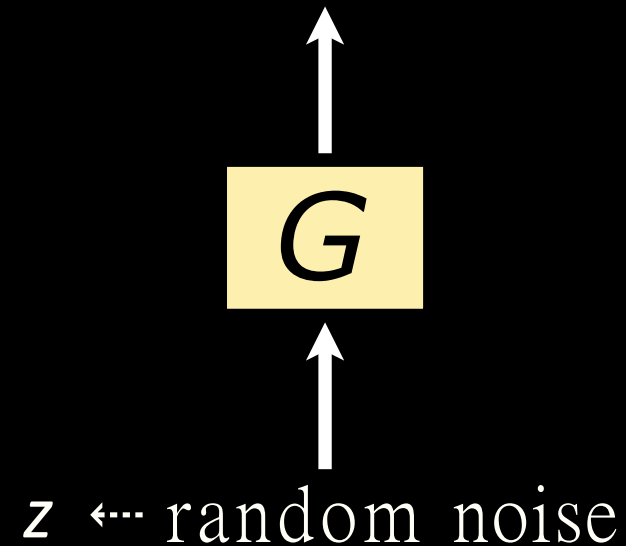
'real' label = 1

'fake' label = 0



While training discriminator, we want:  
real images to be classified as 1  
Fake images to be classified as 0

While training generator, we want the discriminator to classify fake images as real (label=1).





# Two-Player Game



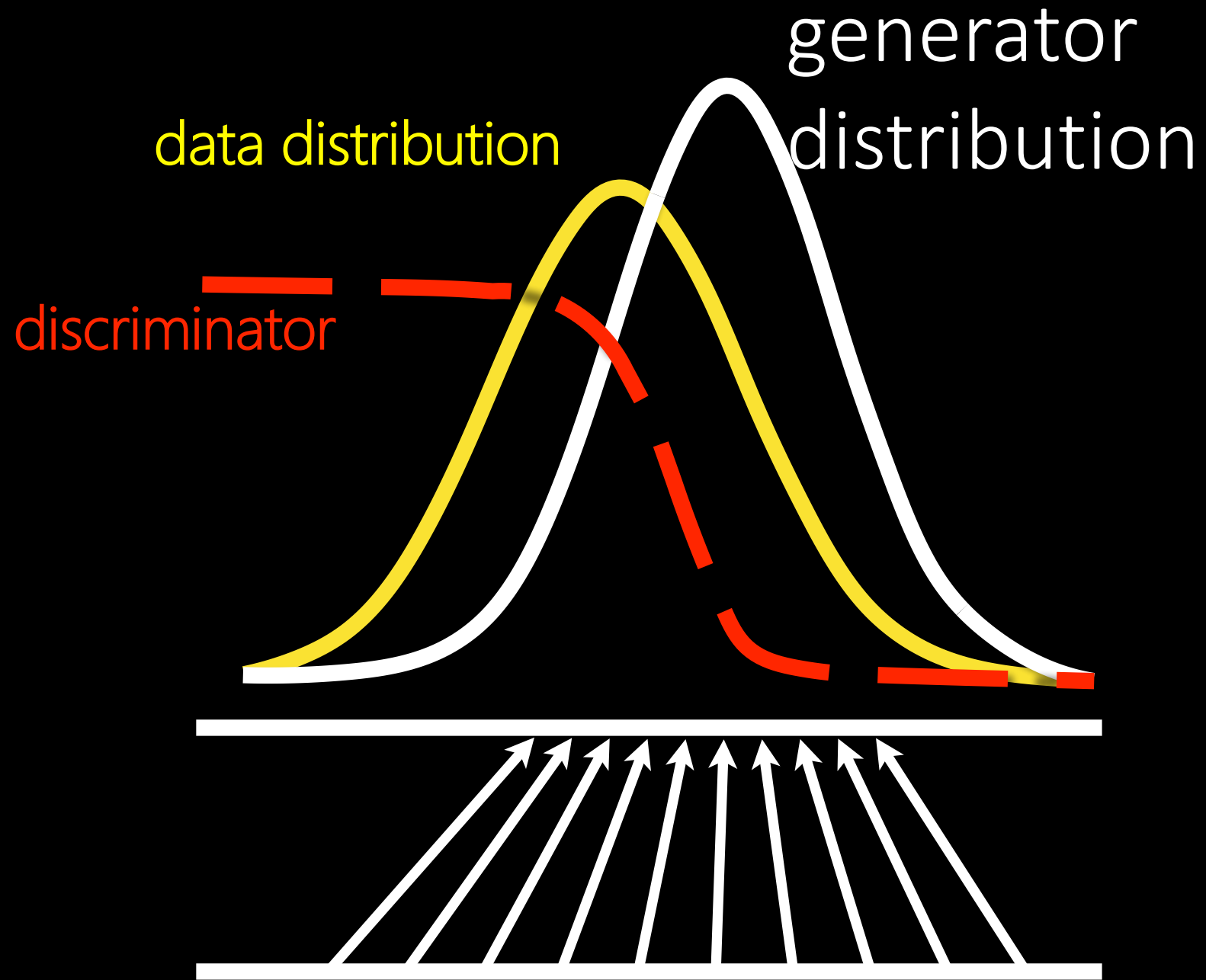


Generator



Generator

Discriminator





minimax  
objective

$$\min_{\theta_g} \max_{\theta_d} \left[ \mathbb{E}_{x \sim p_{\text{data}}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

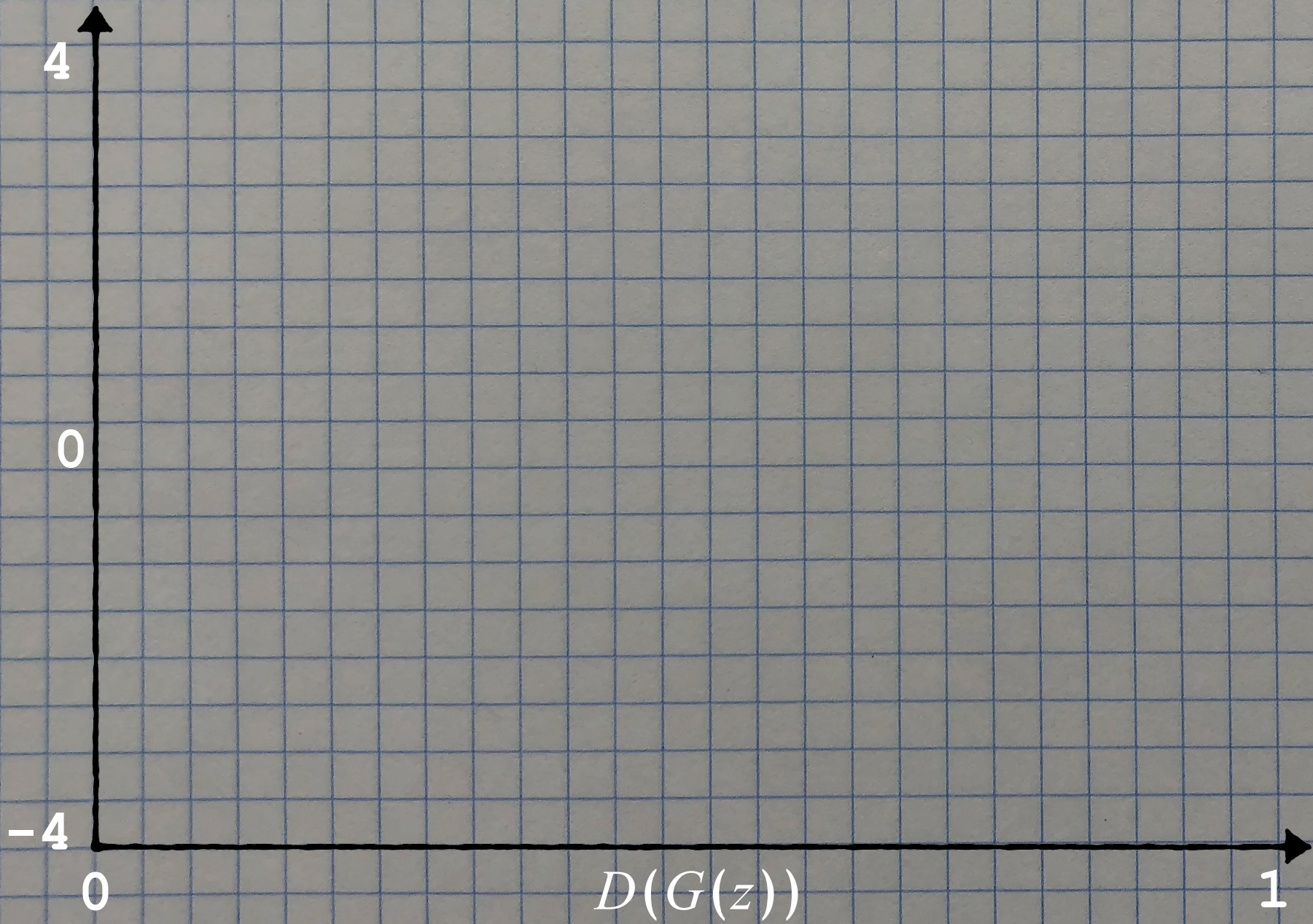
**optimize by alternating between minimizing and maximizing respective sub-objectives.**

