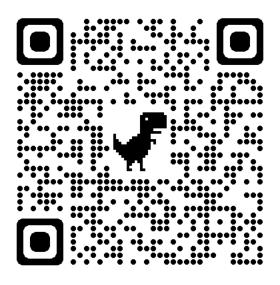
Lecture 7: Features 1

COMP 590/776: Computer Vision

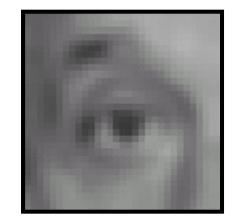
Instructor: Soumyadip (Roni) Sengupta

TA: Mykhailo (Misha) Shvets

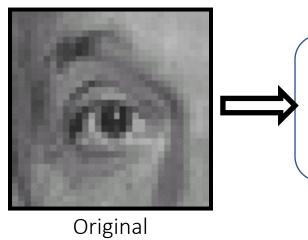


Course Website: Scan Me!

Recap



Blur



Linear Filtering: Cross-correlation & Convolution



Shifted left by 1 pixel



Sharpening

Convolution

 Same as cross-correlation, except that the kernel is "flipped" (horizontally and vertically)

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u,j+v] \qquad \text{Cross-correlation}$$

Convolution is commutative and associative

Aliasing

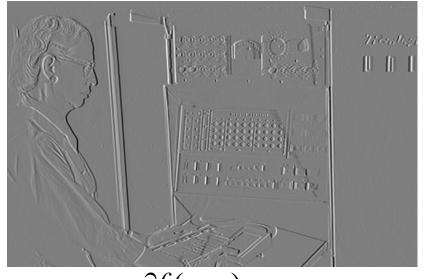
- Images are Signals in 2D
- Signals contain low frequency (smooth regions) and high frequency (sharp changes in intensity)
- To accurately downsample a signal/image, # of samples >= 2*highest frequency in the signal. (Nyquist Rate!)
- If your task is to downsample by 1/4, you do not have enough samples, thus the downsampled image is inaccurate especially in terms of high frequency components.

Partial Derivatives

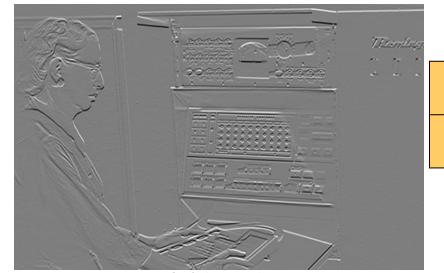
Can be implemented as a convolution operation



-1 1



 $\frac{\partial f(x,y)}{\partial x}$



$$\frac{\partial f(x,y)}{\partial v}$$

or 1

Noise in 2D

Noisy Input Ix via [-1,0,1] Zoom

Source: D. Fouhey

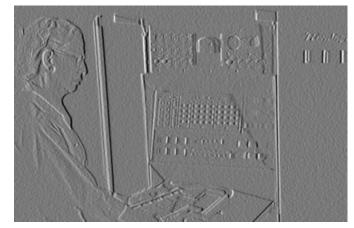
Noise + Smoothing

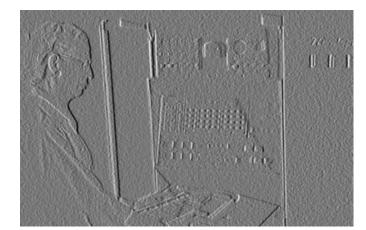
Smoothed Input



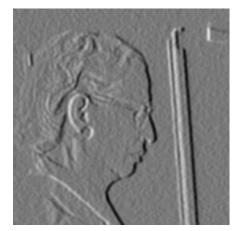
Statement Constitution of the Constitution of

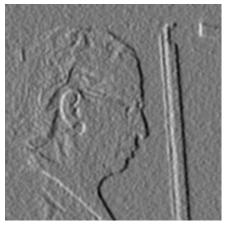
Ix via [-1,0,1]





Zoom





Source: D. Fouhey



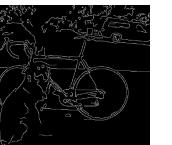
Canny edge detector



- 1. Filter image with derivative of Gaussian
- 2. Find magnitude and orientation of gradient



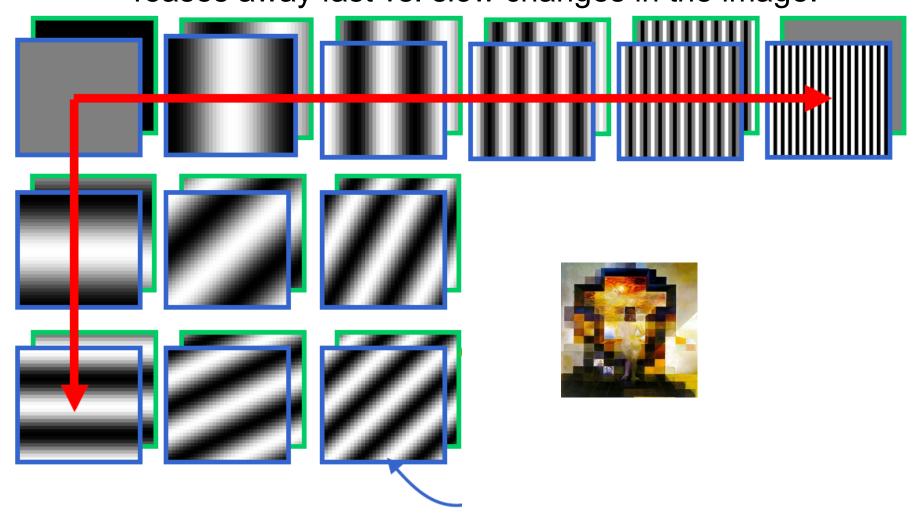
3. Non-maximum suppression



- 4. Linking and thresholding (hysteresis):
 - Define two thresholds: low and high
 - Use the high threshold to start edge curves and the low threshold to continue them

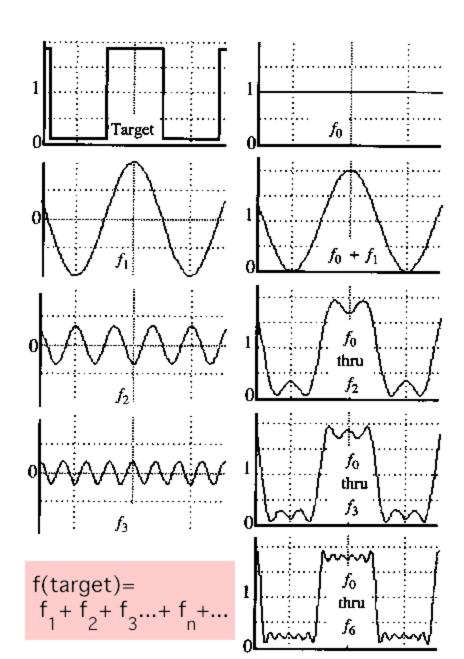
Fourier Transform

Teases away fast vs. slow changes in the image.



A sum of sines

- Our building block:
- $A\sin(\omega x + \phi)$
- Add enough of them to get any signal f(x) you want!



Scary Math

Fourier Transform :
$$F(\omega) = \int_{-\infty}^{+\infty} f(x)e^{-i\omega x} dx$$

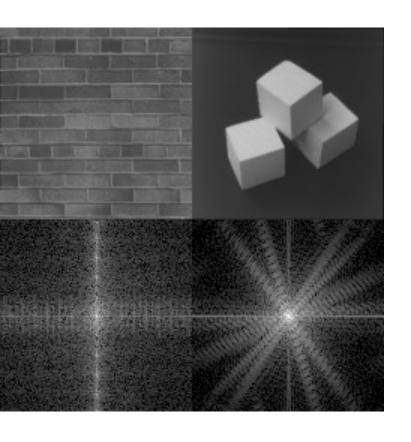
Inverse Fourier Transform :
$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega)e^{i\omega x} d\omega$$

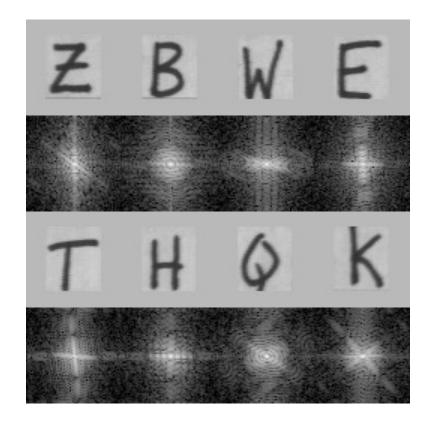
Discrete Fourier Transform
$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi k n/N}.$$

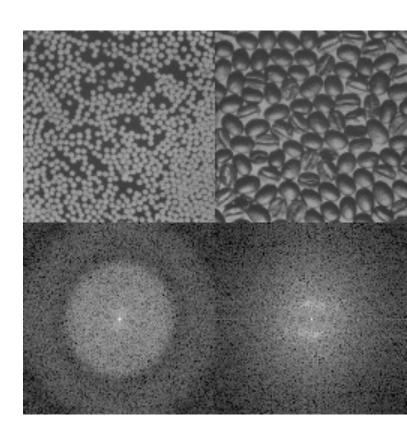
The discrete Fourier transform (DFT) of an image f of size M imes N is an image F of same size defined as:

$$F(u,v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) e^{-j \, 2\pi \left(rac{um}{M} + rac{vn}{N}
ight)}$$

