

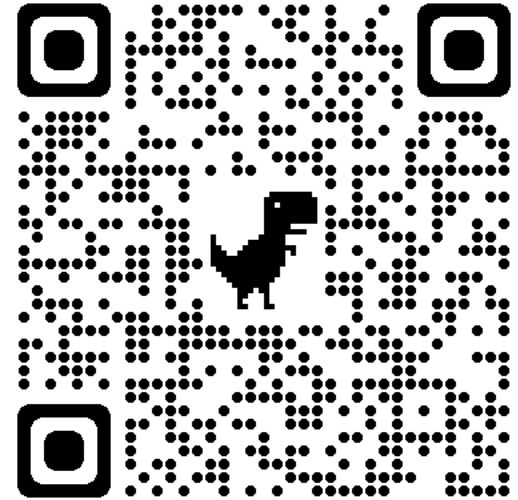
Lecture 16:

Structure from Motion

COMP 590/776: Computer Vision

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TA: Mykhailo (Misha) Shvets



Course Website:
Scan Me!

Recap

Geometry: How do we represent shape of an object?

2.5D representation:

- 1) Depth & Normal map

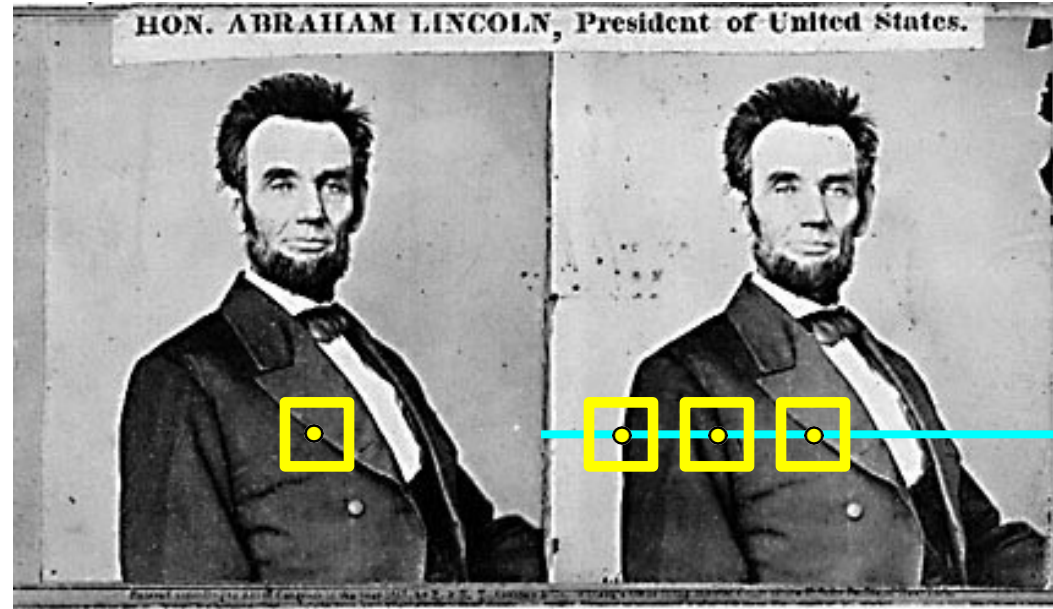
Explicit representation:

- 2) Mesh
- 3) Voxels
- 4) Point Cloud

Implicit representation:

- 5) Surface Representation (SDF)

Stereo

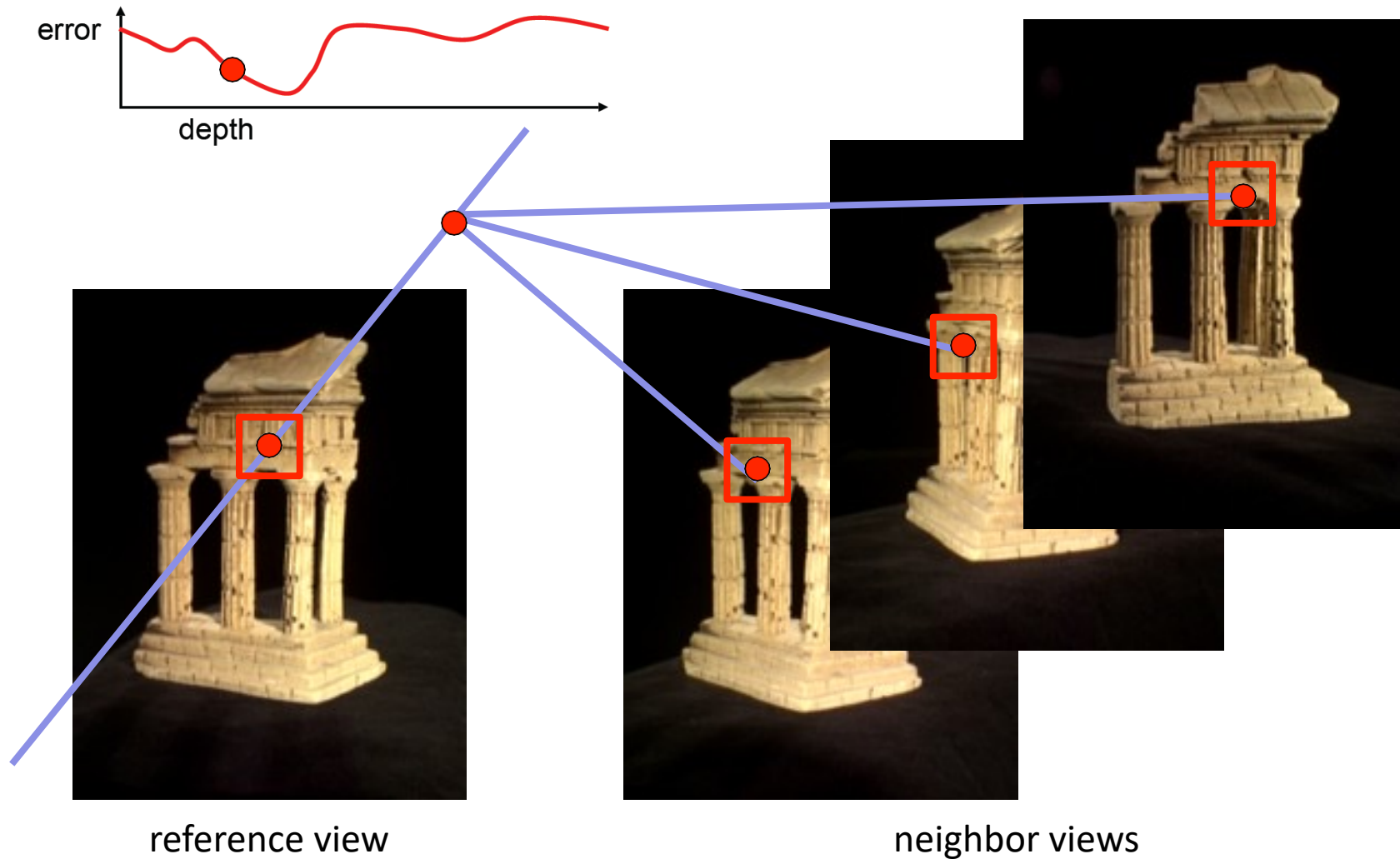


1. Rectify images
(make epipolar lines horizontal)
2. For each pixel
 - a. Find epipolar line
 - b. Scan line for best match
 - c. Compute depth from disparity

$$Z = \frac{bf}{d}$$

How can you make the epipolar lines horizontal?

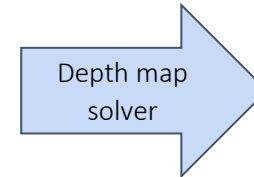
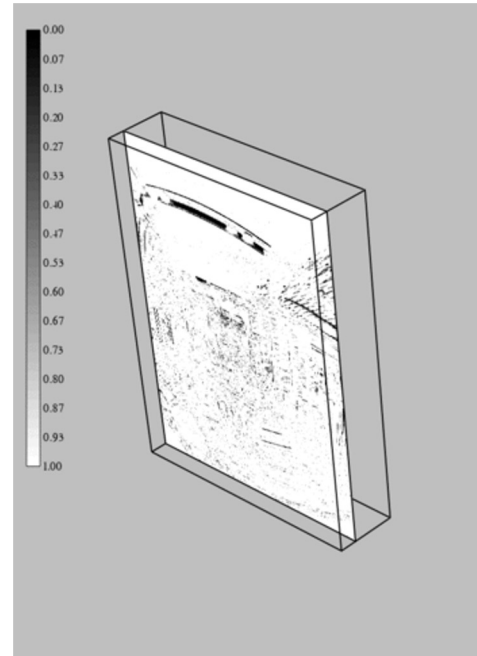
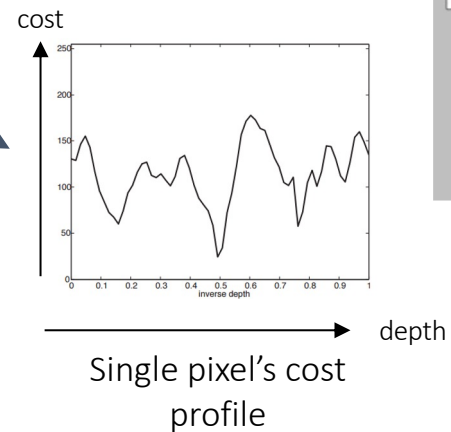
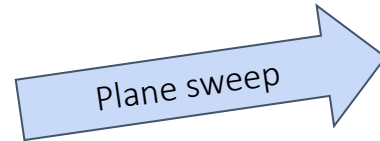
Multi-view stereo: Basic idea



Plane Sweep Stereo: Cost Volumes -> Depth Maps



Reference image



(Belief propagation,
graph cuts, etc.)