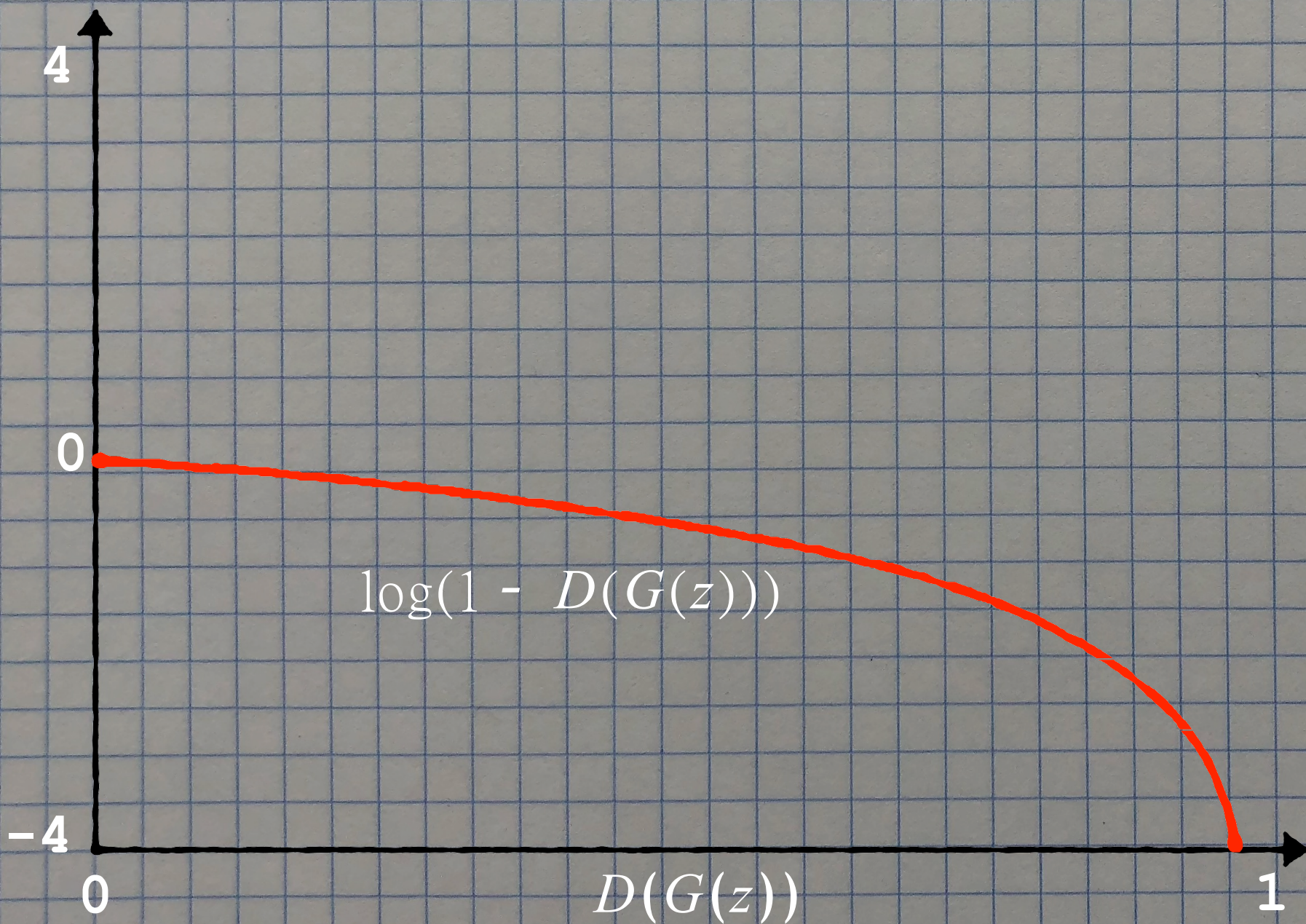
$$\max_{\theta_d, \theta_g} \left[ \mathbb{E}_{x \sim p_{\text{data}}} \log D_{\theta_d}(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z))) \right]$$