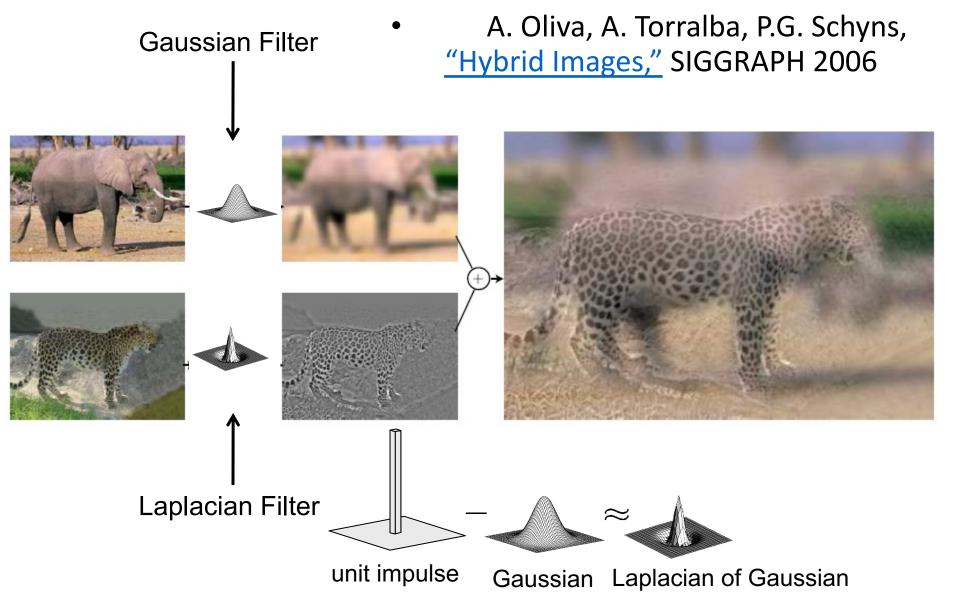
Amplitude Spectrum of DFT





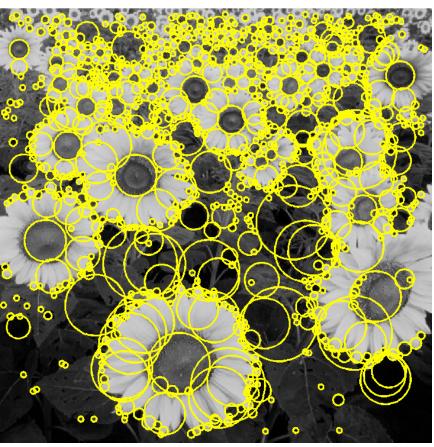


Application: Hybrid Images

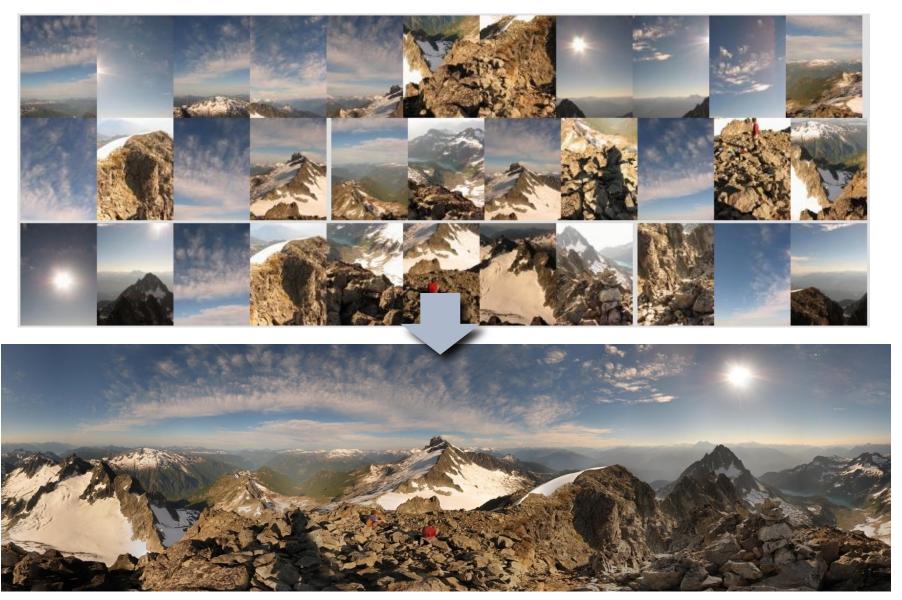


Today: Feature extraction—Corners and blobs





Motivation: Automatic panoramas



Credit: Matt Brown

Motivation: Automatic panoramas



GigaPan:

http://gigapan.com/

Also see Google Zoom Views:

https://www.google.com/culturalinstitute/beta/project/gigapixels

Steps of creating a Panorama (For this & next week)

This is your next homework assignment!

Why extract features?

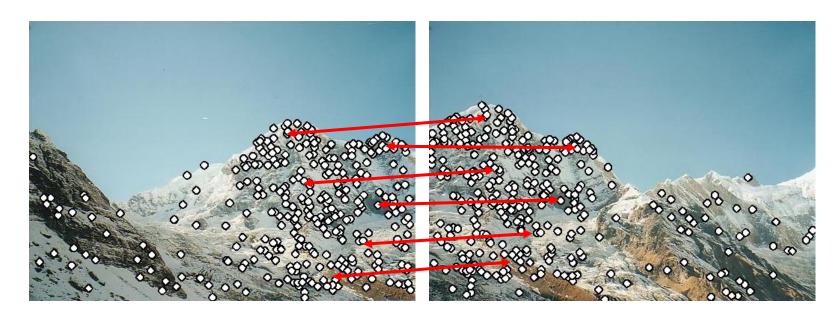
- Motivation: panorama stitching
 - We have two images how do we combine them?





Why extract features?

- Motivation: panorama stitching
 - We have two images how do we combine them?

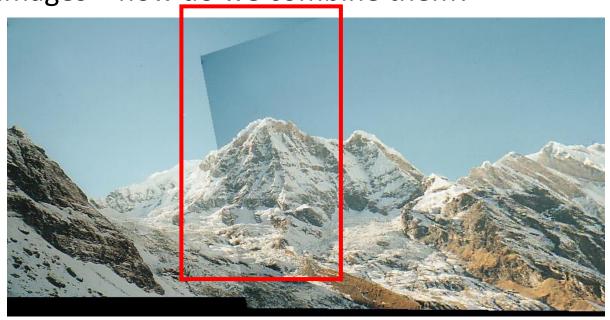


Step 1: extract features Step 2: match features

Why extract features?

Motivation: panorama stitching

• We have two images – how do we combine them?



Step 1: extract features

Step 2: match features

Step 3: align images

Step 4: blending images

This Week

Next Week

Content: Today's class

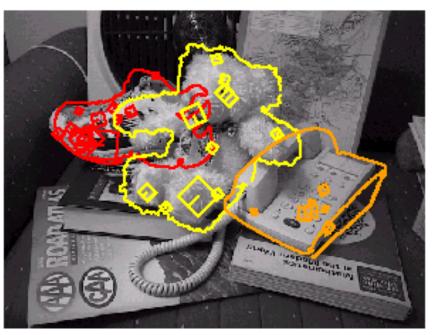
- Why detect features?
- What is a good feature?
- Harris Corner Detector
- Properties of Harris Corner Detector
- Blob Detector

Content: Today's class

- Why detect features?
- What is a good feature?
- Harris Corner Detector
- Properties of Harris Corner Detector
- Blob Detector

Object recognition (David Lowe)





Application: Visual SLAM

• (aka Simultaneous Localization and Mapping)

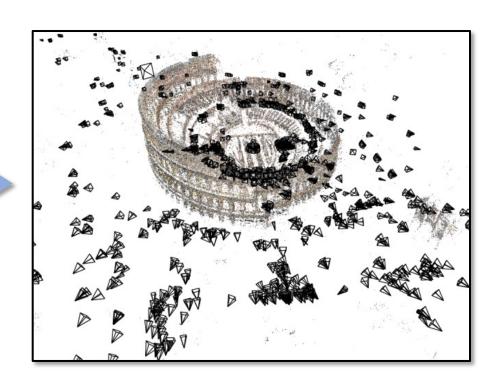




3D Reconstruction



Internet Photos ("Colosseum")