Another example



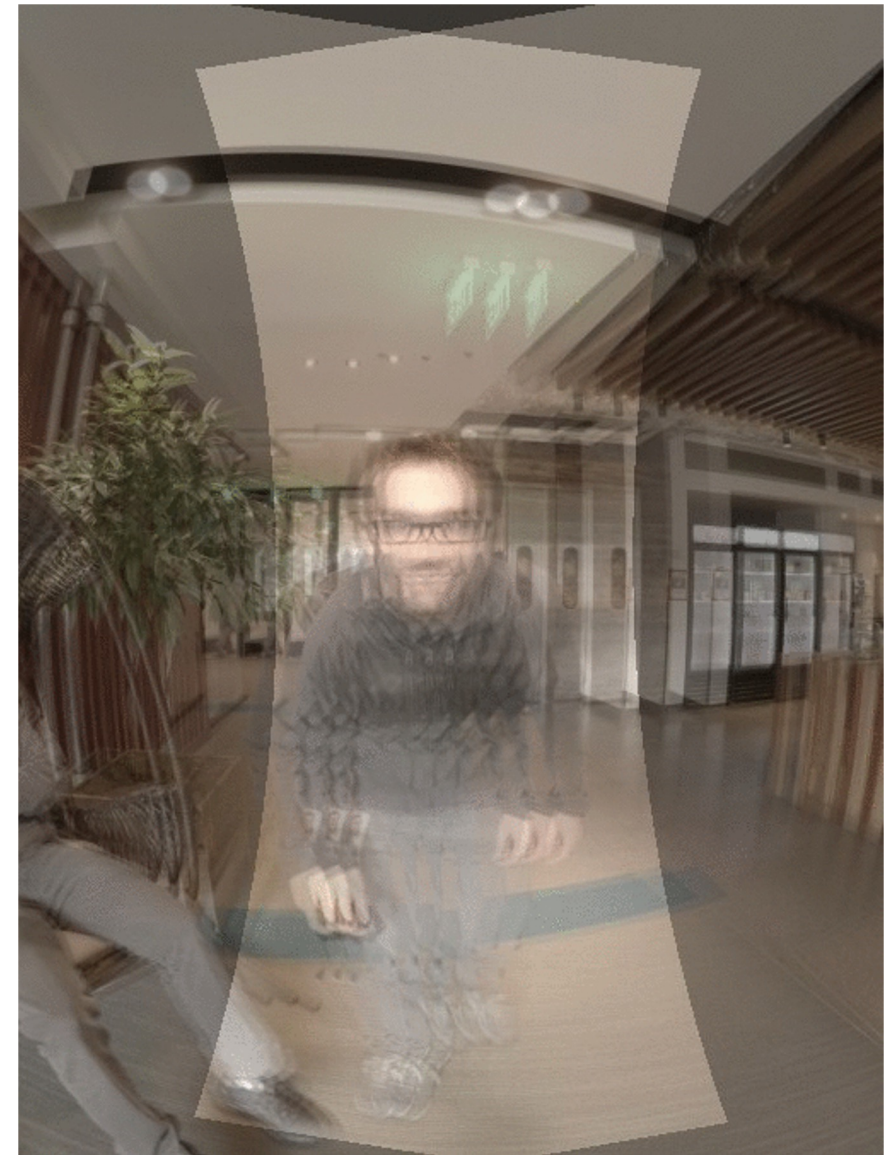
Left neighbor



Reference image



Right neighbor

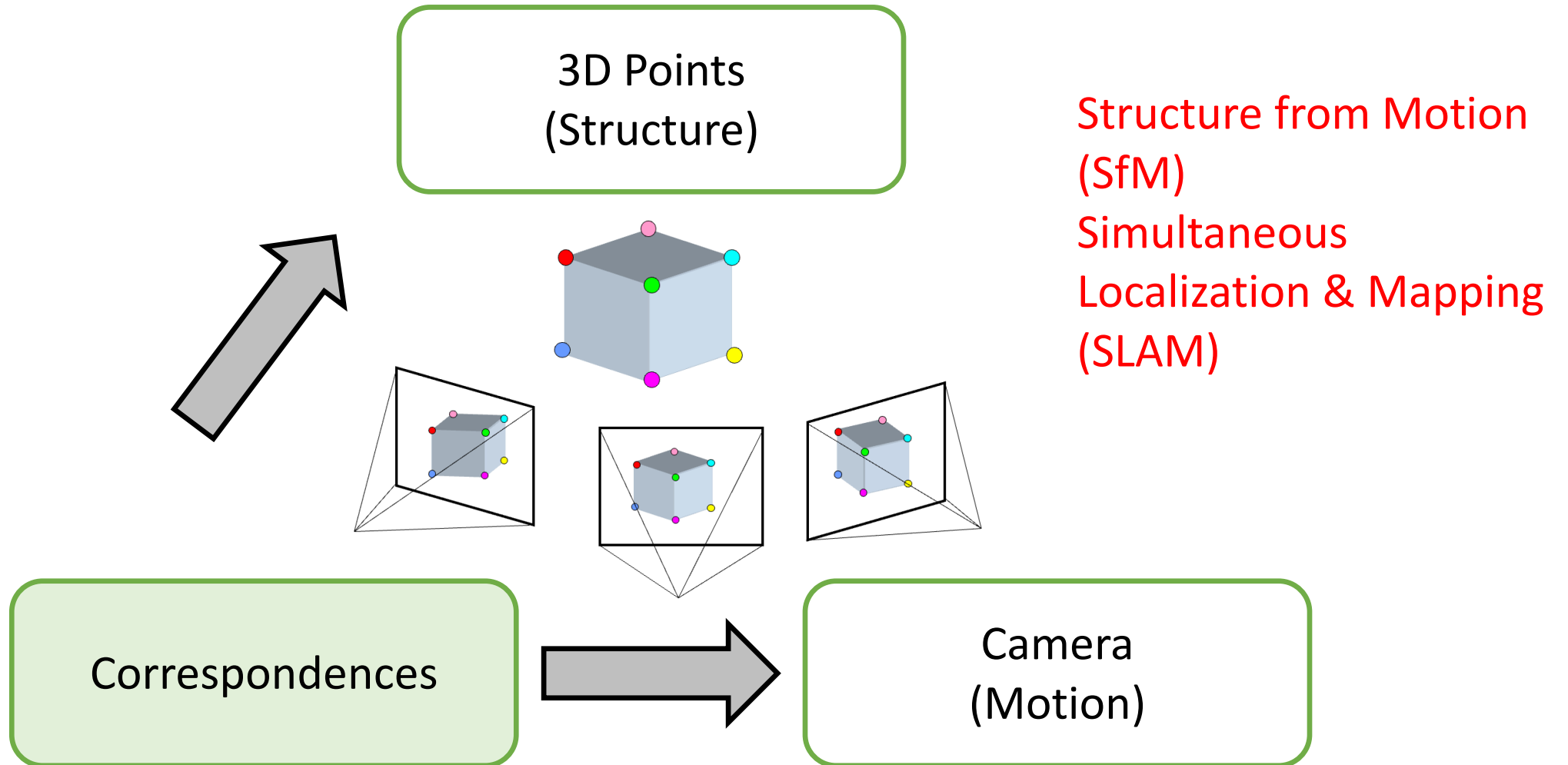


Planar image reprojections swept over depth
(averaged)

For a particular depth sweep, some regions in the average image appear sharp, i.e. photo-consistent.

Notice the face and the EXIT sign appear sharp at different times.

Big picture: 3 key components in 3D



Structure from motion

- SfM solves both of these problems *at once*
- A kind of chicken-and-egg problem
 - (but solvable)

Structure from Motion (SfM)

- Given many images, how can we
 - a) figure out where they were all taken from?
 - b) build a 3D model of the scene?



This is (roughly) the **structure from motion** problem

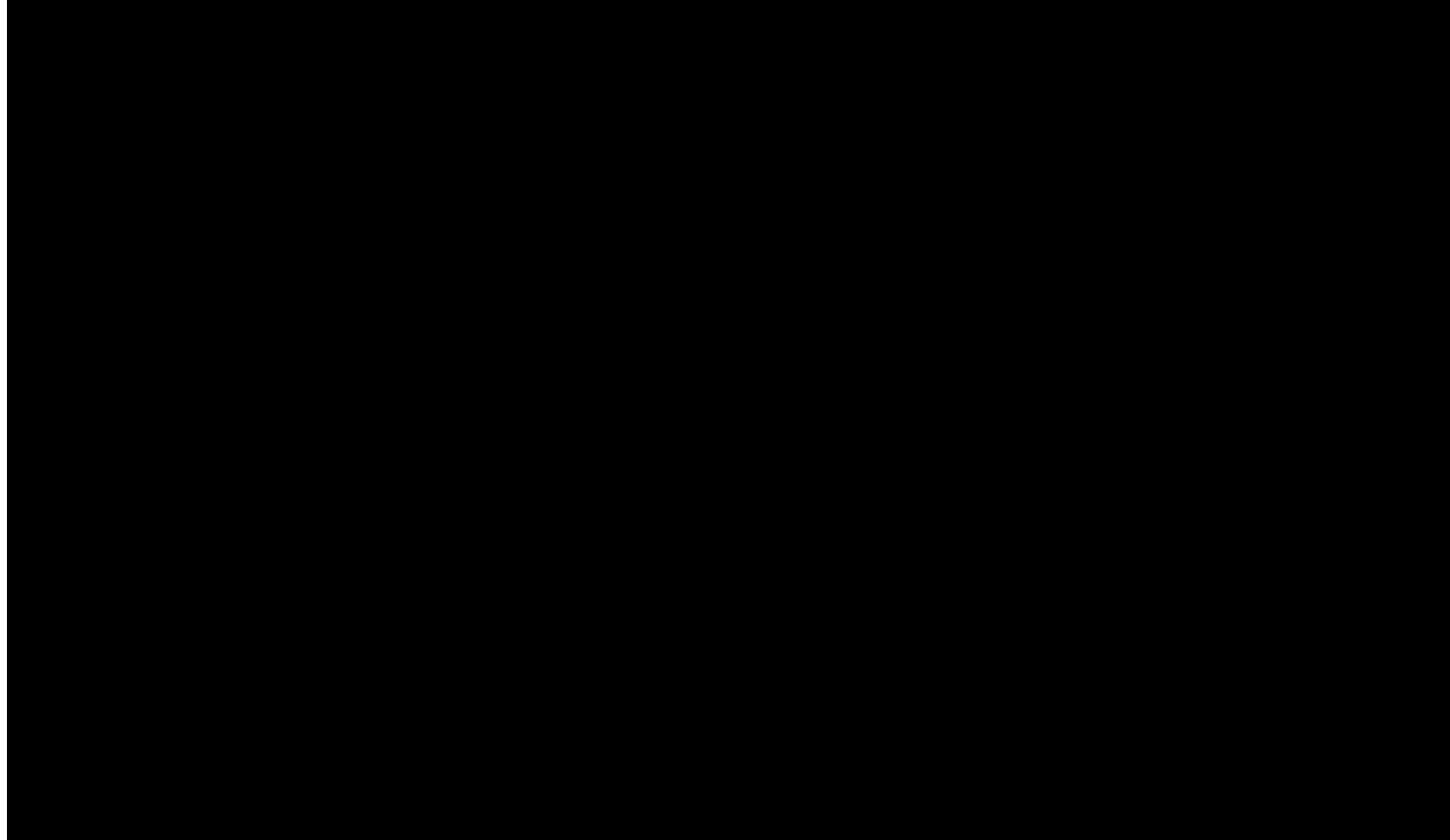
Photo Tourism

Noah Snavely, Steven M. Seitz, Richard Szeliski, "[Photo tourism: Exploring photo collections in 3D](#)," SIGGRAPH 2006



<https://youtu.be/mTBPGuPLI5Y>

Large-scale structure from motion



Dubrovnik, Croatia. 4,619 images (out of an initial 57,845).
Total reconstruction time: 23 hours
Number of cores: 352

Large-scale structure from motion



Rome's Colosseum

Reconstructing the World in Six Days,

Jared Heinly, Johannes L. Schönberger, Enrique Dunn, Jan-Michael Frahm, CVPR 2015.