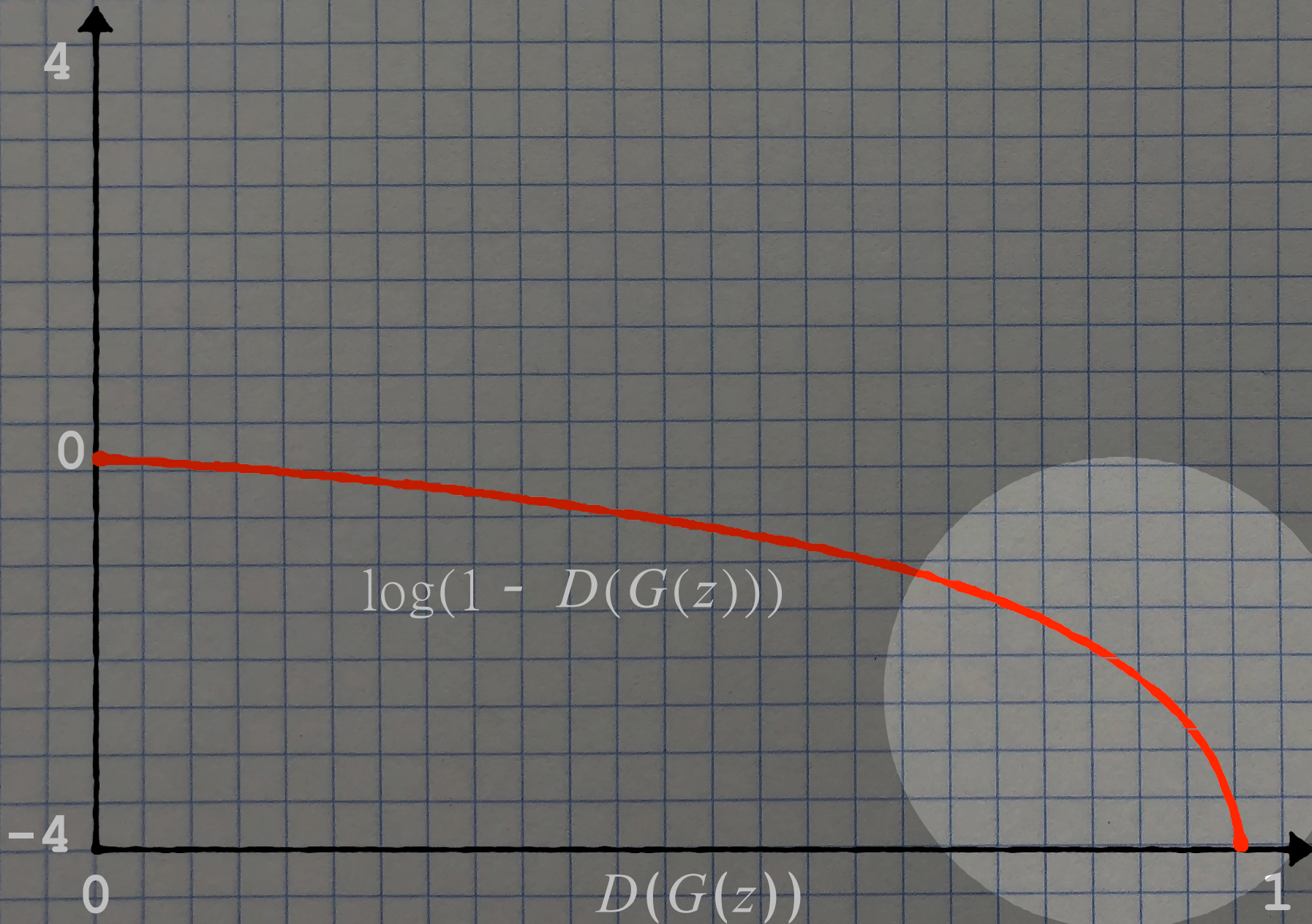
**gradient ascent on discriminator**

$$\min_{\theta_g} \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z)))$$

**gradient descent on generator**







4

$D(G(z))=1$  means the discriminator thinks the fake image is real!