Reconstructed 3D cameras and points

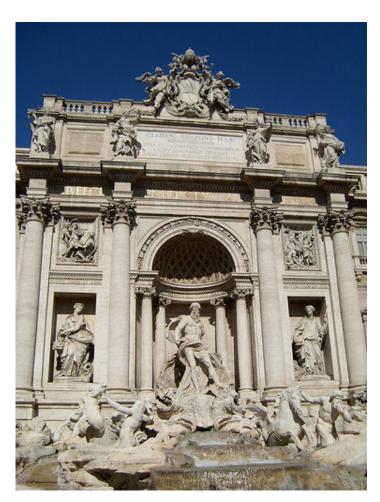
Augmented Reality



Image matching



by <u>Diva Sian</u>



by swashford

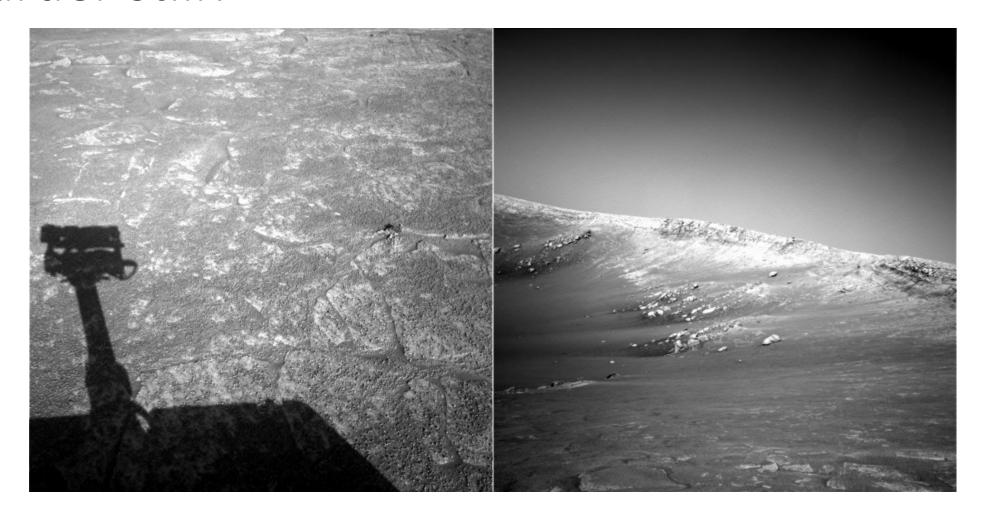
Harder case



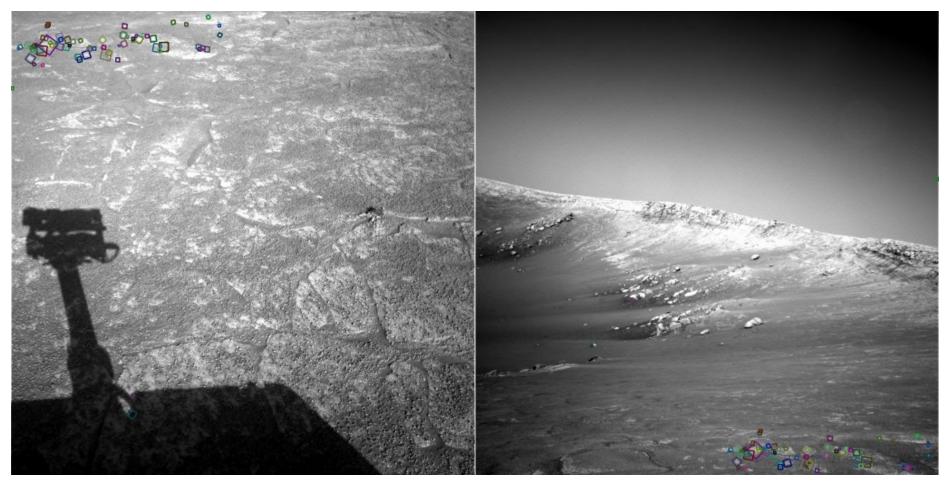


by <u>Diva Sian</u> by <u>scgbt</u>

Harder still?

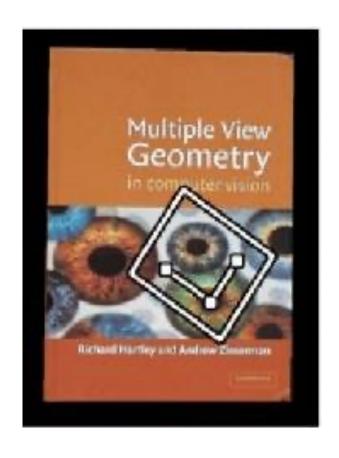


Answer below (look for tiny colored squares...)



NASA Mars Rover images with SIFT feature matches

Feature matching for object search





Feature matching



More motivation...

Feature points are used for:

- Image alignment (e.g., mosaics)
- 3D reconstruction
- Motion tracking (e.g. for AR)
- Object recognition
- Image retrieval
- Robot/car navigation
- ... other



Content: Today's class

- Why detect features?
- What is a good feature?
- Harris Corner Detector
- Properties of Harris Corner Detector
- Blob Detector



Local features: main components

1) Detection: Identify the interest points

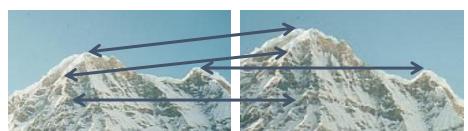


3) Matching: Determine correspondence between descriptors in two views



$$\mathbf{x}_{1} = [x_{1}^{(1)}, \dots, x_{d}^{(1)}]$$

$$\mathbf{x}_{2} = [x_{1}^{(2)}, \dots, x_{d}^{(2)}]$$



Advantages of local features

Locality

features are local, so robust to occlusion and clutter

Quantity

• hundreds or thousands in a single image

Distinctiveness:

can differentiate a large database of objects

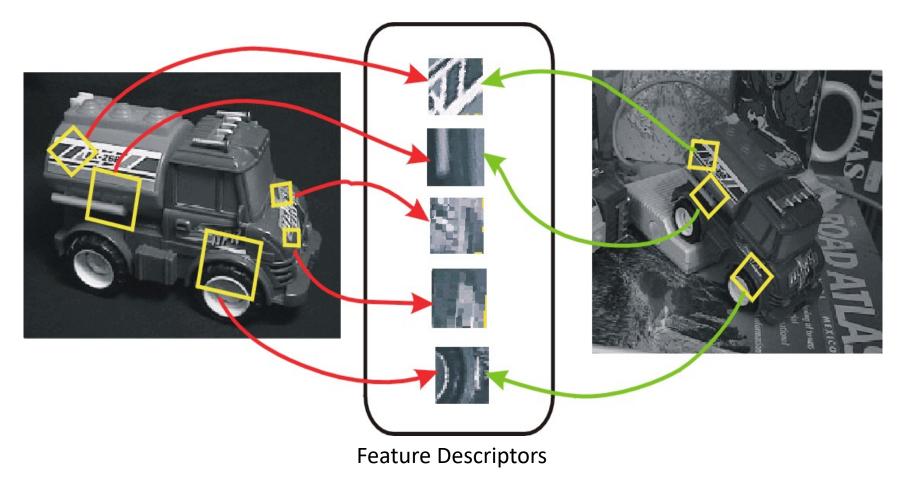
Efficiency

real-time performance achievable

Invariant local features

Find features that are invariant to transformations

- geometric invariance: translation, rotation, scale
- photometric invariance: brightness, exposure, ...



Want uniqueness

Look for image regions that are unusual

• Lead to unambiguous matches in other images

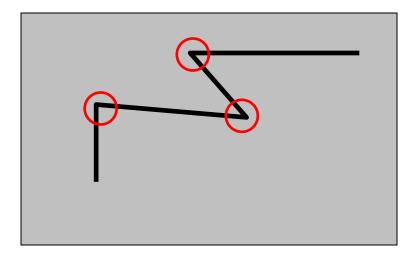
How to define "unusual"?

Content: Today's class

- Why detect features?
- What is a good feature?
- Harris Corner Detector
- Properties of Harris Corner Detector
- Blob Detector

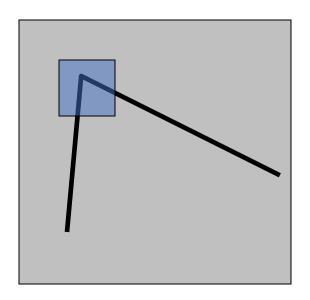
Harris corner detector

• C.Harris, M.Stephens. "A Combined Corner and Edge Detector". 1988



The Basic Idea

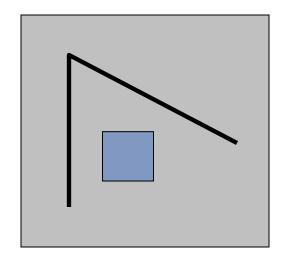
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity

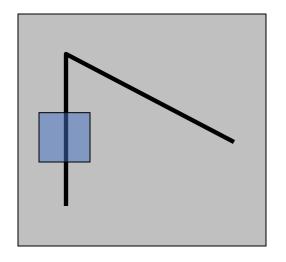


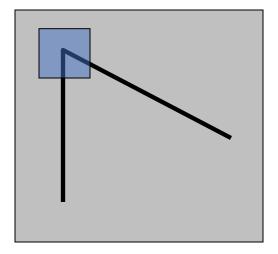
Local measures of uniqueness

Suppose we only consider a small window of pixels

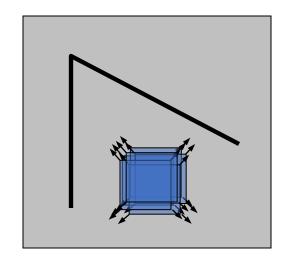
• What defines whether a feature is a good or bad candidate?