Work done at UNC CS!



St. Peter's Basilica, Vatican City

Yahoo Flickr Creative Commons 100M Dataset

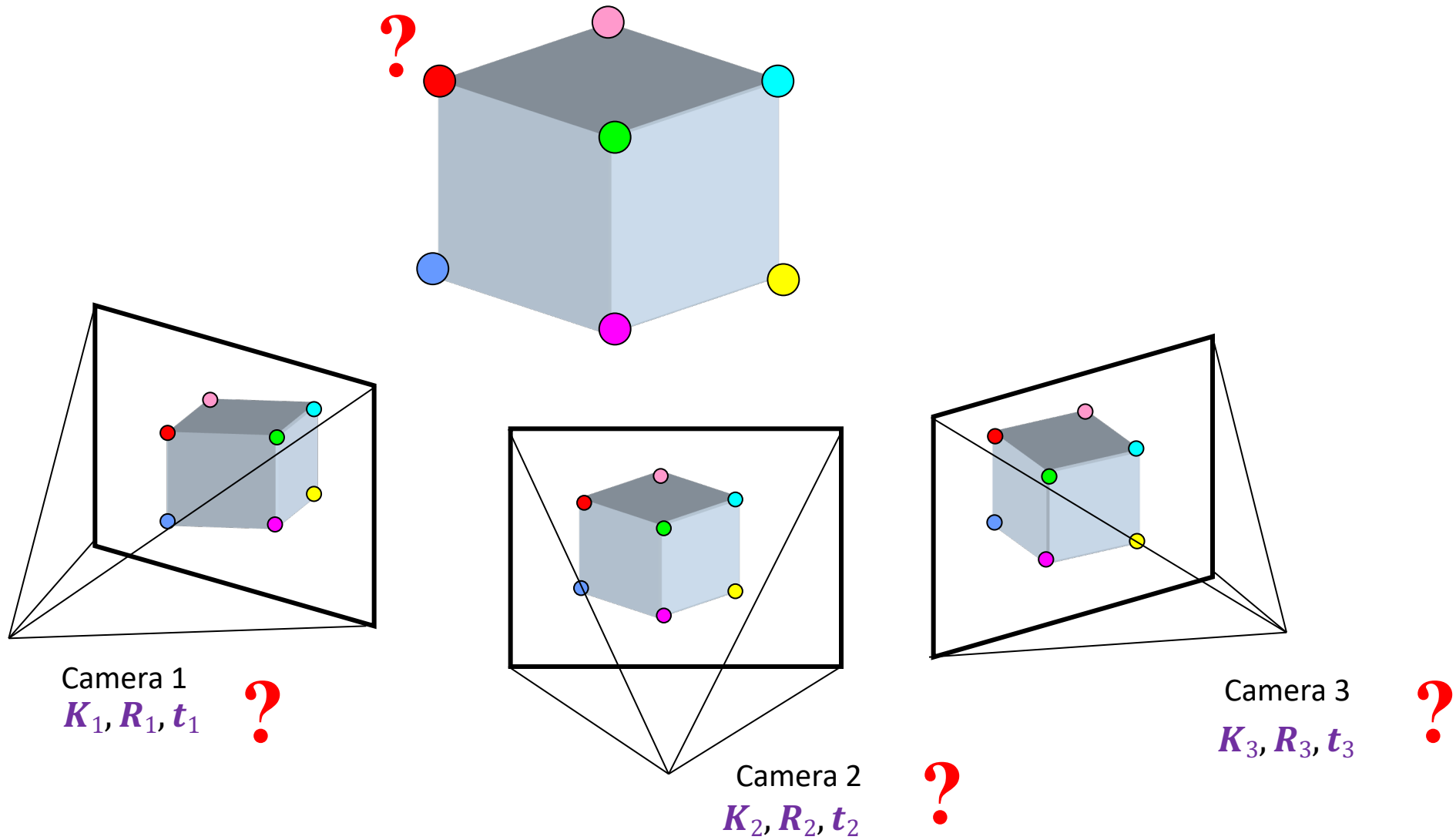
Today's Class

- Ambiguities in SfM
- Affine SfM
- Projective SfM
 - Global SfM
 - Incremental SfM
- Challenges and Applications

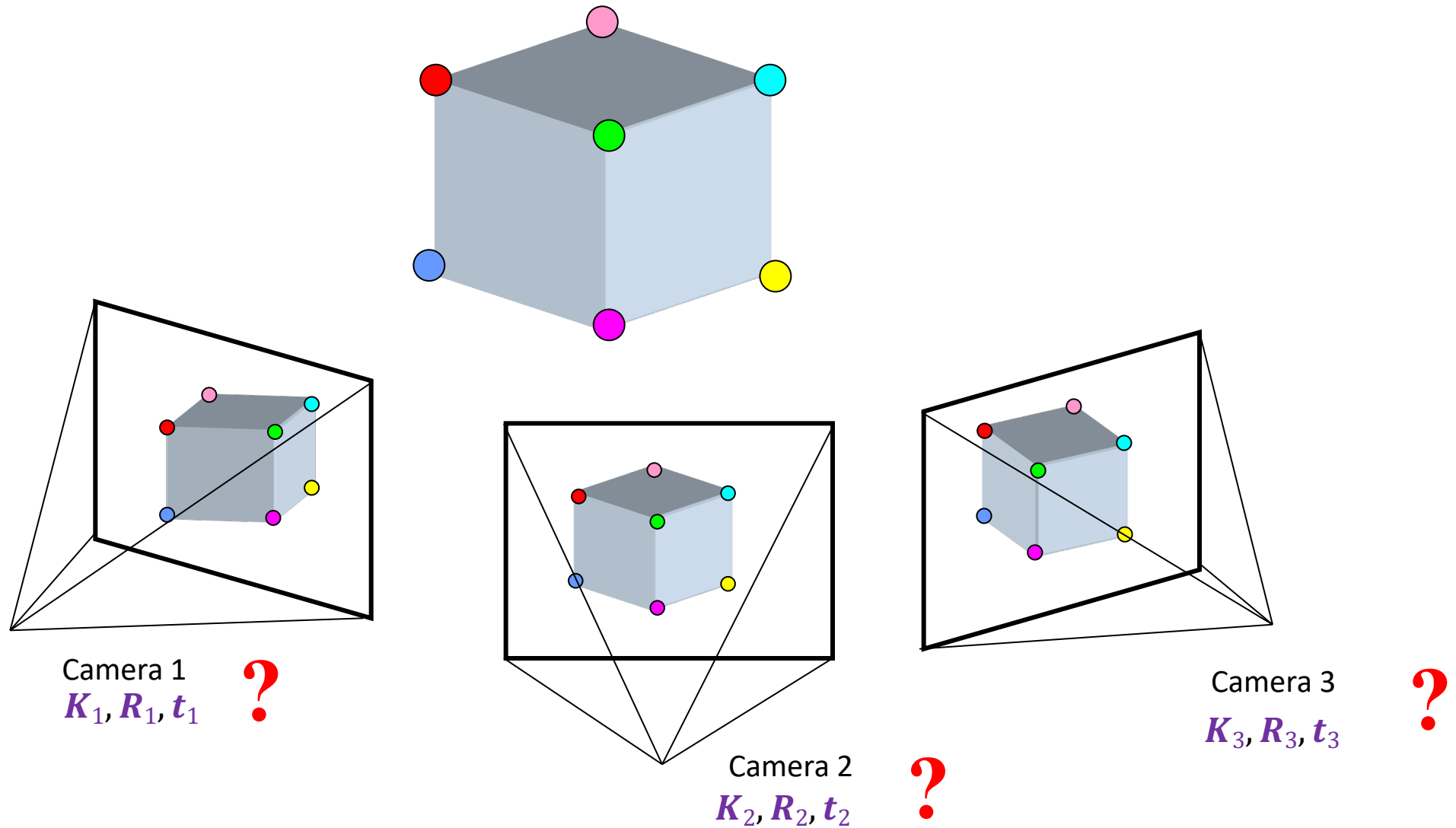
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Structure from motion