gradient signal is strong for good fake samples

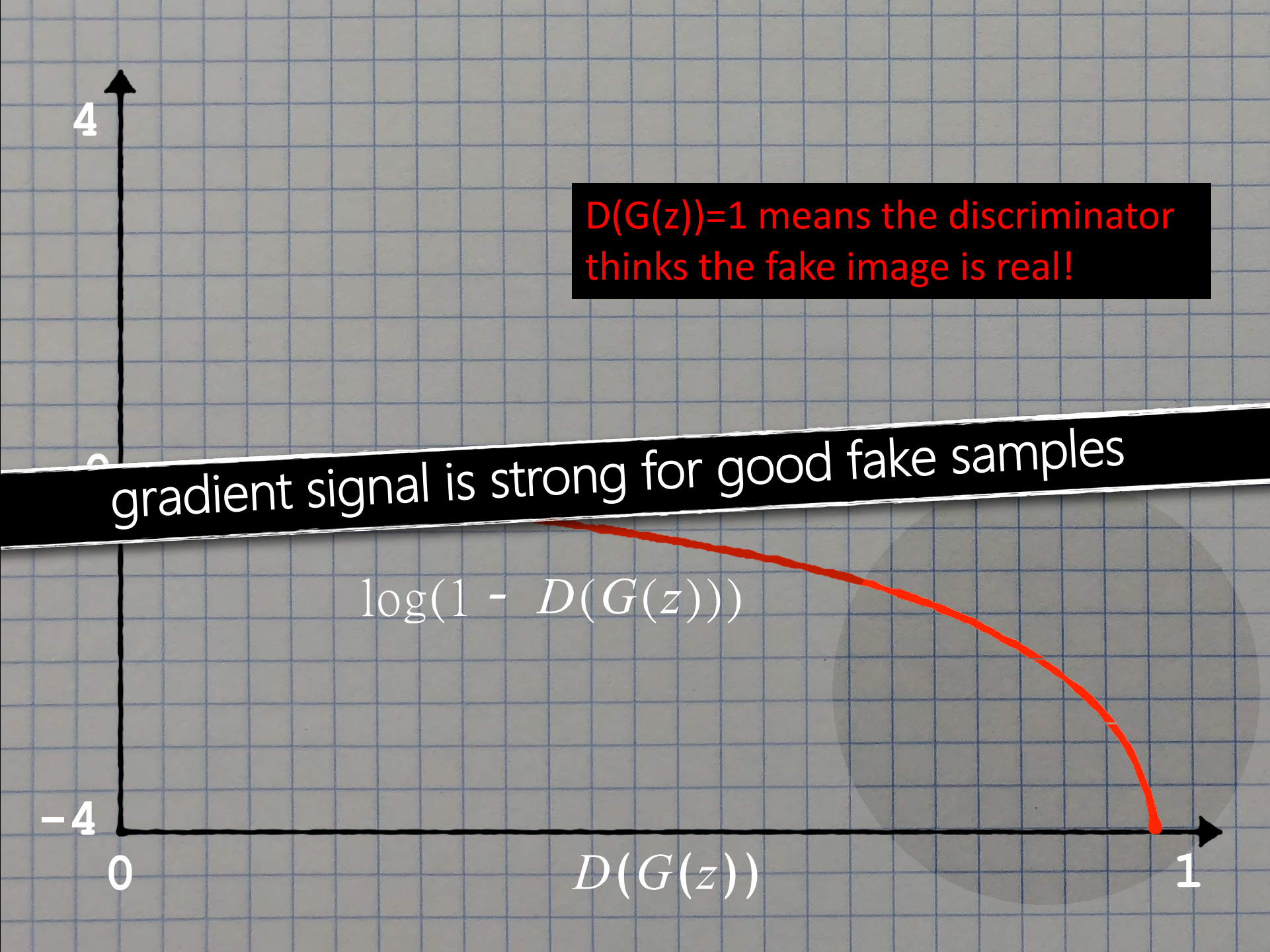
$$\log(1 - D(G(z)))$$

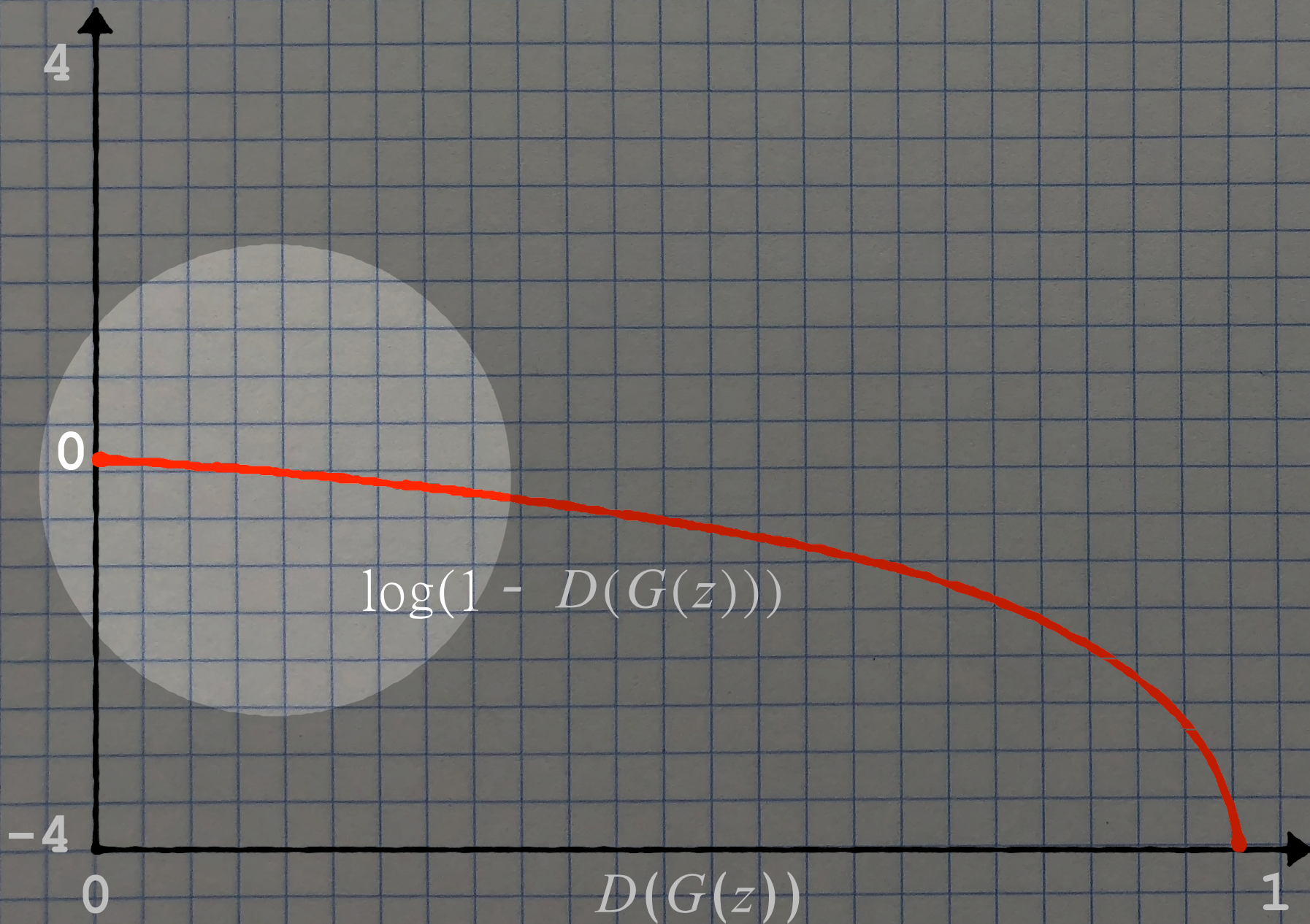
-4

0

$D(G(z))$

1







This kind of problem is often known as vanishing gradient

4

gradient signal is weak for bad fake samples

At the beginning of the training most fake samples are bad!  
Then gradient is small, and the generator do not receive  
much information from discriminator to update itself!