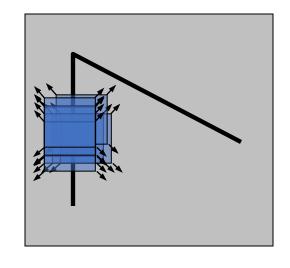


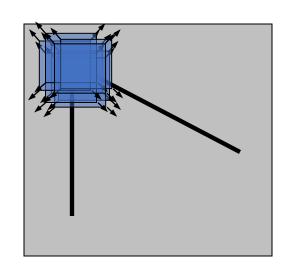




Harris Detector: Basic Idea







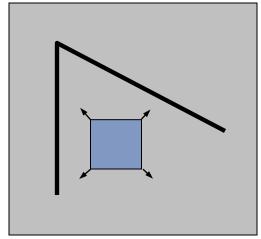
"flat" region: no change in all directions

"edge": no change along the edge direction

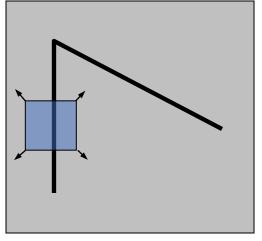
"corner": significant change in all directions

Local measures of uniqueness

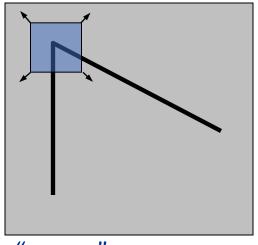
- How does the window change when you shift it?
- Shifting the window in any direction causes a big change



"flat" region: no change in all directions



"edge": no change along the edge direction



"corner":
significant change in all directions

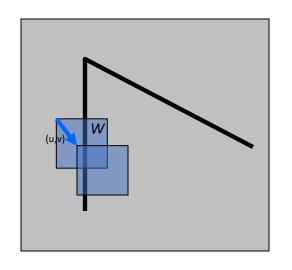
Harris corner detection: the math

Consider shifting the window W by (u,v)

- how do the pixels in W change?
- compare each pixel before and after by summing up the squared differences (SSD)
- this defines an SSD "error" E(u,v):

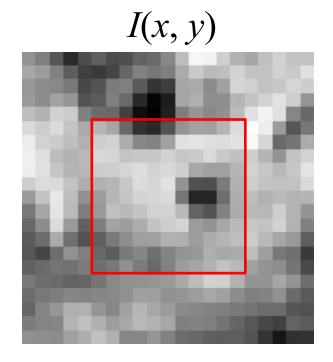
$$E(u,v) = \sum_{(x,y)\in W} [I(x+u,y+v) - I(x,y)]^2$$

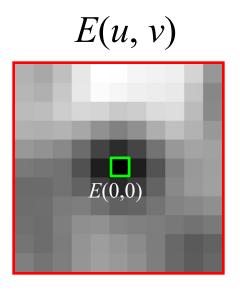
- We are happy if this error is high
- Slow to compute exactly for each pixel and each offset (u,v)



Corner Detection: Mathematics Change in appearance of window W for the shift [u,v]:

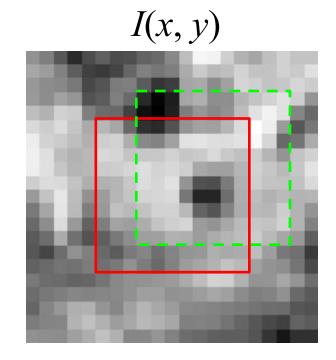
$$E(u,v) = \sum_{(x,y)\in W} [I(x+u,y+v) - I(x,y)]^2$$

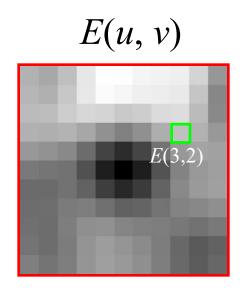




Corner Detection: Mathematics Change in appearance of window W for the shift [u,v]:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u,y+v) - I(x,y)]^{2}$$





Corner Detection: Mathematics Change in appearance of window W for the shift [u,v]:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u,y+v) - I(x,y)]^2$$

We want to find out how this function behaves for small shifts

Small motion assumption

Taylor Series expansion of *I*:

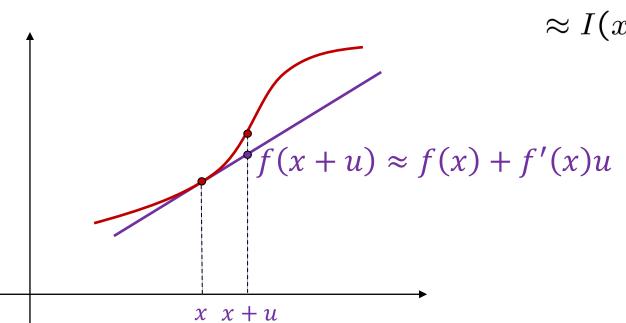
$$I(x+u,y+v) = I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

If the motion (u, v) is small, then first order approximation is good

$$I(x+u,y+v) \approx I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v$$

$$\approx I(x,y) + [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix}$$
short

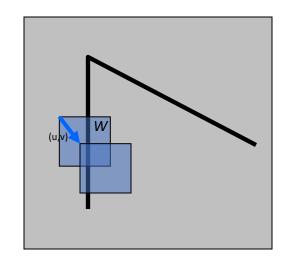
shorthand: $I_x = \frac{\partial I}{\partial x}$



Plugging this into the formula on the previous slide...

Consider shifting the window W by (u,v)

• define an SSD "error" *E(u,v)*:



$$E(u,v) = \sum_{\substack{(x,y) \in W}} [I(x+u,y+v) - I(x,y)]^{2}$$

$$\approx \sum_{\substack{(x,y) \in W}} [I(x,y) + I_{x}u + I_{y}v - I(x,y)]^{2}$$

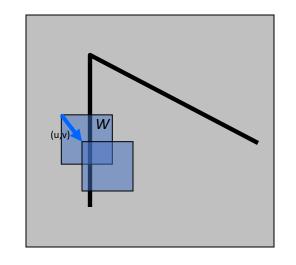
$$\approx \sum_{\substack{(x,y) \in W}} [I_{x}u + I_{y}v]^{2}$$

Consider shifting the window W by (u,v)

• define an SSD "error" *E(u,v)*:

$$E(u,v) \approx \sum_{(x,y)\in W} [I_x u + I_y v]^2$$

$$\approx Au^2 + 2Buv + Cv^2$$



$$A = \sum_{(x,y)\in W} I_x^2 \qquad B = \sum_{(x,y)\in W} I_x I_y \qquad C = \sum_{(x,y)\in W} I_y^2$$

• Thus, E(u,v) is locally approximated as a quadratic error function

The second moment matrix

 $(x,y) \in W$

The surface E(u,v) is locally approximated by a quadratic form.

$$E(u,v) \approx Au^2 + 2Buv + Cv^2$$

$$\approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

$$H$$

$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$

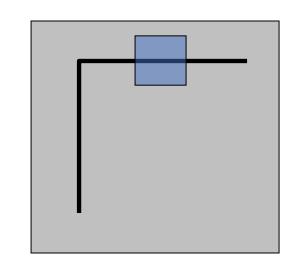
Let's try to understand its shape.

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

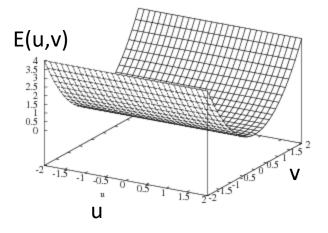
$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$



Horizontal edge:
$$I_x=0$$

$$H = \left| \begin{array}{cc} 0 & 0 \\ 0 & C \end{array} \right|$$

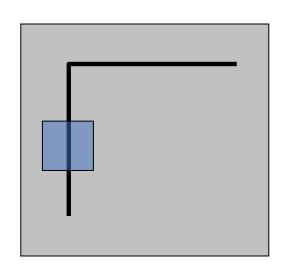


$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

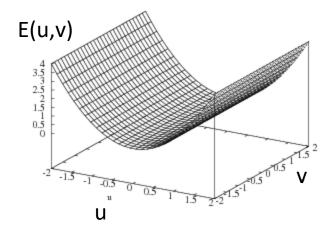
$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$



Vertical edge:
$$I_{y}=0$$

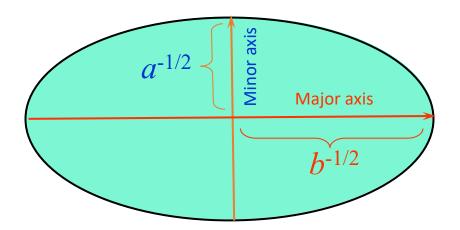
$$H = \left[\begin{array}{cc} A & 0 \\ 0 & 0 \end{array} \right]$$



Interpreting the second moment matrix

 Consider the axis-aligned case (gradients are either horizontal or vertical):

$$\frac{u^2}{\left(a^{-1/2}\right)^2} + \frac{v^2}{\left(b^{-1/2}\right)^2} = 1$$

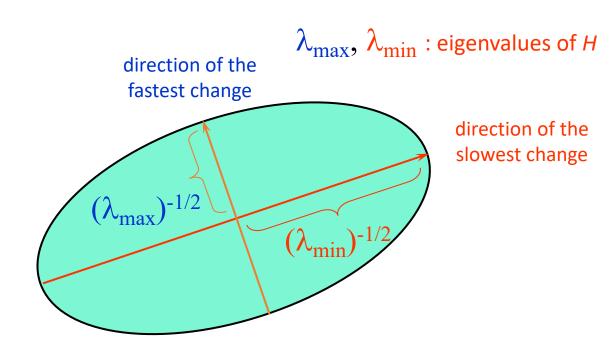


General case

We can visualize *H* as an ellipse with axis lengths determined by the *eigenvalues* of *H* and orientation determined by the *eigenvectors* of *H*



$$\begin{bmatrix} u & v \end{bmatrix} & H & \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$