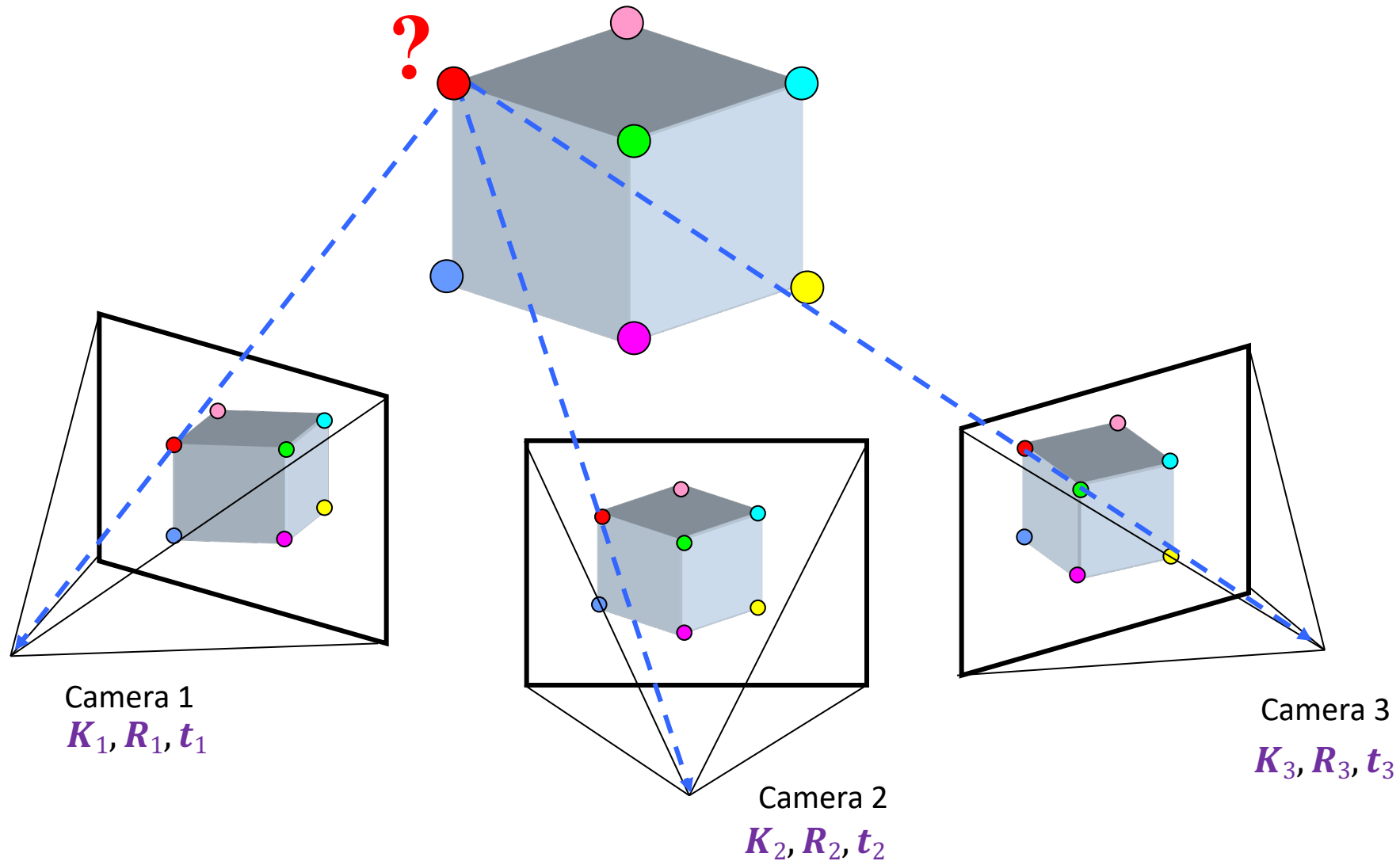


Recall: Calibration



- Given a set of *known* 3D points seen by a camera, compute the camera parameters

Recall: Triangulation



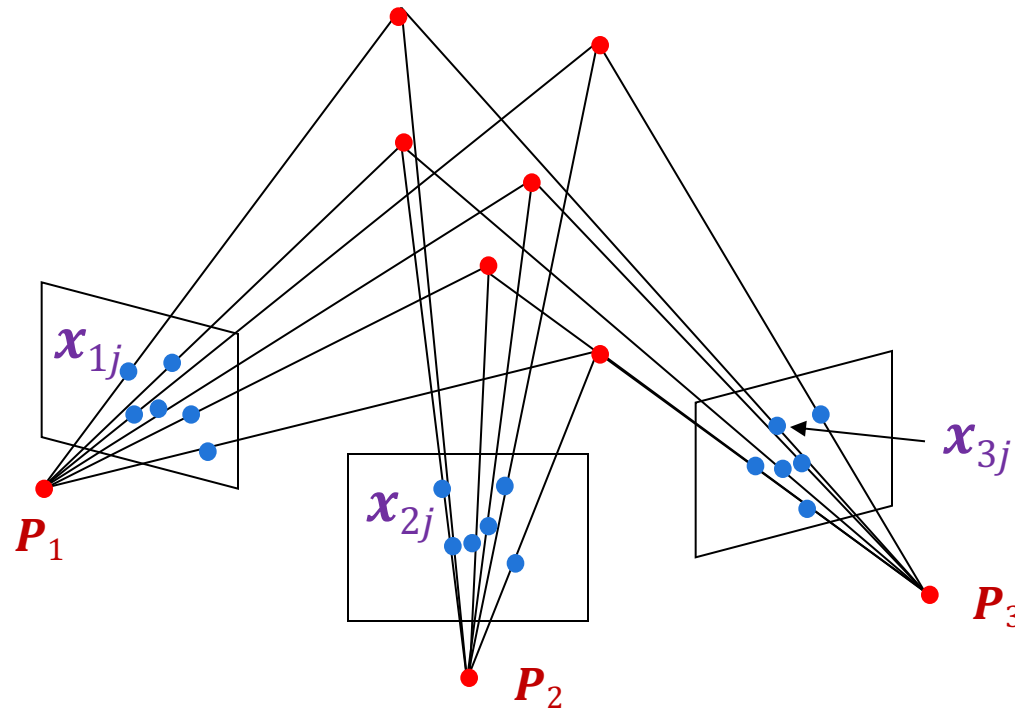
- Given *known cameras* and projections of the same 3D point in two or more images, compute the 3D coordinates of that point

Structure from motion: Problem formulation

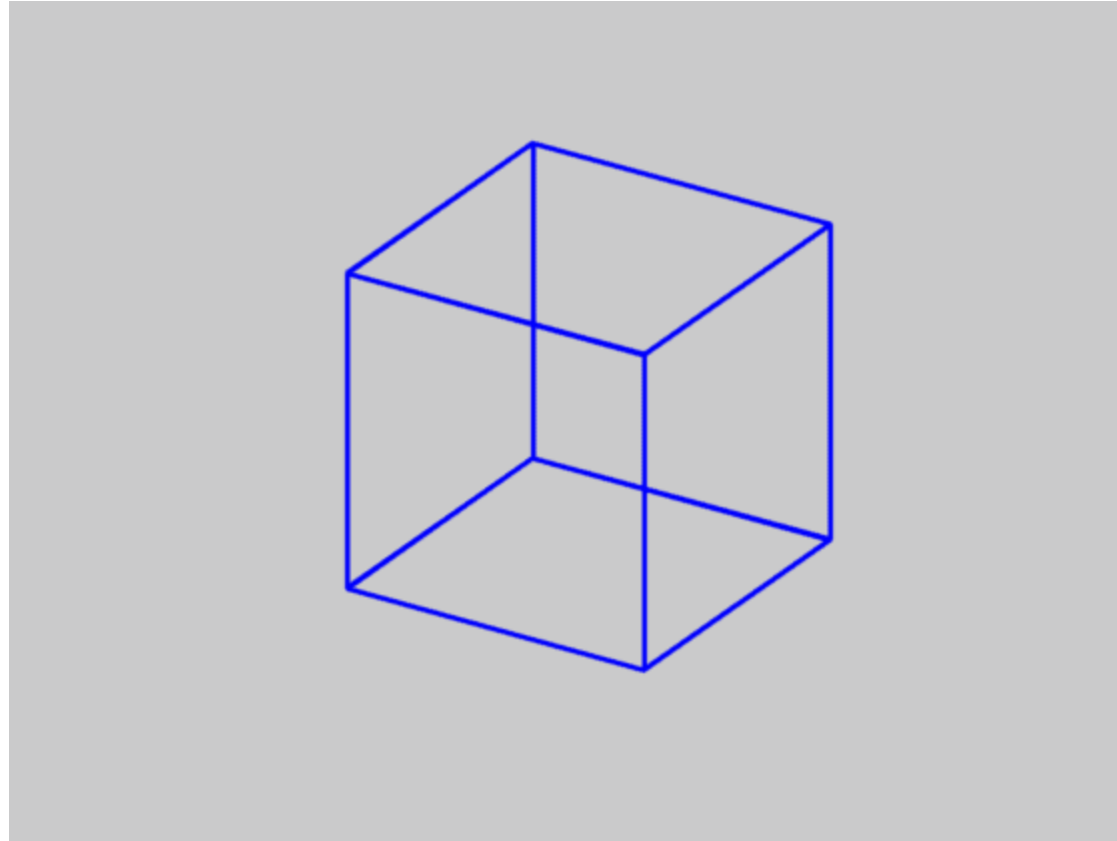
- Given: m images of n fixed 3D points such that (ignoring visibility)

$$\bullet \mathbf{x}_{ij} \cong \mathbf{P}_i \mathbf{X}_j, \quad i = 1, \dots, m, \quad j = 1, \dots, n$$

- Problem: estimate m projection matrices \mathbf{P}_i and n 3D points \mathbf{X}_j from the mn correspondences \mathbf{x}_{ij}



Is SFM always uniquely solvable?



- [Necker cube](#)

Structure from motion ambiguity

- If we scale the entire scene by some factor k and, at the same time, scale the camera matrices by the factor of $1/k$, the projections of the scene points remain exactly the same:

$$\bullet x \cong PX = \left(\frac{1}{k}P\right)(kX)$$

- Without a reference measurement, it is impossible to recover the absolute scale of the scene!
- In general, if we transform the scene using a transformation Q and apply the inverse transformation to the camera matrices, then the image observations do not change:

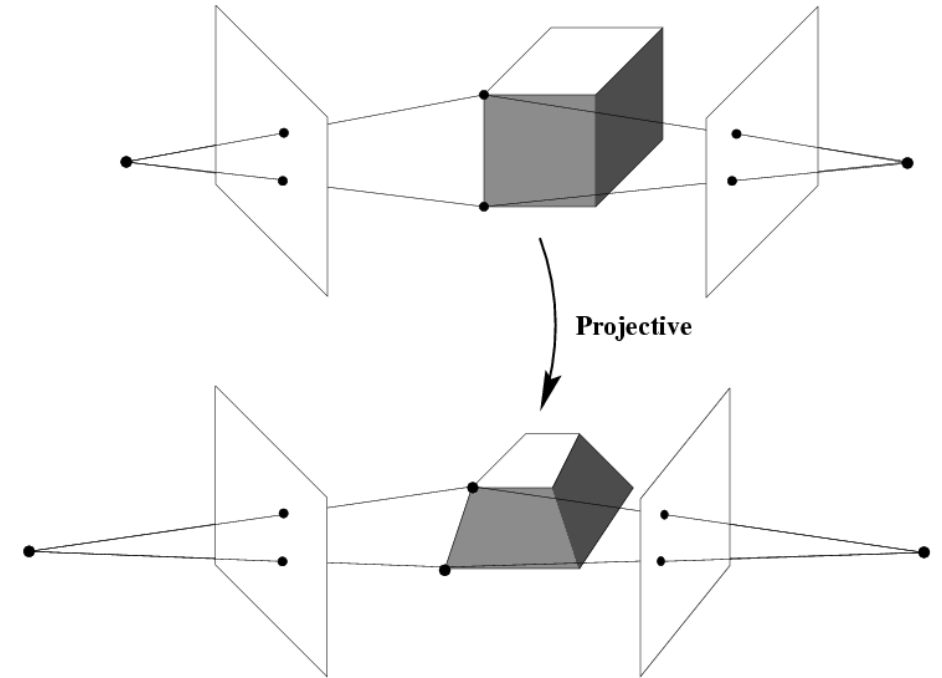
$$\bullet x \cong PX = (PQ^{-1})(QX)$$

Projective ambiguity

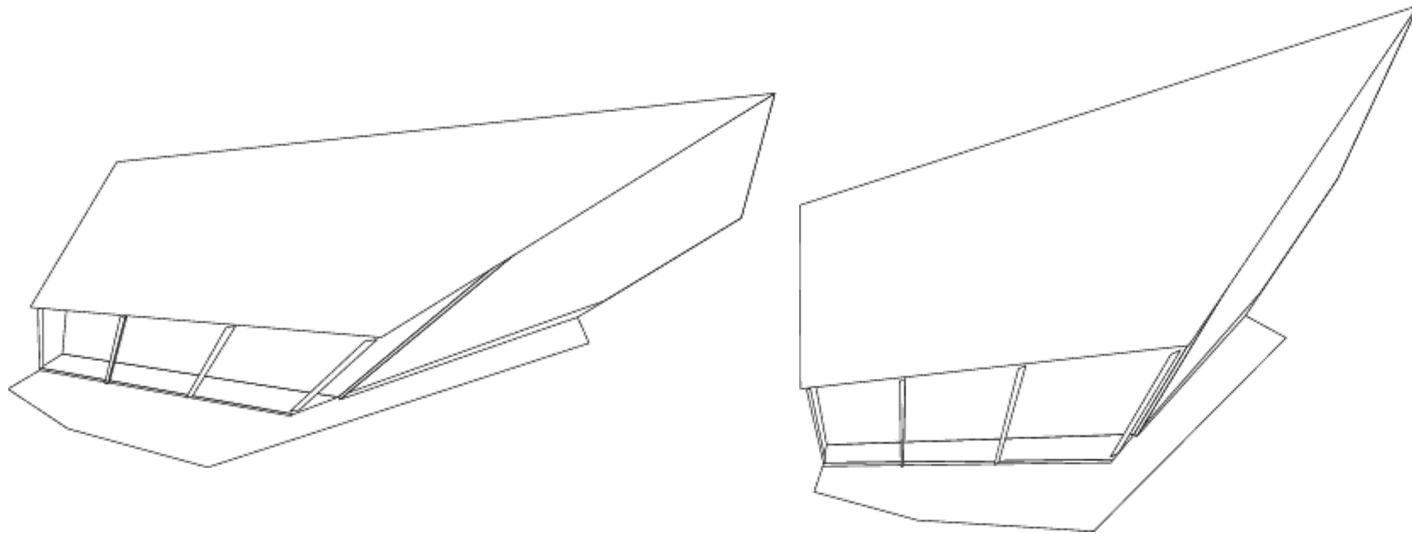
- With no constraints on the camera calibration matrices or on the scene, we can reconstruct up to a *projective* ambiguity:

$$x \cong PX = (PQ^{-1})(QX)$$

Q is a general full-rank 4×4 matrix



Projective ambiguity



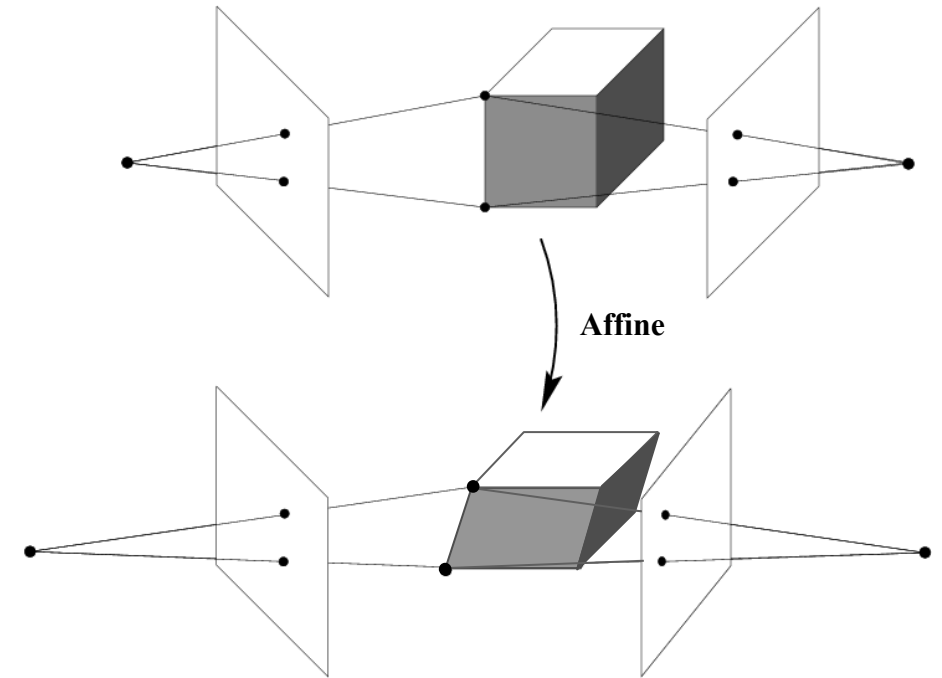
Affine ambiguity

- If we impose parallelism constraints, we can get a reconstruction up to an *affine* ambiguity:

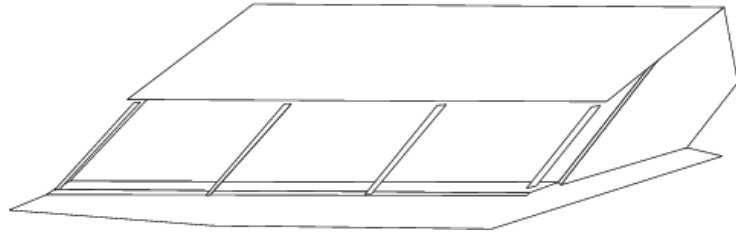
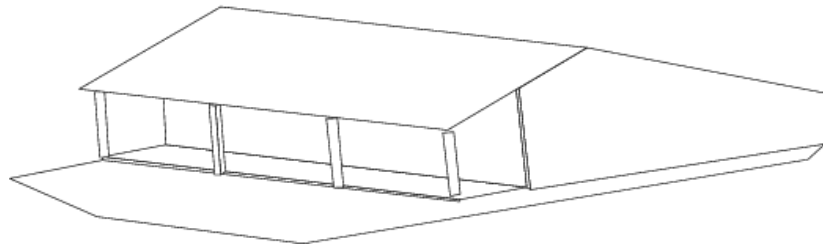
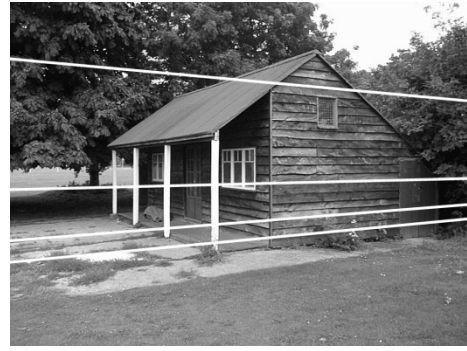
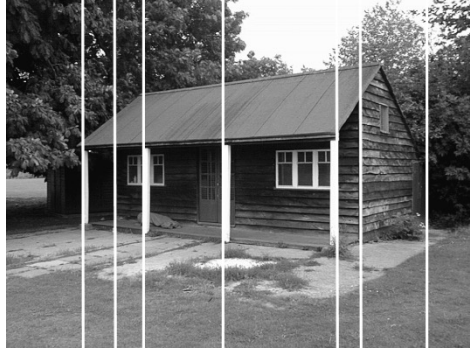
$$x \cong PX = (PQ_A^{-1})(Q_AX)$$

3×3 full-rank matrix 3×1 translation vector

$$Q_A = \begin{bmatrix} A & t \\ \mathbf{0}^T & 1 \end{bmatrix}$$



Affine ambiguity



Similarity ambiguity

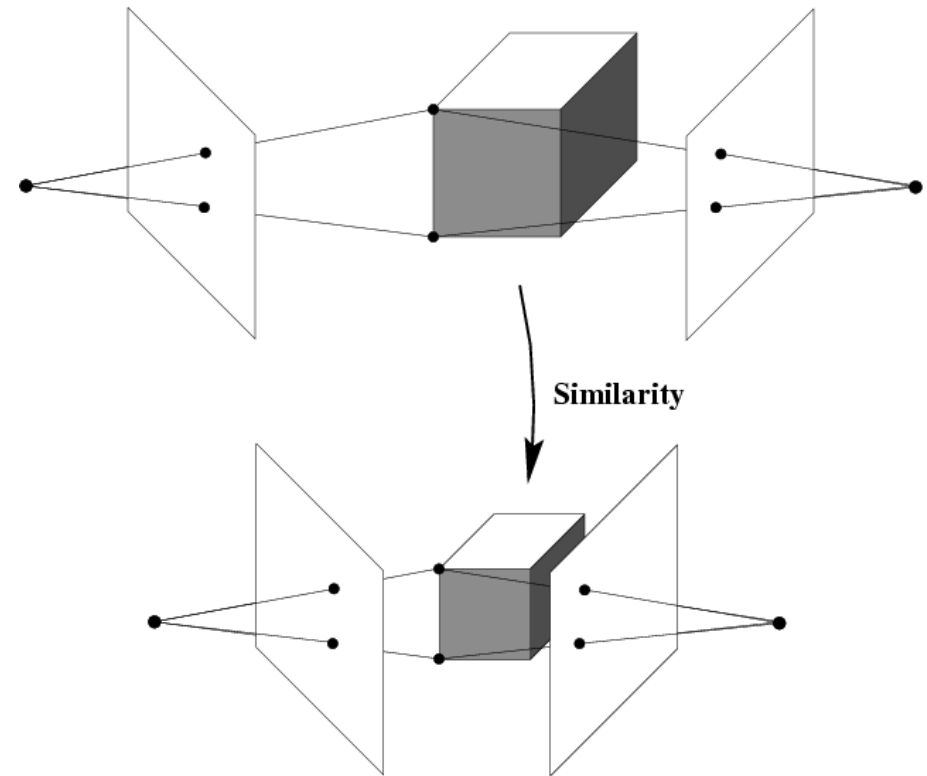
- A reconstruction that obeys orthogonality constraints on camera parameters and/or scene