0

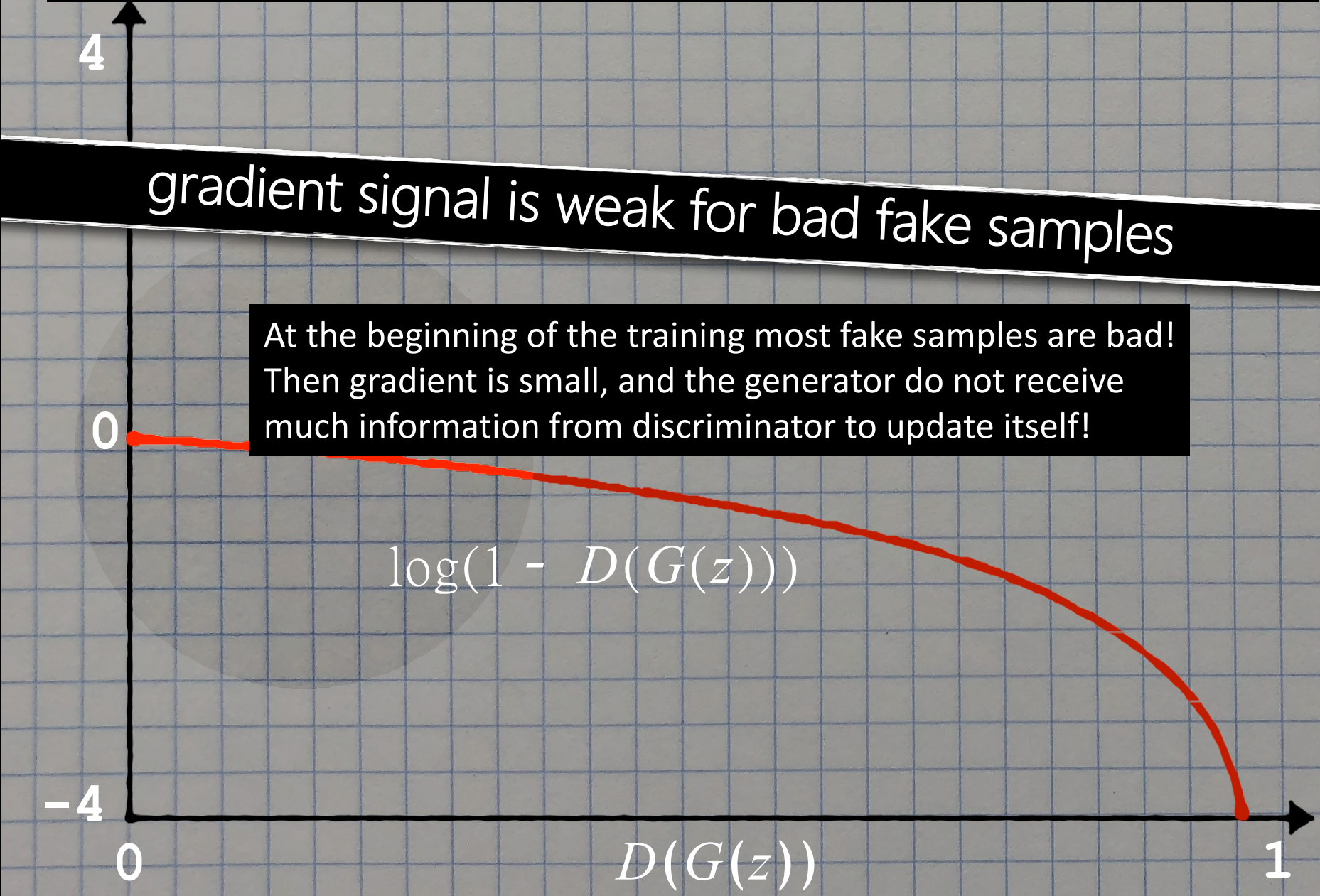
$$\log(1 - D(G(z)))$$

-4

0

$D(G(z))$

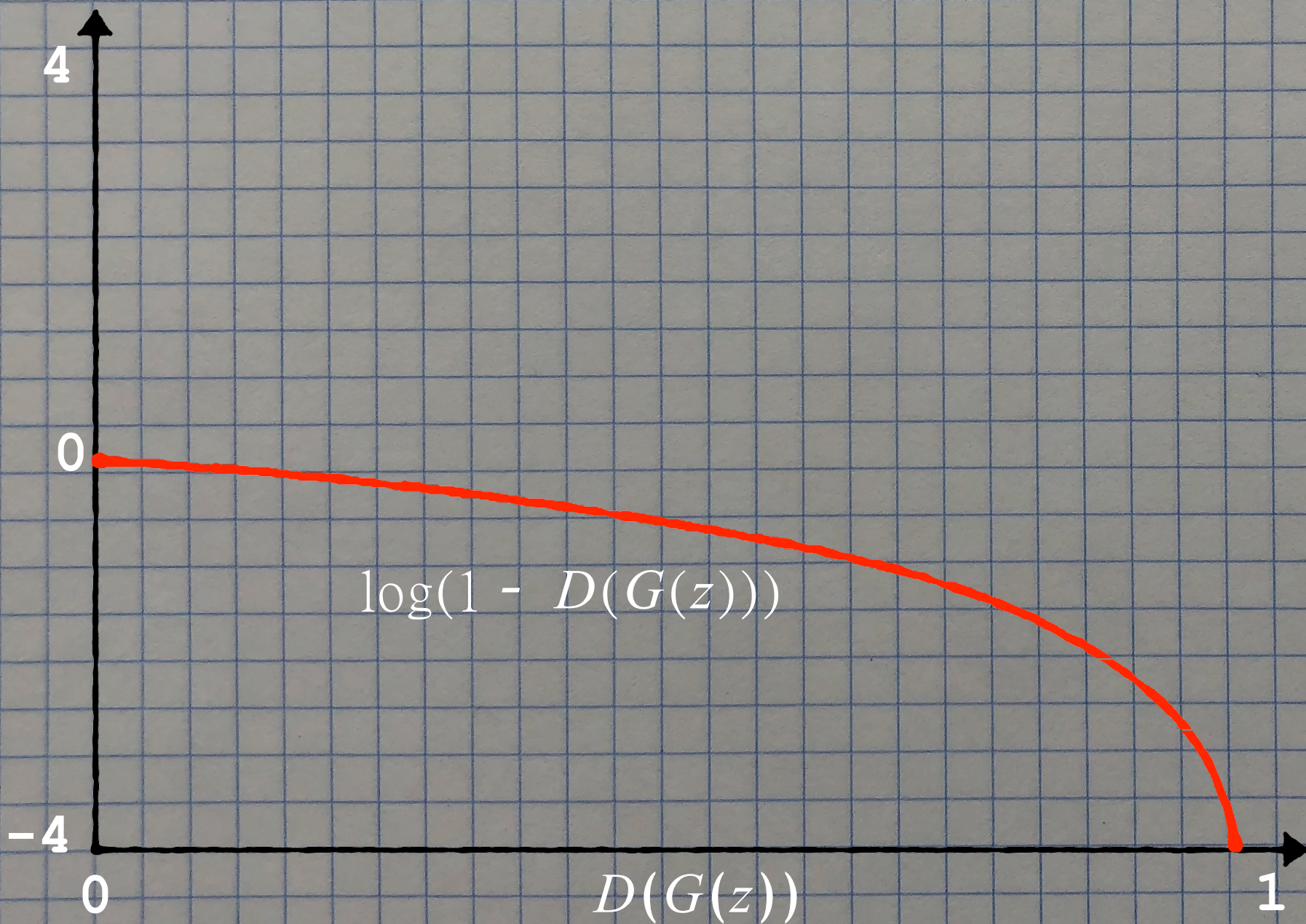
1

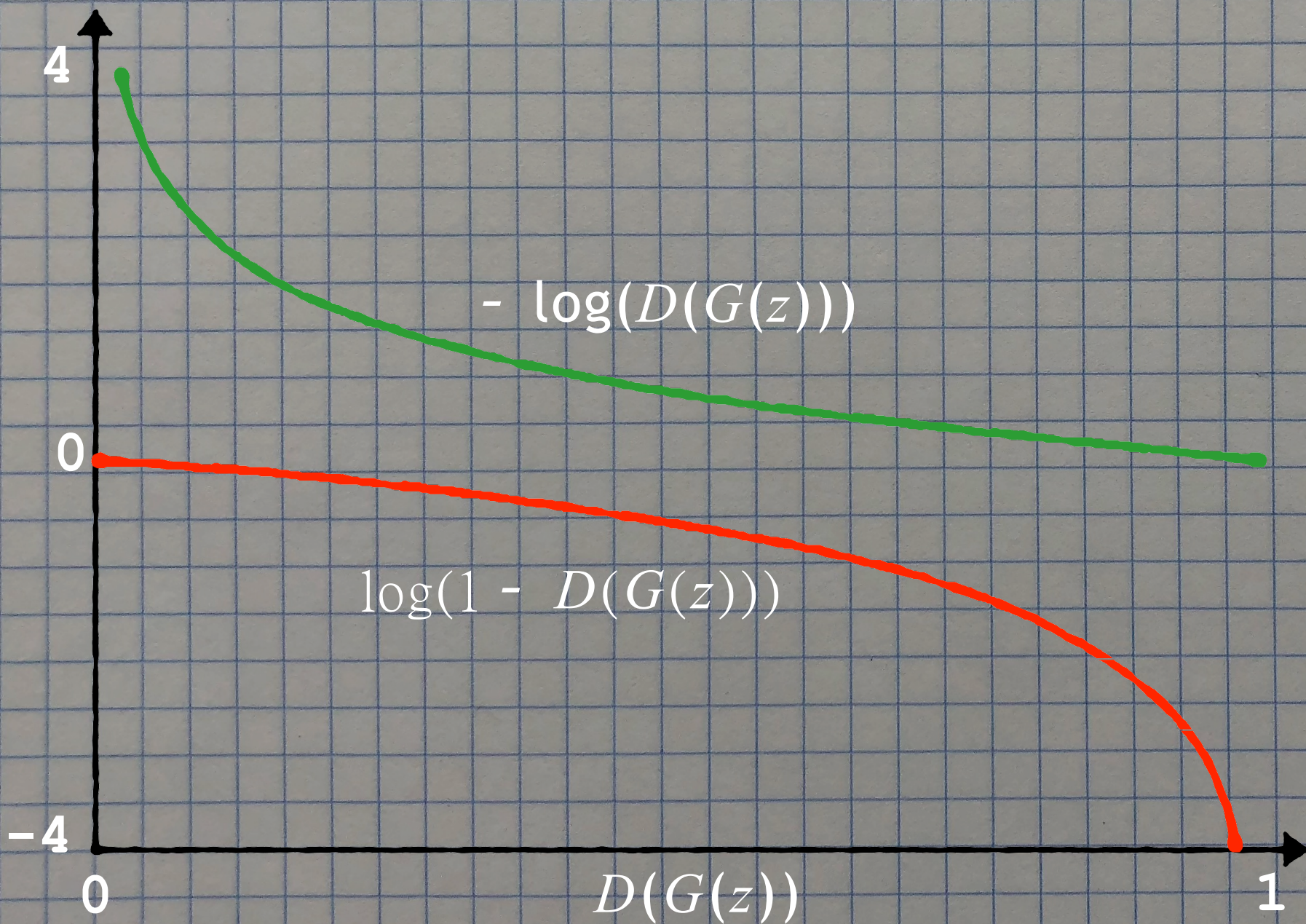


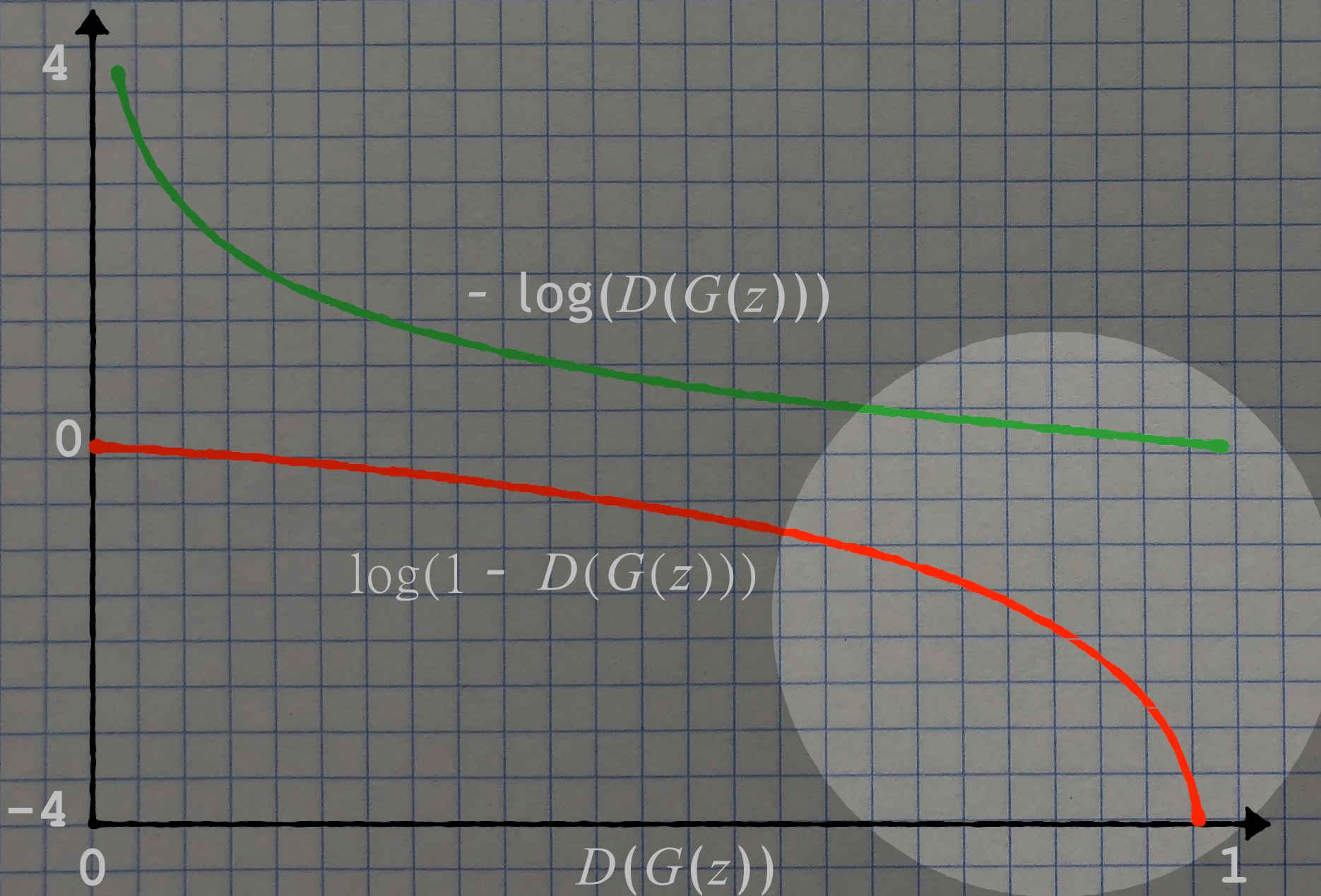
$$\min_{\theta_g} \mathbb{E}_{z \sim p(z)} \log(1 - D_{\theta_d}(G_{\theta_g}(z)))$$