Quick eigenvalue/eigenvector review

The **eigenvectors** of a matrix **A** are the vectors **x** that satisfy:

$$Ax = \lambda x$$

The scalar λ is the **eigenvalue** corresponding to **x**

The eigenvalues are found by solving:

$$det(A - \lambda I) = 0$$

• In our case, **A** = **H** is a 2x2 matrix, so we have

$$\det \begin{bmatrix} h_{11}-\lambda & h_{12} \\ h_{21} & h_{22}-\lambda \end{bmatrix} = 0$$
 • The solution:
$$\lambda_{\pm} = \frac{1}{2} \left[(h_{11}+h_{22}) \pm \sqrt{4h_{12}h_{21} + (h_{11}-h_{22})^2} \right]$$

Once you know λ , you find **x** by solving

$$\begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$E(u,v) pprox \left[egin{array}{c|cccc} u & v \end{array} \right] \left[egin{array}{c|cccc} A & B & u & v \end{array} \right] \left[egin{array}{c|cccc} u & v \end{array} \right] \left[egin{array}{ccccc} u & v \end{array} \right] \left[egin{array}{c|cccc} u & v \end{array} \right] \left[egin{array}{c|ccccc} u & v \end{array} \right] \left[egin{array}{c|cccc} u &$$

Eigenvalues and eigenvectors of H

- Define shift directions with the smallest and largest change in error
- x_{max} = direction of largest increase in E
- λ_{max} = amount of increase in direction x_{max}
- x_{min} = direction of smallest increase in E
- λ_{min} = amount of increase in direction x_{min}

How are λ_{max} , x_{max} , λ_{min} , and x_{min} relevant for feature detection?

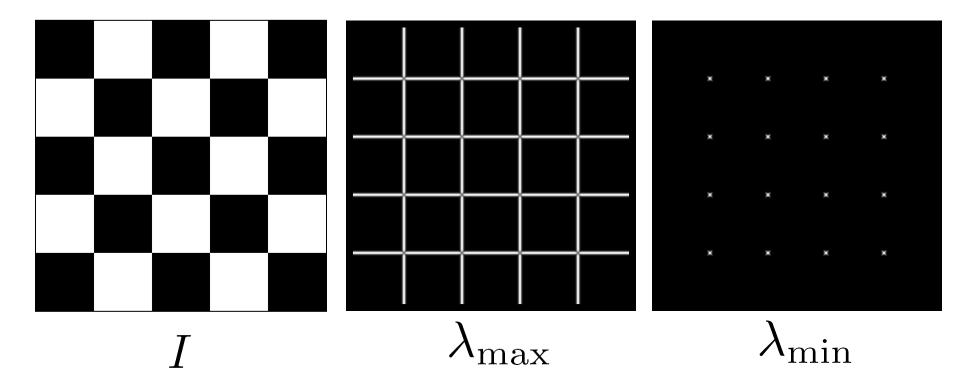
• What's our feature scoring function?

How are λ_{max} , x_{max} , λ_{min} , and x_{min} relevant for feature detection?

What's our feature scoring function?

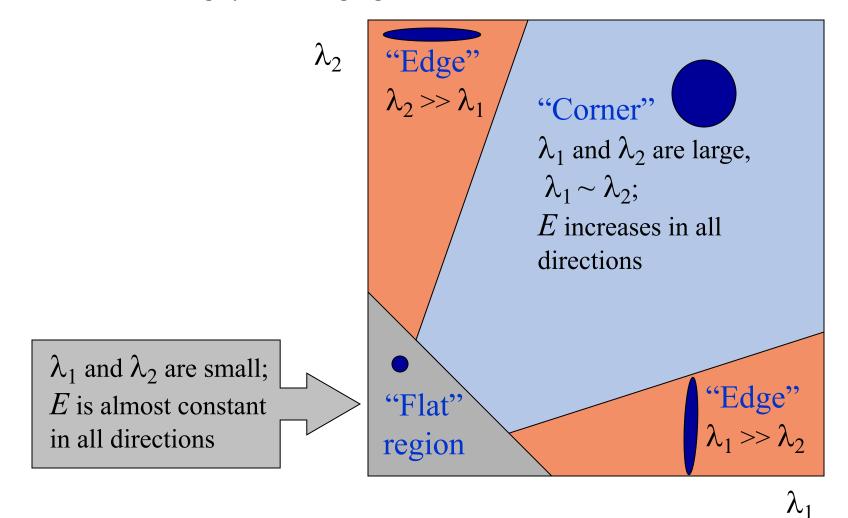
Want E(u,v) to be large for small shifts in all directions

- the minimum of E(u,v) should be large, over all unit vectors $[u \ v]$
- this minimum is given by the smaller eigenvalue (λ_{min}) of H



Interpreting the eigenvalues

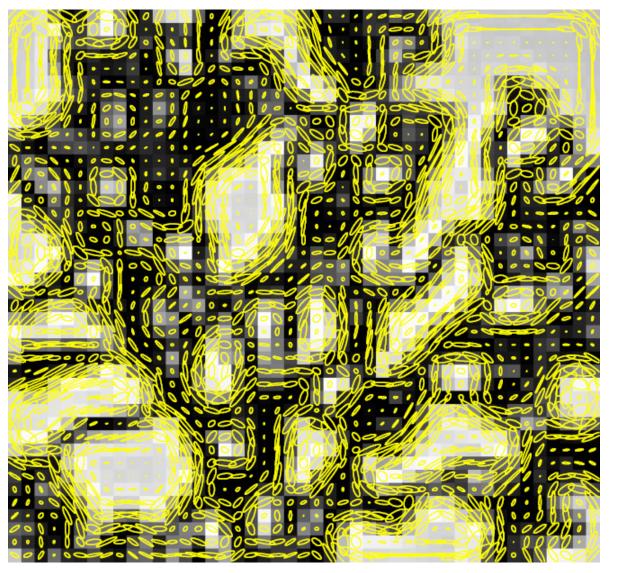
Classification of image points using eigenvalues of *M*:



Visualization of second moment matrices



Visualization of second moment matrices

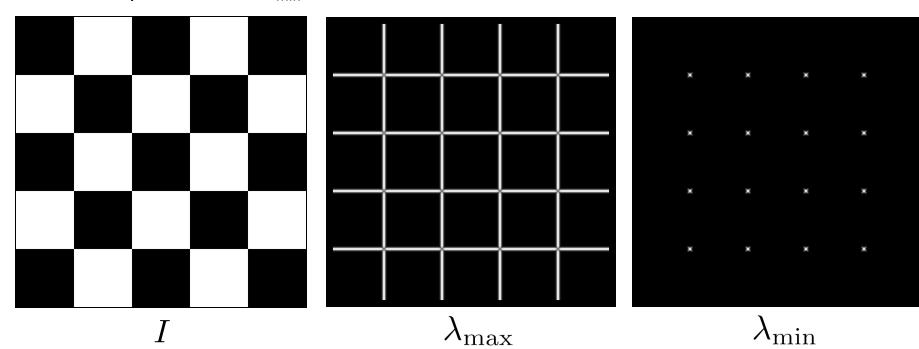


Note: axes are rescaled so ellipse areas are proportional to edge energy (i.e., bigger ellipses correspond to stronger edges)

Corner detection summary

Here's what you do:

- Compute the gradient at each point in the image
- For each pixel:
 - Create the *H* matrix from nearby gradient values
 - Compute the eigenvalues.
 - Find points with large response (λ_{min} > threshold)
- Choose those points where λ_{min} is a local maximum as features

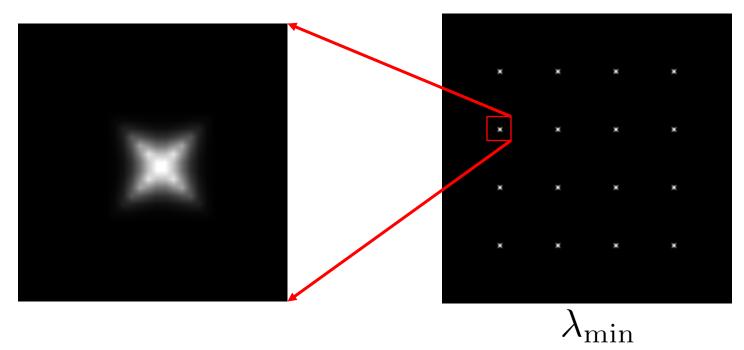


 $H = \sum_{(x,y)\in W} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$

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The Harris operator

 λ_{min} is a variant of the "Harris operator" for feature detection

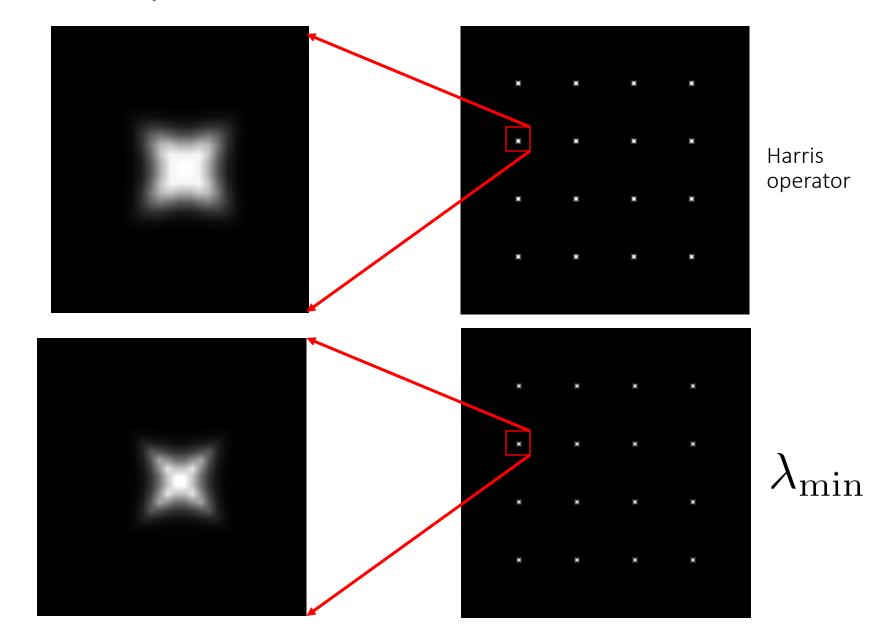
$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$
$$= \frac{determinant(H)}{trace(H)}$$