$$x \cong PX = (PQ_S^{-1})(Q_S X)$$

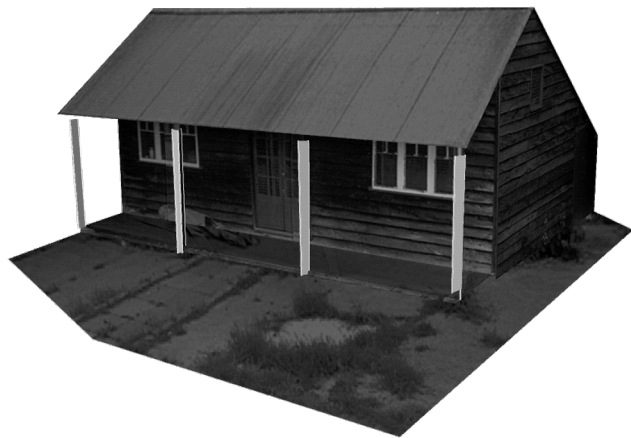
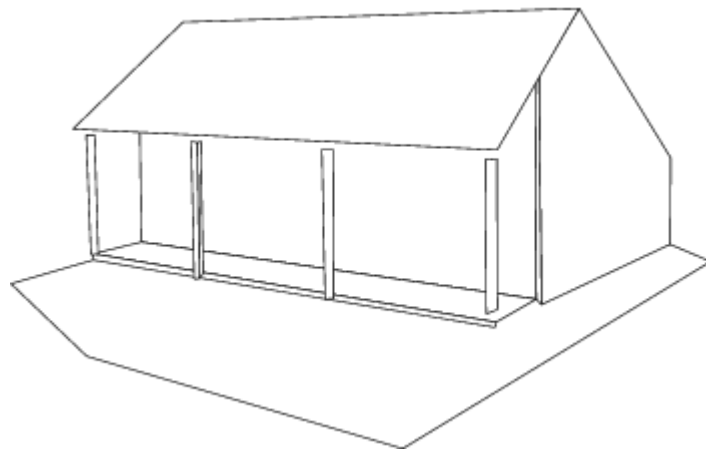
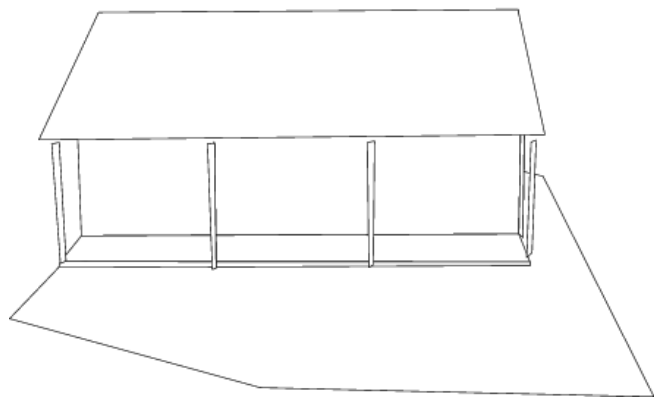
3×3
rotation
matrix

3×1 translation
vector

$$Q_S = \begin{bmatrix} sR & t \\ \mathbf{0}^T & 1 \end{bmatrix}$$



Similarity ambiguity

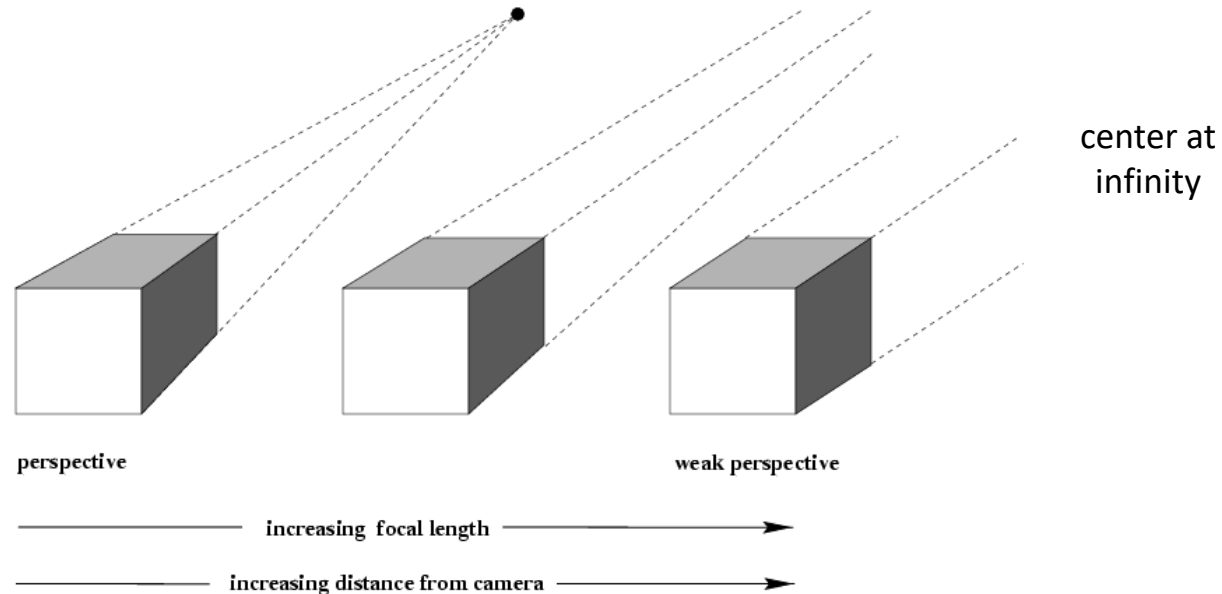


Today's Class

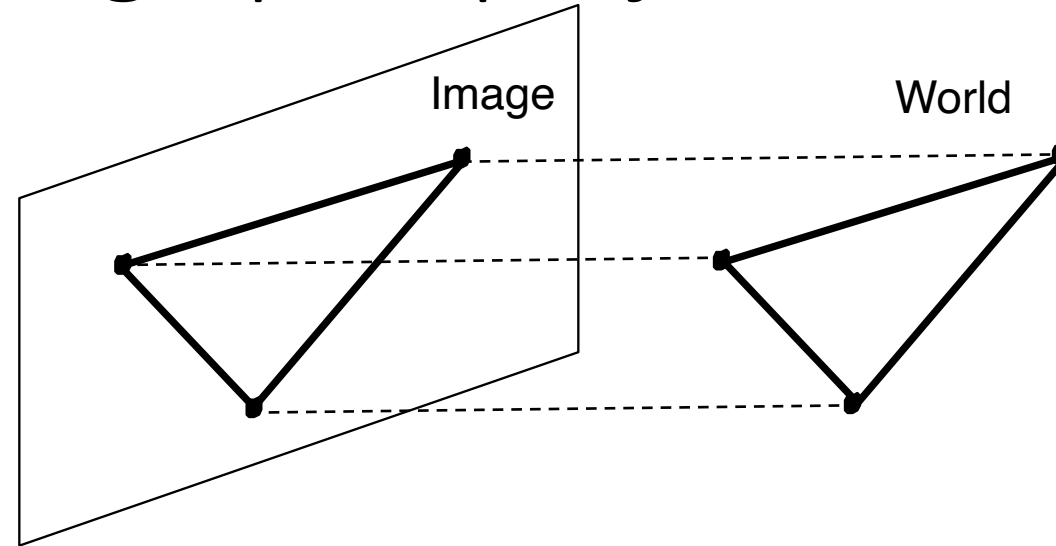
- Ambiguities in SfM
- **Affine SfM**
- Projective SfM
 - Global SfM
 - Incremental SfM
- Challenges and Applications

Affine structure from motion

- Let's start with *affine* or *weak perspective* cameras



Recall: Orthographic projection



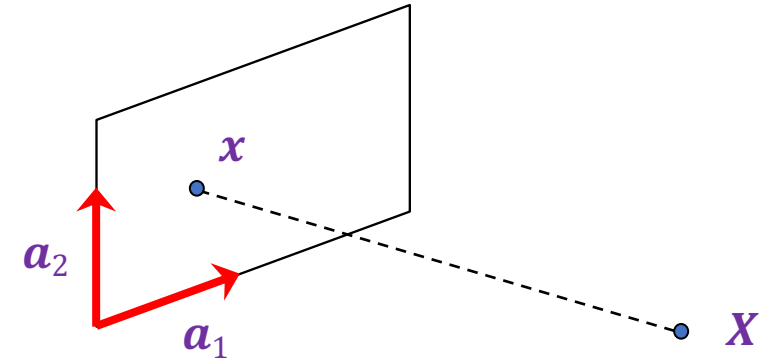
Just drop the z coordinate!

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

General affine projection

- A general affine projection is a 3D-to-2D linear mapping plus translation:

$$P = \begin{bmatrix} a_{11} & a_{12} & a_{13} & t_1 \\ a_{21} & a_{22} & a_{23} & t_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} A & t \\ \mathbf{0}^T & 1 \end{bmatrix}$$



a_1, a_2 : rows of projection matrix

- In non-homogeneous coordinates:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = AX + t$$

Projection of
world origin

Affine structure from motion

- **Given:** m images of n fixed 3D points such that
 - $\mathbf{x}_{ij} = \mathbf{A}_i \mathbf{X}_j + \mathbf{t}_i, \quad i = 1, \dots, m, j = 1, \dots, n$
- **Problem:** use the mn correspondences \mathbf{x}_{ij} to estimate m projection matrices \mathbf{A}_i and translation vectors \mathbf{t}_i , and n points \mathbf{X}_j
- The reconstruction is defined up to an arbitrary *affine* transformation \mathbf{Q} (12 degrees of freedom):

$$\begin{bmatrix} \mathbf{A} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \rightarrow \begin{bmatrix} \mathbf{A} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \mathbf{Q}^{-1}, \quad \begin{pmatrix} \mathbf{X}_j \\ 1 \end{pmatrix} \rightarrow \mathbf{Q} \begin{pmatrix} \mathbf{X}_j \\ 1 \end{pmatrix}$$

- How many knowns and unknowns for m images and n points?
 - $2mn$ knowns and $8m + 3n$ unknowns
 - To be able to solve this problem, we must have $2mn \geq 8m + 3n - 12$ (affine ambiguity takes away 12 dof)
 - E.g., for **two** views, we need **four** point correspondences

Affine structure from motion

- First, center the data by subtracting the centroid of the image points in each view:

$$\begin{aligned}\hat{\mathbf{x}}_{ij} &= \mathbf{x}_{ij} - \frac{1}{n} \sum_{k=1}^n \mathbf{x}_{ik} \\ &= \mathbf{A}_i \mathbf{X}_j + \mathbf{t}_i - \frac{1}{n} \sum_{k=1}^n (\mathbf{A}_i \mathbf{X}_k + \mathbf{t}_i) \\ &= \mathbf{A}_i \left(\mathbf{X}_j - \frac{1}{n} \sum_{k=1}^n \mathbf{X}_k \right) \\ &= \mathbf{A}_i \hat{\mathbf{X}}_j\end{aligned}$$

Affine structure from motion

- After centering, each normalized 2D point $\hat{\mathbf{x}}_{ij}$ is related to the 3D point by

$$\bullet \hat{\mathbf{x}}_{ij} = \mathbf{A}_i \hat{\mathbf{X}}_j$$

- We can get rid of the need to center the 3D data (and the translation ambiguity) by defining the origin of the world coordinate system as the centroid of the 3D points

Affine structure from motion

- Let's create a $2m \times n$ data (measurement) matrix:

$$\bullet D = \begin{bmatrix} \hat{x}_{11} & \hat{x}_{12} & \cdots & \hat{x}_{1n} \\ \hat{x}_{21} & \hat{x}_{22} & \cdots & \hat{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{x}_{m1} & \hat{x}_{m2} & \cdots & \hat{x}_{mn} \end{bmatrix}$$

points (n)

cameras
($2m$)

$$\hat{x}_{ij} = A_i X_j$$

Affine structure from motion

- Let's create a $2m \times n$ data (measurement) matrix:

$$\bullet D = \begin{bmatrix} \hat{x}_{11} & \hat{x}_{12} & \cdots & \hat{x}_{1n} \\ \hat{x}_{21} & \hat{x}_{22} & \cdots & \hat{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{x}_{m1} & \hat{x}_{m2} & \cdots & \hat{x}_{mn} \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{bmatrix} \begin{matrix} [X_1 & X_2 & \cdots & X_n] \\ \mathbf{S} \\ \text{points } (3 \times n) \end{matrix}$$

M
cameras
($2m \times 3$)

- What must be the rank of the measurement matrix $D = MS$?