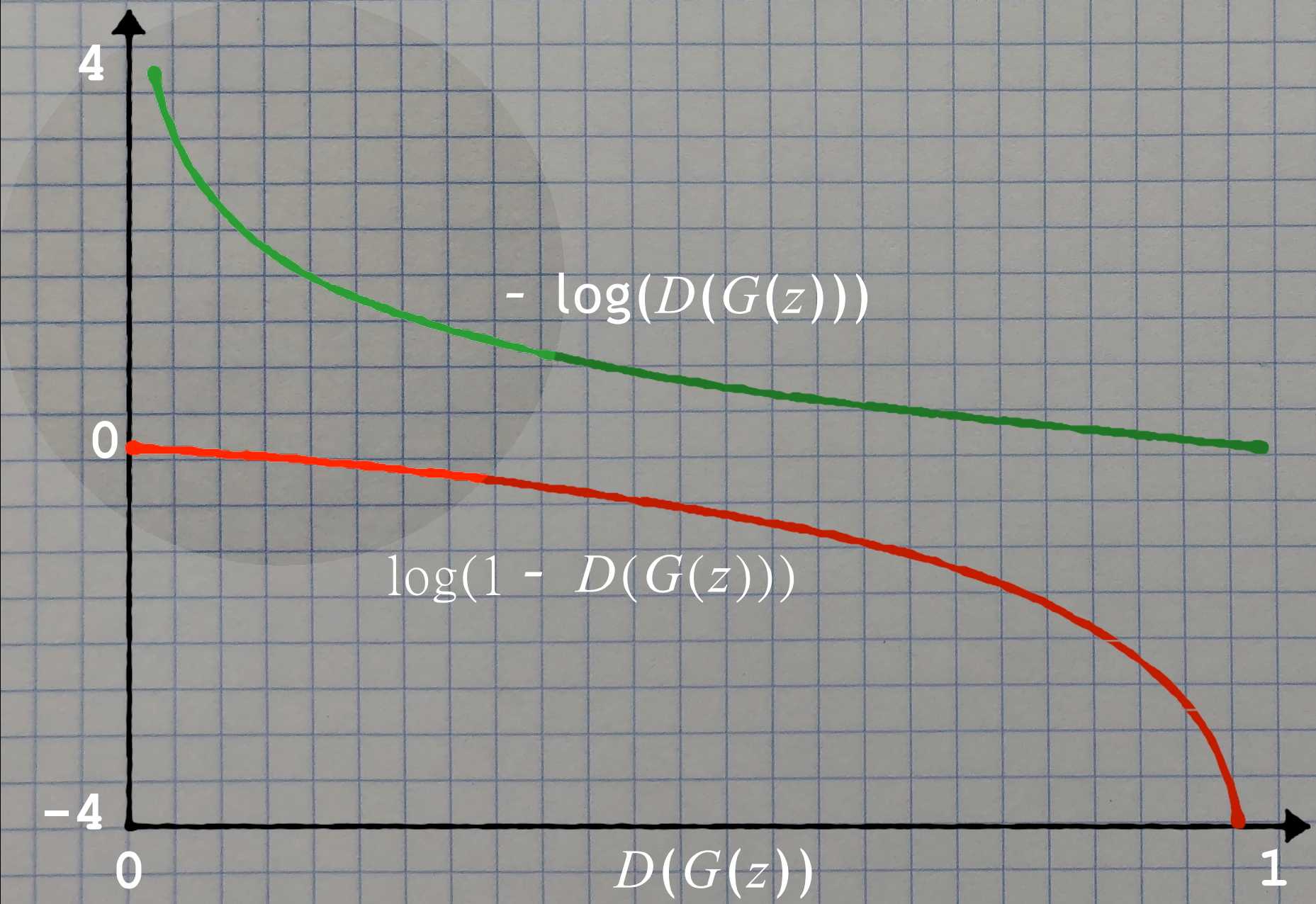
**replace with**

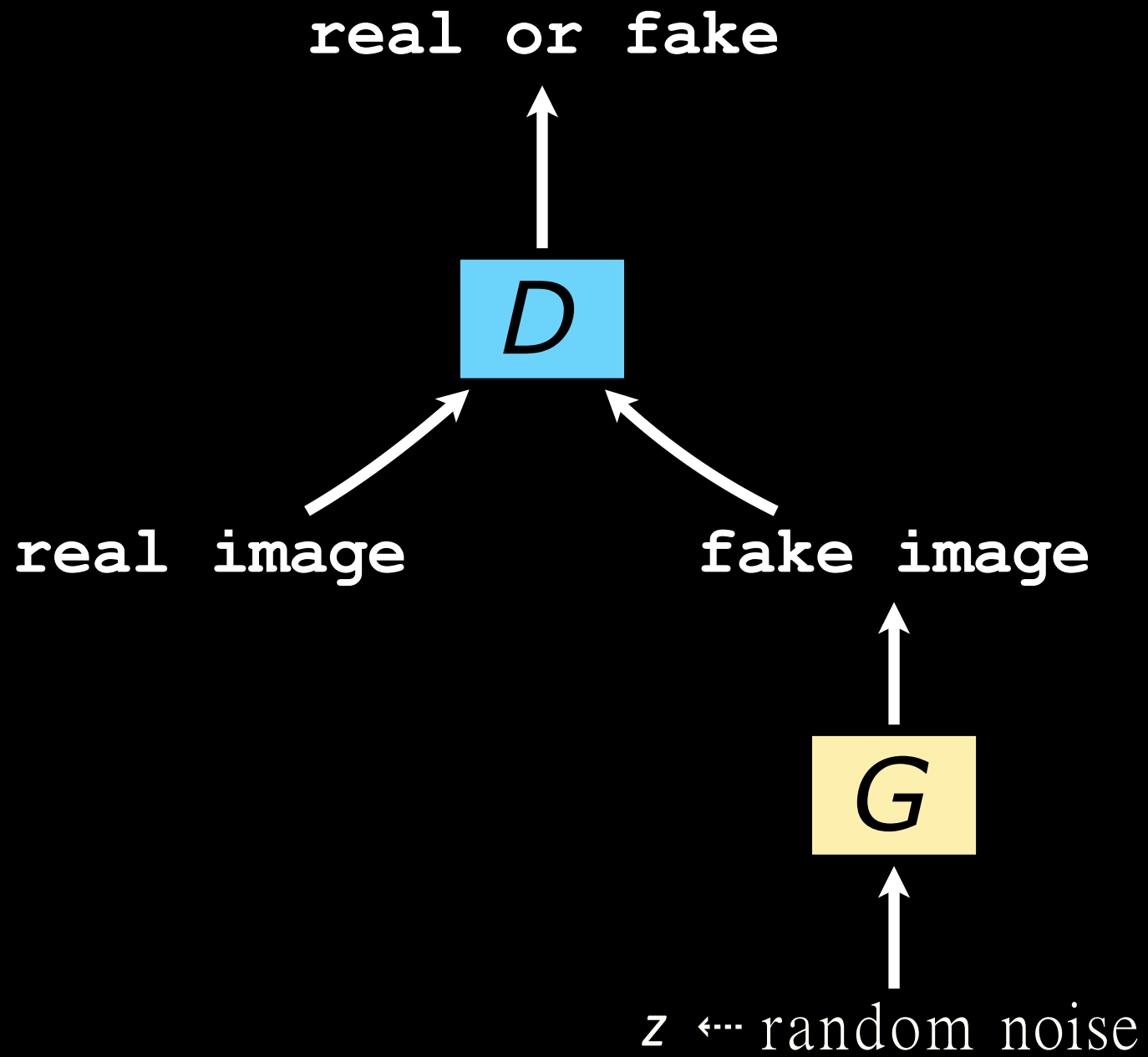
$$\max_{\theta_g} \mathbb{E}_{z \sim p(z)} \log(D_{\theta_d}(G_{\theta_g}(z)))$$