- The *trace* is the sum of the diagonals, i.e., $trace(H) = h_{11} + h_{22}$
- Very similar to λ_{min} but less expensive (no square root)
- Called the *Harris Corner Detector* or *Harris Operator*
- Lots of other detectors, this is one of the most popular

Harris detector: Steps

- 1. Compute Gaussian derivatives at each pixel
- 2. Compute second moment matrix *H* in a Gaussian window around each pixel
- 3. Compute corner response function *f* or *R*
- 4. Threshold *f* or *R*
- 5. Find local maxima of response function (nonmaximum suppression)

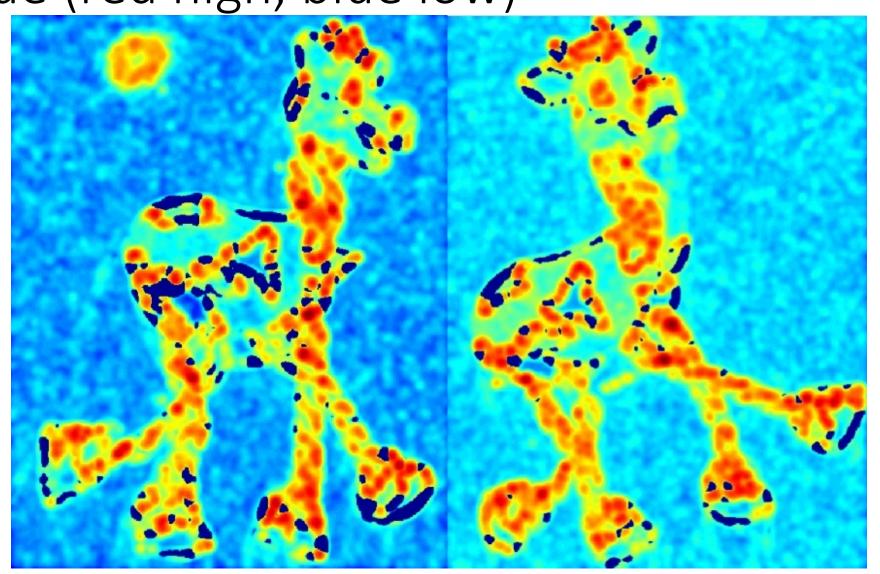
The Harris operator



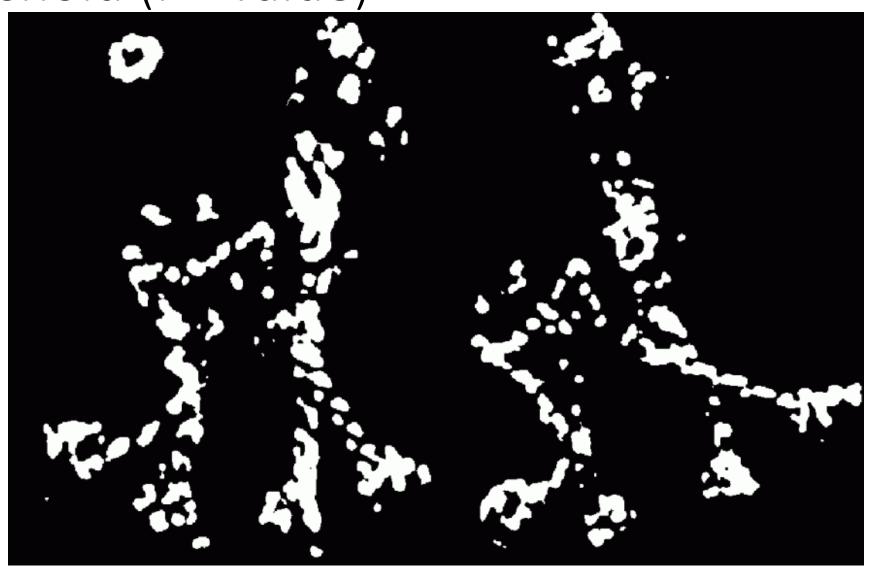
Harris detector example



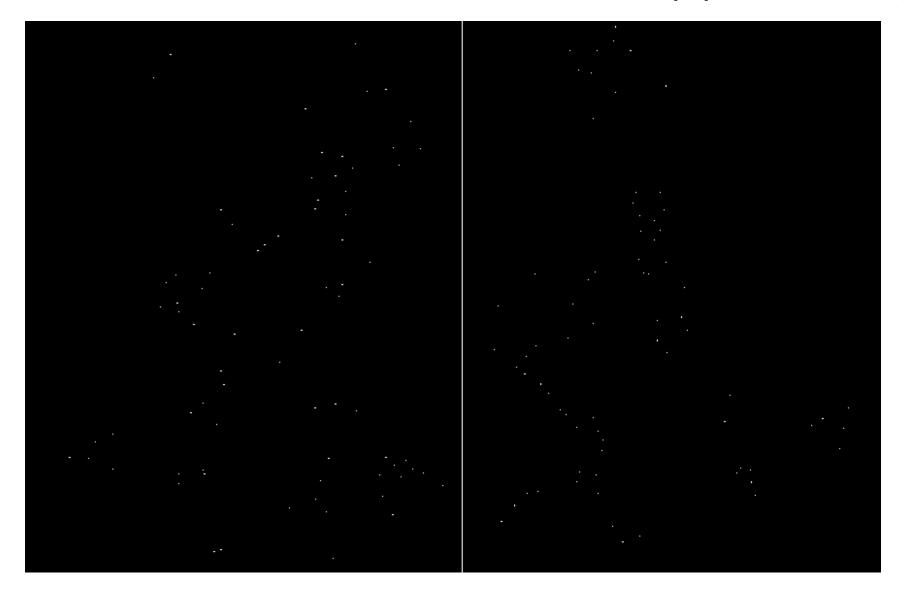
f value (red high, blue low)



Threshold (f > value)



Find local maxima of f (non-max suppression)

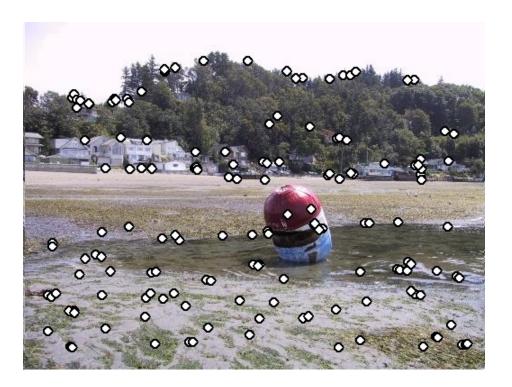


Harris features (in red)



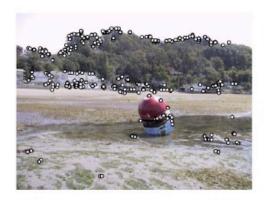
Feature selection

• Distribute points evenly over the image



Adaptive Non-maximal Suppression

- Desired: Fixed # of features per image
 - Want evenly distributed spatially...
 - Sort points by non-maximal suppression radius [Brown, Szeliski, Winder, CVPR'05]



(a) Strongest 250



(b) Strongest 500



(c) ANMS 250, r = 24



(d) ANMS 500, r = 16

Weighting the derivatives

• In practice, using a simple window W doesn't work too well

$$H = \sum_{(x,y)\in W} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

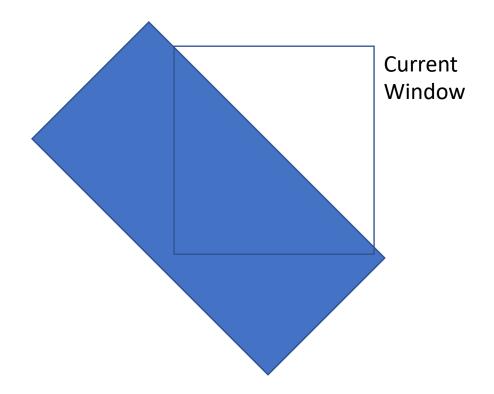
• Instead, we'll weight each derivative value based on its distance from the center pixel

$$H = \sum_{(x,y)\in W} w_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Harris Corners – Why so complicated?