C. Tomasi and T. Kanade. [Shape and motion from image streams under orthography: A factorization method.](#)
IJCV, 9(2):137-154, November 1992.

Factorizing the measurement matrix

- We want:

The diagram illustrates the factorization of a measurement matrix D into two matrices M and S . Matrix D is represented by a blue rectangle with a height of $2m$ and a width of n . It is equal to the product of matrix M (a green rectangle with a width of 3) and matrix S (an orange rectangle). The dimensions of M and S are not explicitly labeled, but their widths are indicated by arrows below them.

$$\begin{matrix} \text{height } 2m \\ \downarrow \\ \text{blue rectangle } D \\ \leftarrow \text{width } n \end{matrix} = \begin{matrix} \text{green rectangle } M \\ \leftarrow \text{width } 3 \end{matrix} \times \begin{matrix} \text{orange rectangle } S \end{matrix}$$

Factorizing the measurement matrix

- Perform SVD of \mathbf{D} :

The diagram illustrates the SVD factorization of matrix \mathbf{D} . It consists of four colored rectangles representing matrices, connected by mathematical symbols. From left to right: a blue rectangle for \mathbf{D} with dimensions $2m \times n$, an equals sign, a green rectangle for \mathbf{U} with dimensions $2m \times n$, a multiplication sign, an orange rectangle for $\mathbf{\Sigma}$ with dimensions $n \times n$, another multiplication sign, and a final orange rectangle for \mathbf{V}^T with dimensions $n \times n$.

$$\mathbf{D}_{2m \times n} = \mathbf{U}_{2m \times n} \times \mathbf{\Sigma}_{n \times n} \times \mathbf{V}^T_{n \times n}$$

Factorizing the measurement matrix

- Keep top 3 singular values:
- This is the closest approximation of D with a rank-3 matrix in terms of Frobenius norm

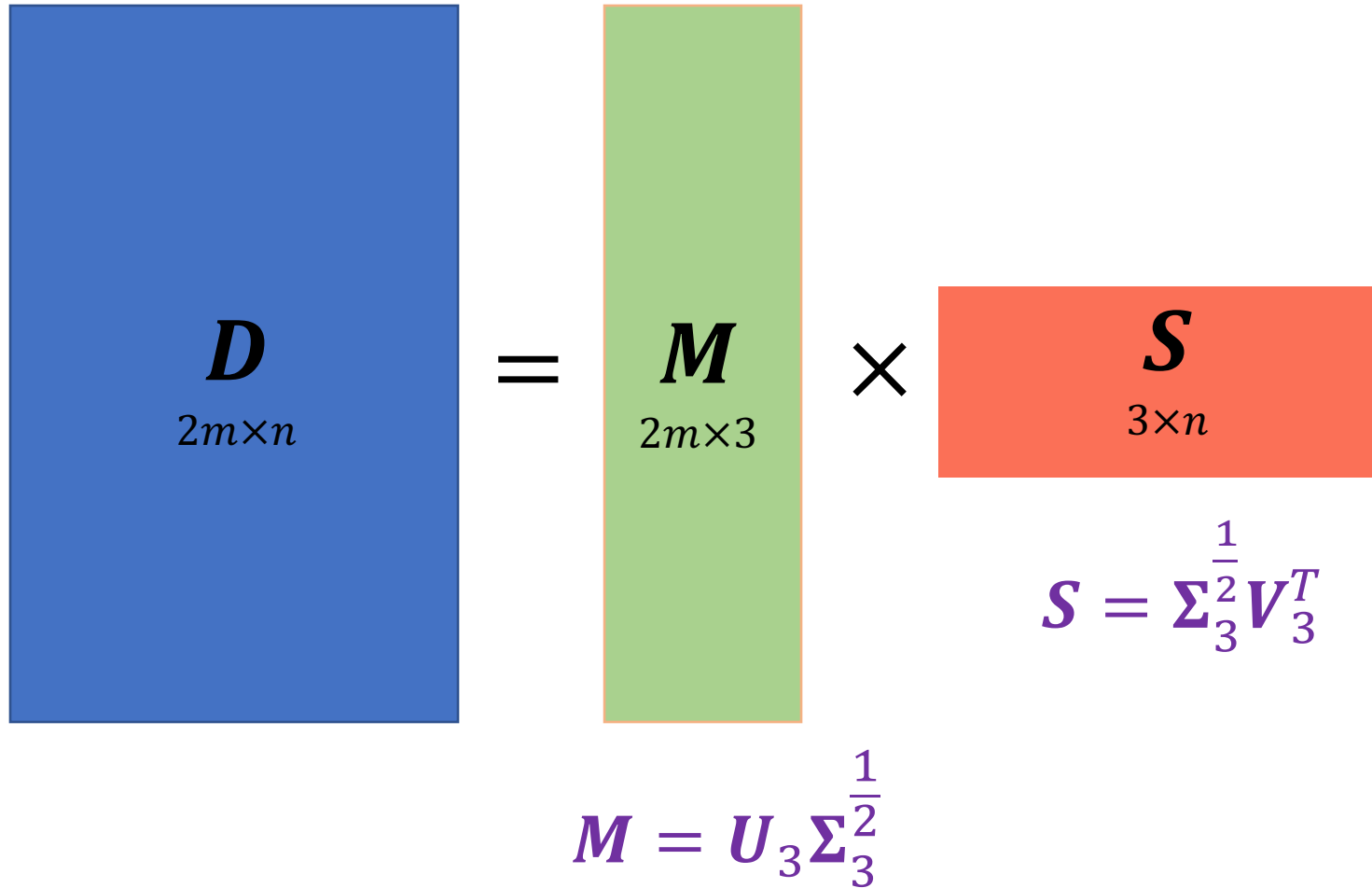
The diagram illustrates the factorization of a matrix D into three components. On the left is a blue rectangle representing matrix D with dimensions $2m \times n$. To its right is an equals sign. Next is a green rectangle representing matrix U_3 with dimensions $2m \times 3$. This is followed by a multiplication symbol \times and a white rectangle with a red border representing matrix Σ_3 with dimensions 3×3 . The top-left corner of this rectangle is shaded red and contains the label Σ_3 and 3×3 . This is followed by another multiplication symbol \times and a white rectangle with a red border representing matrix V_3^T with dimensions $3 \times n$. The top-left corner of this rectangle is shaded red and contains the label V_3^T and $3 \times n$.

$$\begin{matrix} D \\ 2m \times n \end{matrix} = \begin{matrix} U_3 \\ 2m \times 3 \end{matrix} \times \begin{matrix} \Sigma_3 \\ 3 \times 3 \end{matrix} \times \begin{matrix} V_3^T \\ 3 \times n \end{matrix}$$

- What to do about Σ_3 ?
- One solution: $M = U_3 \Sigma_3^{\frac{1}{2}}, S = \Sigma_3^{\frac{1}{2}} V_3^T$

Factorizing the measurement matrix

- One possible solution:



The diagram illustrates the factorization of the measurement matrix D into two matrices, M and S . On the left, a large blue rectangle represents the matrix D with dimensions $2m \times n$. To its right is an equals sign, followed by a tall green rectangle representing the matrix M with dimensions $2m \times 3$. To the right of M is a multiplication symbol, followed by a horizontal red rectangle representing the matrix S with dimensions $3 \times n$. Below the green rectangle M is the equation $M = U_3 \Sigma_3^{\frac{1}{2}}$ in purple. Below the red rectangle S is the equation $S = \Sigma_3^{\frac{1}{2}} V_3^T$ in purple.

$$\begin{matrix} \text{Blue box} & = & \text{Green box} & \times & \text{Red box} \\ \begin{matrix} \mathbf{D} \\ 2m \times n \end{matrix} & & \begin{matrix} \mathbf{M} \\ 2m \times 3 \end{matrix} & & \begin{matrix} \mathbf{S} \\ 3 \times n \end{matrix} \end{matrix}$$
$$\mathbf{M} = \mathbf{U}_3 \Sigma_3^{\frac{1}{2}}$$
$$\mathbf{S} = \Sigma_3^{\frac{1}{2}} \mathbf{V}_3^T$$

Factorizing the measurement matrix

- Other possible solutions (Ambiguity in Reconstruction)

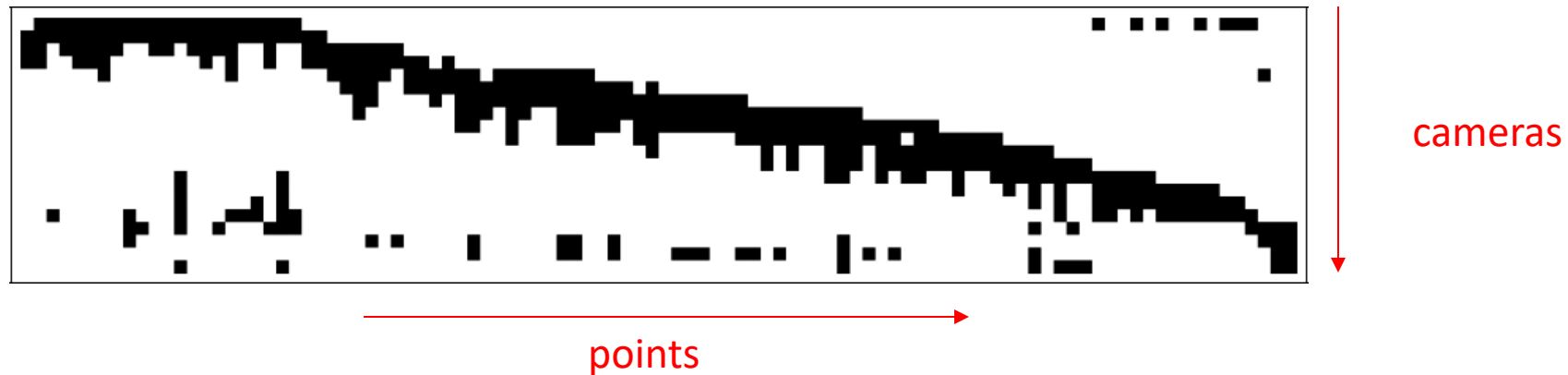
How to eliminate ambiguity?
Assume certain special structure of the projection matrix.
Assume certain conditions about the 3D structure.

$$\begin{matrix} \text{Blue box} \\ \mathbf{D} \\ 2m \times n \end{matrix} = \begin{matrix} \text{Green box} \\ \mathbf{M} \\ 2m \times 3 \end{matrix} \times \begin{matrix} \text{Pink box} \\ \mathbf{Q} \\ 3 \times 3 \end{matrix} \times \begin{matrix} \text{Pink box} \\ \mathbf{Q}^{-1} \\ 3 \times 3 \end{matrix} \times \begin{matrix} \text{Red box} \\ \mathbf{S} \\ 3 \times n \end{matrix}$$

We can estimate \mathbf{Q} to give the camera matrices in \mathbf{M} desirable properties, like orthographic projection

Dealing with missing data

- So far, we have assumed that all points are visible in all views
- In reality, the measurement matrix typically looks something like this:



- These kind of problems are called Low-rank Matrix Completion problems (aka the Netflix Problem). Solved with convex/non-convex optimizations.
- Very popular before deep learning era!