GAN  
training

```
for number of training iterations do
```

```
  for k steps do
```

```
    sample m noise samples from noise prior
```

```
    sample m real examples from dataset
```

```
    update the discriminator by gradient ascent
```

```
  end for
```

```
  sample m noise samples from noise prior
```

```
  update generator by stochastic gradient ascent
```

```
end for
```



```
for number of training iterations do
```

```
  for k steps do
```

```
    sample m noise samples from noise prior
```

```
    sample m real examples from dataset
```

```
    update the discriminator by gradient ascent
```

```
  end for
```

```
  sample m noise samples from noise prior
```

```
  update generator by stochastic gradient ascent
```

```
end for
```

```
for number of training iterations do
```

```
  for k steps do
```

```
    sample m noise samples from noise prior
```

```
    sample m real examples from dataset
```

```
    update the discriminator by gradient ascent
```

```
  end for
```

```
  sample m noise samples from noise prior
```

```
  update generator by stochastic gradient ascent
```

```
end for
```

```
for number of training iterations do
```

```
  for k steps do
```

```
    sample m noise samples from noise prior
```

```
    sample m real examples from dataset
```

```
    update the discriminator by gradient ascent
```

```
  end for
```

```
    update discriminator
```

```
    update generator by stochastic gradient descent
```

```
end for
```

```
for number of training iterations do
```

```
  for k steps do
```

```
    sample m noise samples from noise prior
```

```
    sample m real examples from dataset
```

```
    update the discriminator by gradient ascent
```

```
  end for
```

```
  sample m noise samples from noise prior
```

```
  update generator by stochastic gradient ascent
```

```
end for
```

```
for number of training iterations do
```

```
  for k steps do
```

```
    sample m noise samples from noise prior
```

```
    sample m real examples from dataset
```

```
    update the discriminator by gradient ascent
```

```
  end for
```

```
  sample m noise samples from noise prior
```

```
  update generator by stochastic gradient ascent
```

```
end for
```

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^m \left[ \log D_{\theta_d}(x^{(i)}) + \log(1 - D_{\theta_d}(G_{\theta_g}(z^{(i)}))) \right]$$

for k step, do  
sample m noise samples from noise prior

update the discriminator by gradient ascent

```
for number of training iterations do
```

```
  for k steps do
```

```
    sample m noise samples from noise prior
```

```
    sample m real examples from dataset
```

```
    update the discriminator by gradient ascent
```

```
  end for
```

```
  sample m noise samples from noise prior
```

```
  update generator by stochastic gradient ascent
```

```
end for
```

```
for number of training iterations do
```

```
  for k steps do
```

```
    sample m noise samples from noise prior
```

```
    sample m real examples from dataset
```

```
    update the discriminator by gradient ascent
```

```
  end for
```

```
  sample m noise samples from noise prior
```

```
  update generator by stochastic gradient ascent
```

```
  update generator using modified objective
```



```
for number of training iterations do
```

```
  for k steps do
```

```
    sample m noise samples from noise prior
```

```
    sample m real examples from dataset
```

```
    update the discriminator by gradient ascent
```

```
  end for  $\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^m \log(D_{\theta_d}(G_{\theta_g}(z^{(i)})))$ 
```

```
    sample m noise samples from noise prior
```

```
    update generator by stochastic gradient ascent
```

```
end for
```



GAN  
training

```
for number of training iterations do  
  
  for k steps do  
    sample m noise samples from noise prior  
    sample m real examples from dataset  
    update the discriminator by gradient ascent  
  end for  
  
  sample m noise samples from noise prior  
  update generator by stochastic gradient ascent  
  
end for
```

# Generative Adversarial Networks

PYTORCH



GAN  
training

```
z_dim, data_dim = 8, 2  
hidden_dim = 100  
batch_size = 10  
lr = 1e-3
```

```
G_model = nn.Sequential(nn.Linear(z_dim, hidden_dim),  
                        nn.ReLU(),  
                        nn.Linear(hidden_dim, data_dim))
```

```
D_model = nn.Sequential(nn.Linear(data_dim, hidden_dim),  
                        nn.ReLU(),  
                        nn.Linear(hidden_dim, 1),  
                        nn.Sigmoid())
```

```
G_optimizer = optim.Adam(G_model.parameters(),  
                          lr = lr)
```

```
D_optimizer = optim.Adam(D_model.parameters(),  
                          lr = lr)
```

```
z_dim, data_dim = 8, 2
hidden_dim = 100
batch_size = 10
lr = 1e-3
```

```
G_model = nn.Sequential(nn.Linear(z_dim, hidden_dim),
                        nn.ReLU(),
                        nn.Linear(hidden_dim, data_dim))
```

```
D_model = nn.Sequential(nn.Linear(data_dim, hidden_dim),
                        nn.ReLU(),
                        nn.Linear(hidden_dim, 1),
                        nn.Sigmoid())
```

```
G_optimizer = optim.Adam(G_model.parameters(),
                          lr = lr)
```

```
D_optimizer = optim.Adam(D_model.parameters(),
                          lr = lr)
```

```
z_dim, data_dim = 8, 2
hidden_dim = 100
batch_size = 10
lr = 1e-3
```

```
G_model = nn.Sequential(nn.Linear(z_dim, hidden_dim),
                        nn.ReLU(),
                        nn.Linear(hidden_dim, data_dim))
```

```
D_model = nn.Sequential(nn.Linear(data_dim, hidden_dim),
                        nn.ReLU(),
                        nn.Linear(hidden_dim, 1),
                        nn.Sigmoid())
```

```
G_optimizer = optim.Adam(G_model.parameters(),
                          lr = lr)
```

```
D_optimizer = optim.Adam(D_model.parameters(),
                          lr = lr)
```


```
z_dim, data_dim = 8, 2
hidden_dim = 100
batch_size = 10
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```
G_model = nn.Sequential(nn.Linear(z_dim, hidden_dim),
                        nn.ReLU(),
                        nn.Linear(hidden_dim, data_dim))
```

```
D_model = nn.Sequential(nn.Linear(data_dim, hidden_dim),
                        nn.ReLU(),
                        nn.Linear(hidden_dim, 1),
                        nn.Sigmoid())
```

```
G_optimizer = optim.Adam(G_model.parameters(),
                          lr = lr)
```

```
D_optimizer = optim.Adam(D_model.parameters(),
                          lr = lr)
```