- Can't we just check for regions with lots of gradients in the x and y directions?
 - No! A diagonal line would satisfy that criteria



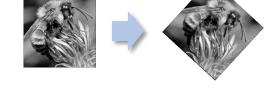
Content: Today's class

- Why detect features?
- What is a good feature?
- Harris Corner Detector
- Properties of Harris Corner Detector
- Blob Detector

Image transformations

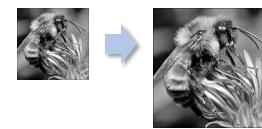
• Geometric

Rotation



Scale

Photometric
 Intensity change







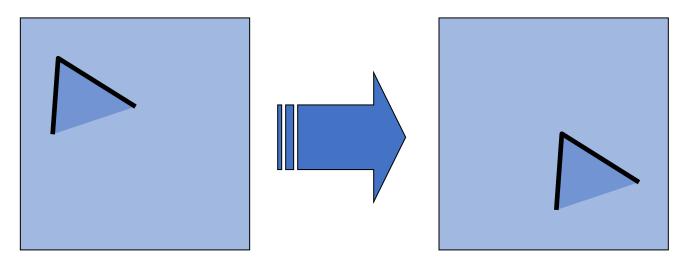


Invariance and equivariance

- We want corner locations to be invariant to photometric transformations and equivariant to geometric transformations
 - Invariance: image is transformed and corner locations do not change
 - Equivariance: if we have two transformed versions of the same image, features should be detected in corresponding locations
 - (Sometimes "invariant" and "equivariant" are both referred to as "invariant")
 - (Sometimes "equivariant" is called "covariant")



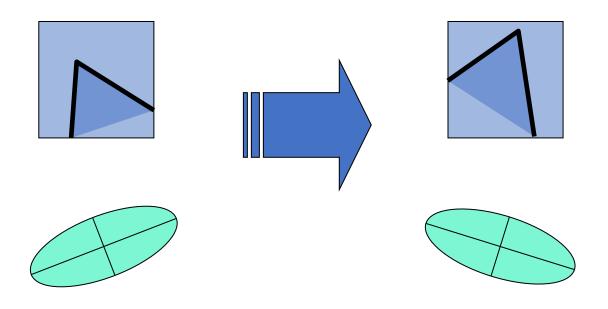
Harris detector invariance properties: image translation



• Derivatives and window function are equivariant

Corner location is equivariant w.r.t. translation

Harris detector invariance properties: image rotation



Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

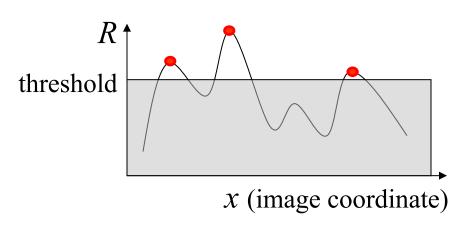
Corner location is equivariant w.r.t. image rotation

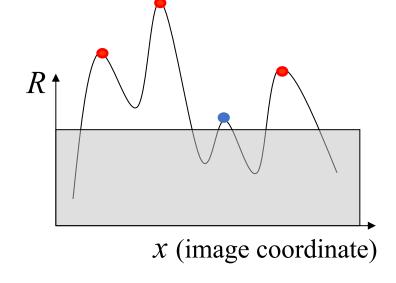
Harris detector invariance properties: Affine intensity change



• Only derivatives are used \rightarrow invariance to intensity shift $I \rightarrow I + b$

• Intensity scaling: $I \rightarrow a I$

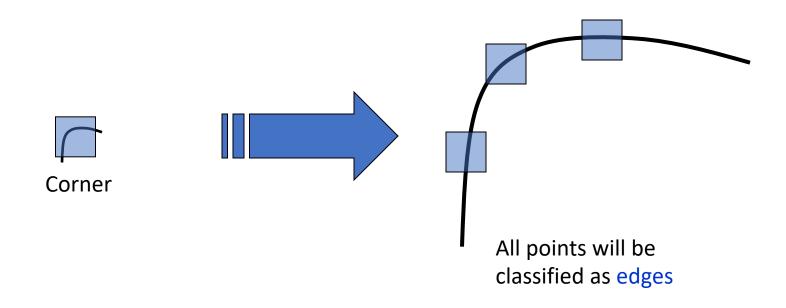




 $I \rightarrow a I + b$

Partially invariant to affine intensity change

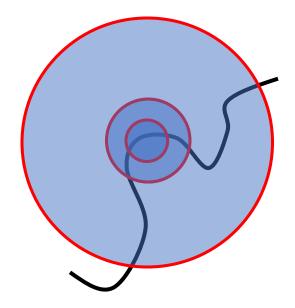
Harris detector invariance properties: scaling



Neither invariant nor equivariant to scaling

Scale invariant detection

Suppose you're looking for corners

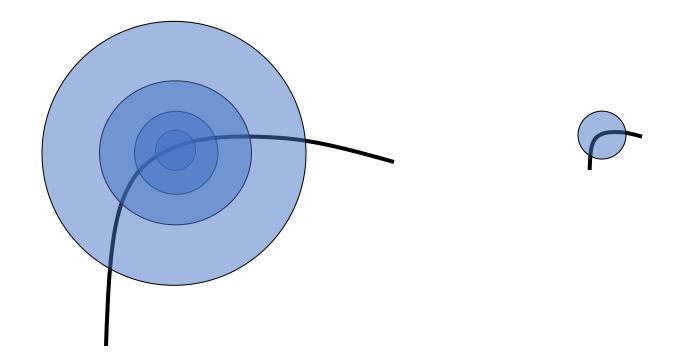


Key idea: find scale that gives local maximum of f

- in both position and scale
- One definition of *f*: the Harris operator

Scale Invariant Detection

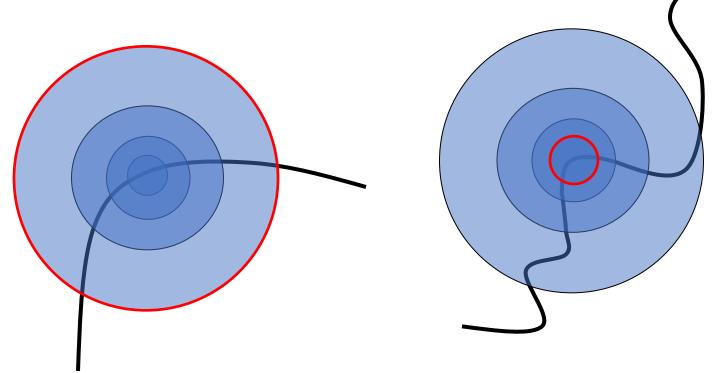
- Consider regions (e.g. circles) of different sizes around a point
- Regions of corresponding sizes will look the same in both images



Scale Invariant Detection

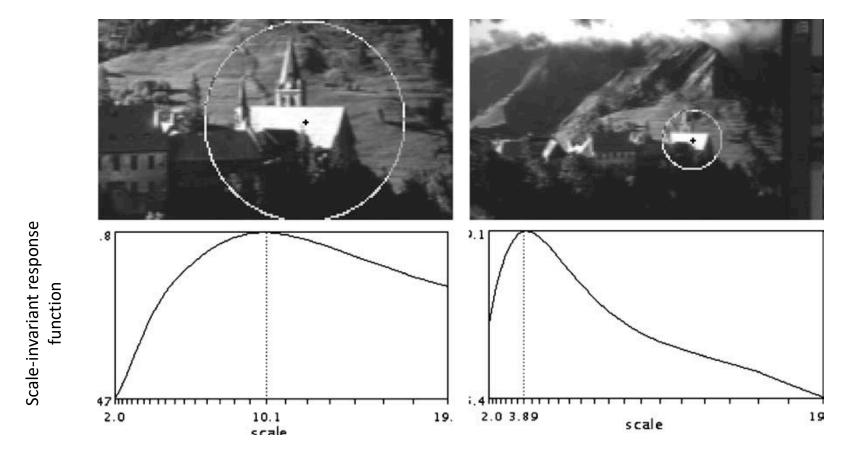
• The problem: how do we choose corresponding circles *independently* in each image?

• Choose the scale of the "best" corner



Keypoint detection with scale selection

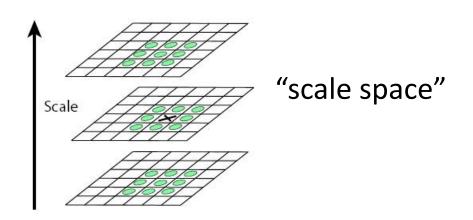
 We want to extract keypoints with characteristic scales that are equivariant (or covariant) w.r.t. to scaling of the image

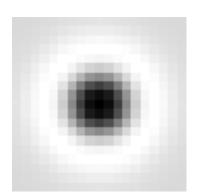


K. Mikolajczyk and C. Schmid. <u>Indexing based on scale invariant interest points</u>. ICCV 2001 T. Lindeberg, <u>Feature detection with automatic scale selection</u>, *IJCV* 30(2), pp. 77-116, 1998

Keypoint detection with scale selection

- We want to extract keypoints with characteristic scales that are equivariant (or covariant) w.r.t. to scaling of the image
- Approach: compute a *scale-invariant* response function over neighborhoods centered at each location (x, y) and a range of scales (σ) , find *scale-space locations* (x, y, σ) where this function reaches a local maximum
- A particularly convenient response function is given by the scalenormalized Laplacian of Gaussian (LoG) filter:



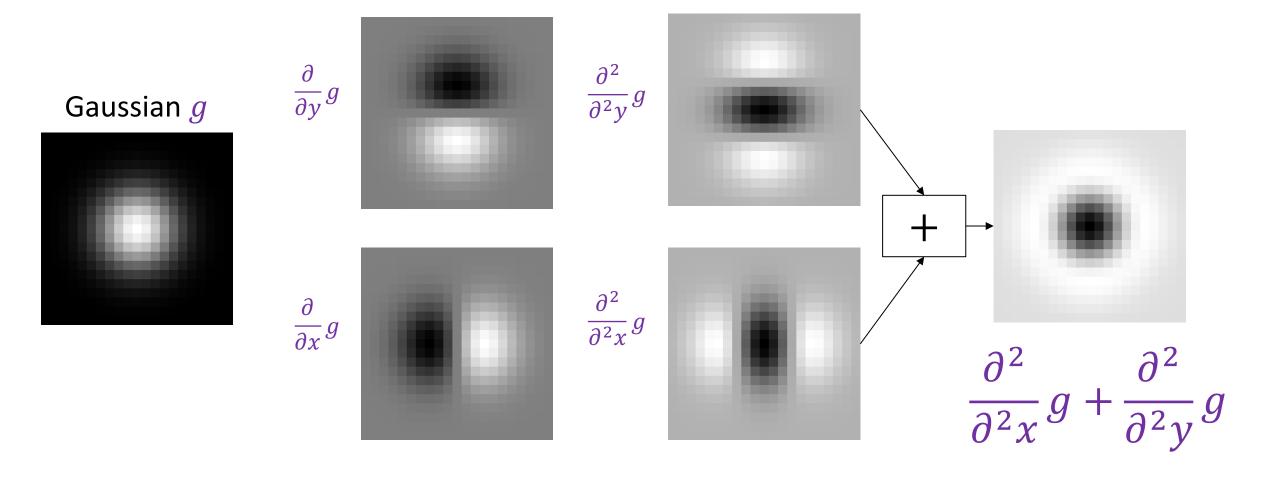


$$\nabla_{\text{norm}}^2 = \sigma^2 \left(\frac{\partial^2}{\partial x^2} g + \frac{\partial^2}{\partial^2 y} g \right)$$

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- Properties of Harris Corner Detector
- Blob Detector

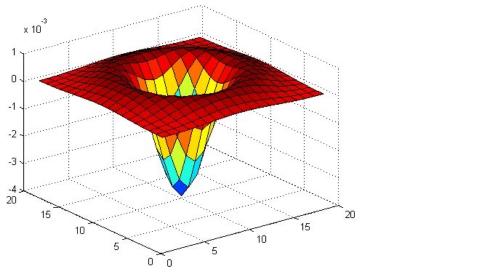
Laplacian of Gaussian



Source: J. Johnson and D. Fouhey

Scale-normalized Laplacian

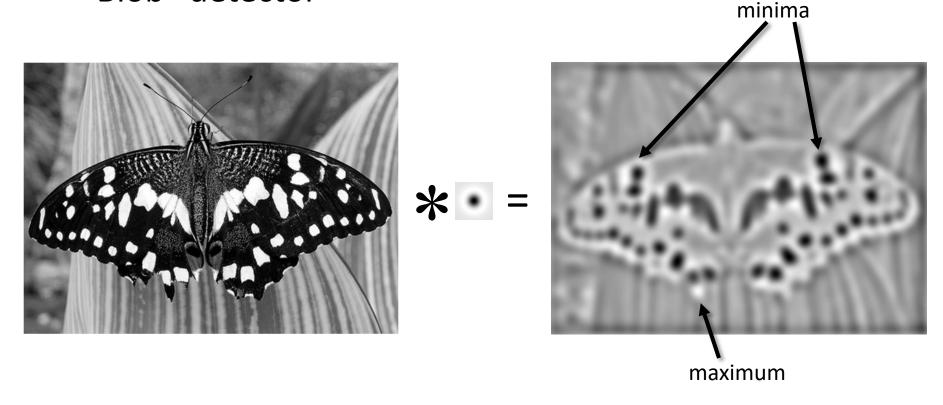
• You need to multiply the LoG by σ^2 to make responses comparable across scales



$$\nabla_{\text{norm}}^2 = \sigma^2 \left(\frac{\partial^2}{\partial x^2} g + \frac{\partial^2}{\partial^2 y} g \right)$$

Laplacian of Gaussian

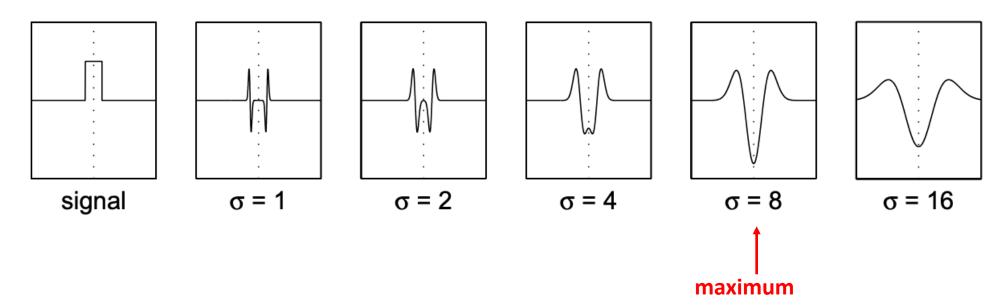
• "Blob" detector



• Find maxima and minima of LoG operator in space and scale

Scale selection: Characteristic Scale

• We can find the *characteristic scale* of the blob by convolving it with *scale-normalized* Laplacians at several scales (σ) and looking for the maximum response



Scale-space blob detector: Example

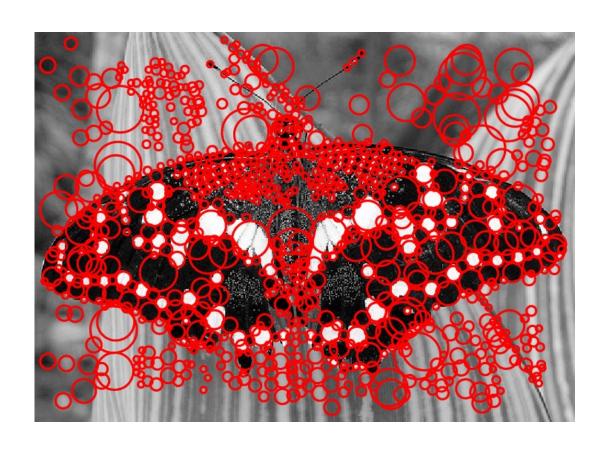


Scale-space blob detector: Example

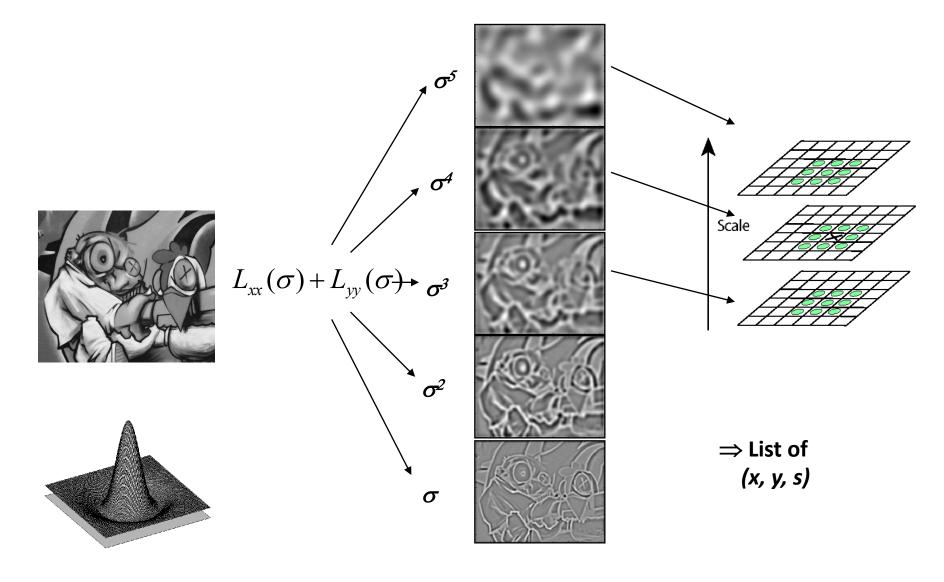


sigma = 11.9912

Scale-space blob detector: Example



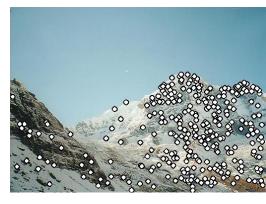
Find local maxima in 3D position-scale space



Local features: main components

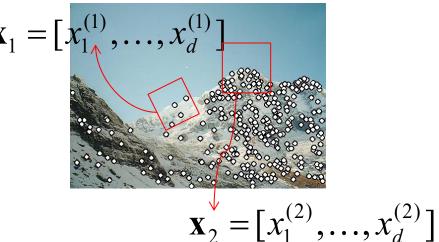
This Class

1) Detection: Identify the interest points



Next Class:
We will learn
about what is SIFT
feature! The most
famous feature in
Computer Vision!!

Description: Extract vector $\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$ feature descriptor surrounding each interest point.



3) Matching: Determine correspondence between descriptors in two views



Slide Credits

- <u>CS5670, Introduction to Computer Vision</u>, Cornell Tech, by Noah Snavely.
- <u>CS 194-26/294-26</u>: Intro to Computer Vision and Computational Photography, UC Berkeley, by Alyosha Efros.
- Fall 2022 CS 543/ECE 549: Computer Vision, UIUC, by Svetlana Lazebnik.