GAN  
training

```
z_dim, data_dim = 8, 2
hidden_dim = 100
batch_size = 10
lr = 1e-3
```

```
G_model = nn.Sequential(nn.Linear(z_dim, hidden_dim),
                        nn.ReLU(),
                        nn.Linear(hidden_dim, data_dim))
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```
D_model = nn.Sequential(nn.Linear(data_dim, hidden_dim),
                        nn.ReLU(),
                        nn.Linear(hidden_dim, 1),
                        nn.Sigmoid())
```

```
G_optimizer = optim.Adam(G_model.parameters(),
                          lr = lr)
```

```
D_optimizer = optim.Adam(D_model.parameters(),
                          lr = lr)
```



```
lr = lr)
```

```
for iters in range(epochs_num):
```

```
    for t, real_batch in \
        enumerate(real_samples.split(batch_size)):

        z = real_batch.new_empty((real_batch.size(0),
                                   z_dim)).normal_()
        fake_batch = G_model(z)

        real_D_scores = D_model(real_batch)
        fake_D_scores = D_model(fake_batch)

        if t%2 == 0:
            loss = -fake_D_scores.log().mean()
            G_optimizer.zero_grad()
            loss.backward()
            G_optimizer.step()
        else:
            loss = (- (1 - fake_D_scores).log().mean()
                   - real_D_scores.log().mean())
            D_optimizer.zero_grad()
```

```
for t, real_batch in \
    enumerate(real_samples.split(batch_size)):

    z = real_batch.new_empty((real_batch.size(0),
                              z_dim)).normal_()
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    else:
        loss = (- (1 - fake_D_scores).log().mean()
                - real_D_scores.log().mean())
        D_optimizer.zero_grad()
        loss.backward()
        D_optimizer.step()
```

```
for t, real_batch in \
    enumerate(real_samples.split(batch_size)):

    z = real_batch.new_empty((real_batch.size(0),
                              z_dim)).normal_()
    fake_batch = G_model(z)

    real_D_scores = D_model(real_batch)
    fake_D_scores = D_model(fake_batch)

    if t%2 == 0:
        loss = -fake_D_scores.log().mean()
        G_optimizer.zero_grad()
        loss.backward()
        G_optimizer.step()
    else:
        loss = (- (1 - fake_D_scores).log().mean()
                - real_D_scores.log().mean())
        D_optimizer.zero_grad()
        loss.backward()
        D_optimizer.step()
```

```
for t, real_batch in \
    enumerate(real_samples.split(batch_size)):
```

```
    z = real_batch.new_empty((real_batch.size(0),
                              z_dim)).normal_()
```

```
    fake_batch = G_model(z)
```

**generate random latent vectors**

```
    fake_D_scores = D_model(fake_batch)
```

```
    if t%2 == 0:
```

```
        loss = -fake_D_scores.log().mean()
```

```
        G_optimizer.zero_grad()
```

```
        loss.backward()
```

```
        G_optimizer.step()
```

```
    else:
```

```
        loss = (- (1 - fake_D_scores).log().mean()
```

```
                - real_D_scores.log().mean())
```

```
        D_optimizer.zero_grad()
```

```
        loss.backward()
```

```
        D_optimizer.step()
```

```
for t, real_batch in \
    enumerate(real_samples.split(batch_size)):

    z = real_batch.new_empty((real_batch.size(0),
                              z_dim)).normal_()
```

```
fake_batch = G_model(z)
```

```
real_D_scores = D_model(real_batch)
fake_D_scores = D_model(fake_batch)
```

```
if t%2 == 0:
    loss = -fake_D_scores.log().mean()
    G_optimizer.zero_grad()
    loss.backward()
    G_optimizer.step()
else:
    loss = (- (1 - fake_D_scores).log().mean()
            - real_D_scores.log().mean())
    D_optimizer.zero_grad()
    loss.backward()
    D_optimizer.step()
```

```
for t, real_batch in \
    enumerate(real_samples.split(batch_size)):

    z = real_batch.new_empty((real_batch.size(0),
                              z_dim)).normal_()
    fake_batch = G_model(z)
```

```
real_D_scores = D_model(real_batch)
fake_D_scores = D_model(fake_batch)
```

```
if t%2 == 0:
    loss = -fake_D_scores.log().mean()
    G_optimizer.zero_grad()
    loss.backward()
    G_optimizer.step()
else:
    loss = (- (1 - fake_D_scores).log().mean()
            - real_D_scores.log().mean())
    D_optimizer.zero_grad()
    loss.backward()
    D_optimizer.step()
```

```
real_D_scores = D_model(real_batch)
fake_D_scores = D_model(fake_batch)
```

```
if t%2 == 0:
    loss = -fake_D_scores.log().mean()
    G_optimizer.zero_grad()
    loss.backward()
    G_optimizer.step()
else:
    loss = (- (1 - fake_D_scores).log().mean()
            - real_D_scores.log().mean())
    D_optimizer.zero_grad()
    loss.backward()
    D_optimizer.step()
```

```
real_D_scores = D_model(real_batch)
fake_D_scores = D_model(fake_batch)
```

```
if t%2 == 0:
    loss = -fake_D_scores.log().mean()
    G_optimizer.zero_grad()
    loss.backward()
    G_optimizer.step()
```

**update generator using modified objective**

```
loss = -real_D_scores.log().mean()
D_optimizer.zero_grad()
loss.backward()
D_optimizer.step()
```



```
real_D_scores = D_model(real_batch)
fake_D_scores = D_model(fake_batch)
```

```
if t%2 == 0:
    loss = -fake_D_scores.log().mean()
    G_optimizer.zero_grad()
    loss.backward()
    G_optimizer.step()
```

```
else:
    loss = (- (1 - fake_D_scores).log().mean()
            - real_D_scores.log().mean())
    D_optimizer.zero_grad()
    loss.backward()
    D_optimizer.step()
```

**update discriminator**

```
z_dim, data_dim = 8, 2
hidden_dim = 100
batch_size = 10
lr = 1e-3

G_model = nn.Sequential(nn.Linear(z_dim, hidden_dim),
                        nn.ReLU(),
                        nn.Linear(hidden_dim, data_dim))

D_model = nn.Sequential(nn.Linear(data_dim, hidden_dim),
                        nn.ReLU(),
                        nn.Linear(hidden_dim, 1),
                        nn.Sigmoid())

G_optimizer = optim.Adam(G_model.parameters(),
                          lr = lr)
D_optimizer = optim.Adam(D_model.parameters(),
                          lr = lr)

for iters in range(epochs_num):

    for t, real_batch in \
        enumerate(real_samples.split(batch_size)):

        z = real_batch.new_empty((real_batch.size(0),
                                  z_dim)).normal_()
        fake_batch = G_model(z)

        real_D_scores = D_model(real_batch)
        fake_D_scores = D_model(fake_batch)

        if t%2 == 0:
            loss = -fake_D_scores.log().mean()
            G_optimizer.zero_grad()
            loss.backward()
            G_optimizer.step()
        else:
            loss = (- (1 - fake_D_scores).log().mean()
                   - real_D_scores.log().mean())
            D_optimizer.zero_grad()
            loss.backward()
            D_optimizer.step()
```



GAN  
training

# How is the quality of generated images assessed?

Two simple properties for evaluation metric:

- **Fidelity**: We want our GAN to generate *high* quality images.
- **Diversity**: Our GAN should generate images that are inherent in the training dataset.

## Feature Distance:

- Use a pre-trained image classification model (neural network).
- Pass an image through the model and use the activation of intermediate layers as features.
- Calculate any distance metric (L2/L1) between the features of generated image and GT real image.
- LPIPS metric (Learned Perceptual Image Patch Similarity).

But often, we do not have the GT image to compare with.

What do we do?

# FID (Frechet Inception Distance)

Frechet Distance between two univariate gaussian distribution

$$d(X, Y) = (\mu_X - \mu_Y)^2 + (\sigma_X - \sigma_Y)^2$$

Frechet Distance between two multi-variate gaussian distribution

$$\text{FID} = \|\mu_X - \mu_Y\|^2 - \text{Tr}(\Sigma_X + \Sigma_Y - 2 \Sigma_X \Sigma_Y)$$

Frechet Inception Distance (FID), X and Y are features of Inception V3 classification model for real and fake images respectively.

Note: The loss is between set of real and fake images, not individual real and fake image!



results typically assessed visually

# Important Deadlines

- 590: Assignment 2 announced, due Sept 8.
- 590/790: Paper presentation/review schedule announced
- 790: Deadline to register your project group, Sept 1! 1 points deducted per late day!
- 790: Project Proposal presentation is due Sept 20!

# Slide Credits

- EECS 6322 Deep Learning for Computer Vision, Kosta Derpanis (York University)
- EECS 498 Deep Learning for Computer Vision, Justin Johnson (U. Michigan)
- Many amazing research papers!