Today's Class

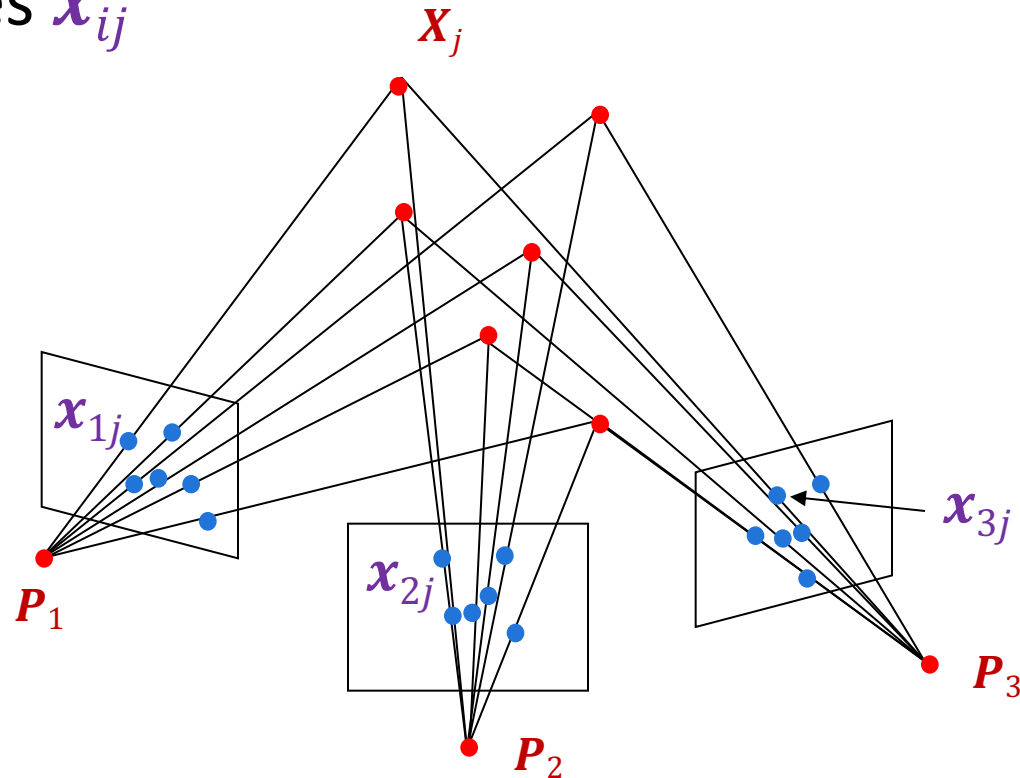
- Ambiguities in SfM
- Affine SfM
- Projective SfM
 - Global SfM
 - Incremental SfM
- Challenges and Applications

Projective structure from motion

- **Given:** m images of n fixed 3D points such that (ignoring visibility):

$$\bullet \mathbf{x}_{ij} \cong \mathbf{P}_i \mathbf{X}_j, \quad i = 1, \dots, m, \quad j = 1, \dots, n$$

- **Problem:** estimate m projection matrices \mathbf{P}_i and n 3D points \mathbf{X}_j from the mn correspondences \mathbf{x}_{ij}



Projective structure from motion

- **Given:** m images of n fixed 3D points such that (ignoring visibility):

$$\bullet \mathbf{x}_{ij} \cong \mathbf{P}_i \mathbf{X}_j, \quad i = 1, \dots, m, \quad j = 1, \dots, n$$

- **Problem:** estimate m projection matrices \mathbf{P}_i and n 3D points \mathbf{X}_j from the mn correspondences \mathbf{x}_{ij}
- With no calibration info, cameras and points can only be recovered up to a 4×4 projective transformation \mathbf{Q} :

$$\bullet \mathbf{X} \rightarrow \mathbf{QX}, \mathbf{P} \rightarrow \mathbf{PQ}^{-1}$$

- We can solve for structure and motion when $2mn \geq 11m + 3n - 15$
- For two cameras, at least 7 points are needed

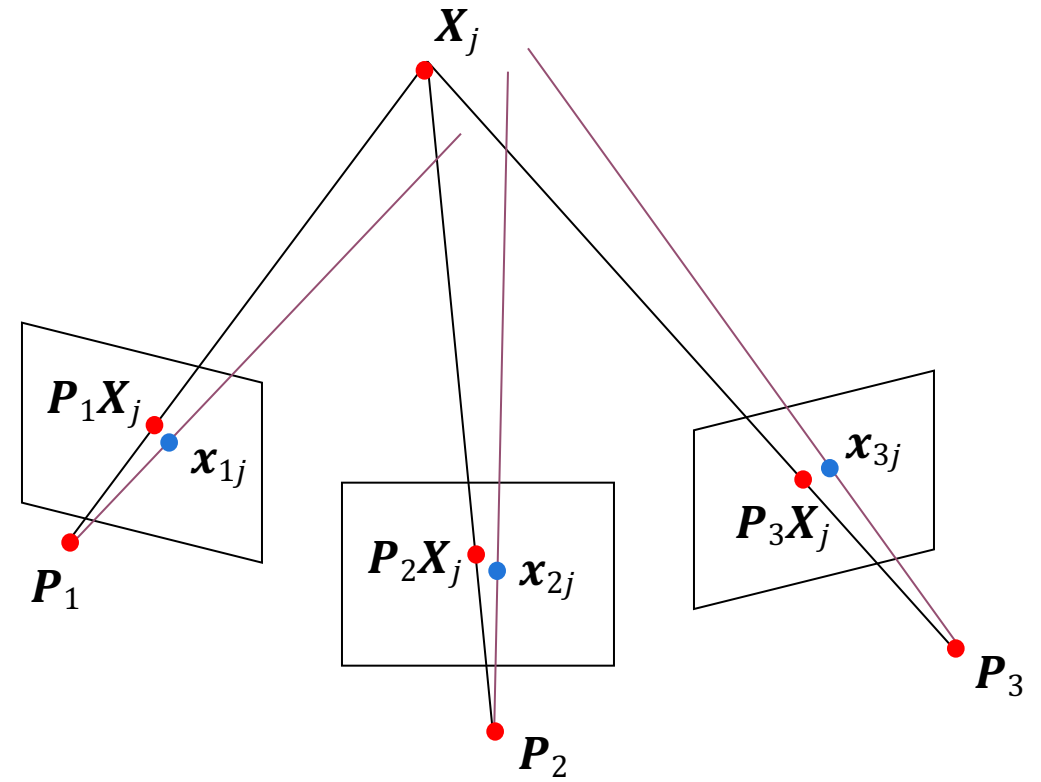
Bundle adjustment

- Non-linear method for refining structure and motion
- Minimize reprojection error (with lots of bells and whistles):

$$\bullet \sum_{i=1}^m \sum_{j=1}^n w_{ij} d \left(\mathbf{x}_{ij} - \text{proj}(\mathbf{P}_i \mathbf{X}_j) \right)^2$$



visibility flag:
is point j visible in view i ?



Global Structure from Motion

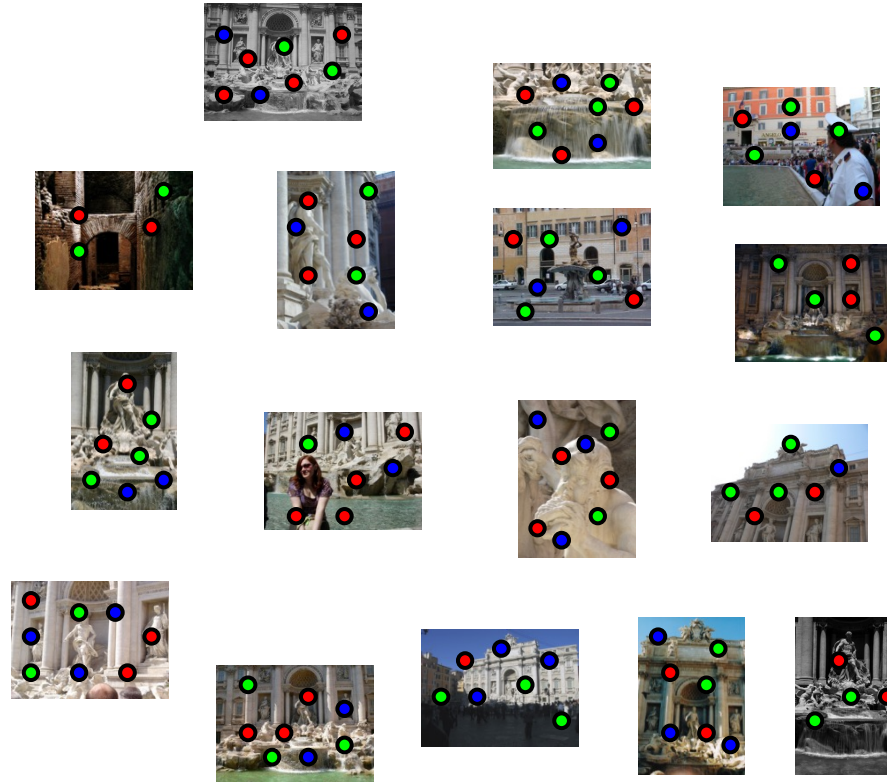
Feature detection

Detect features using SIFT [Lowe, IJCV 2004]



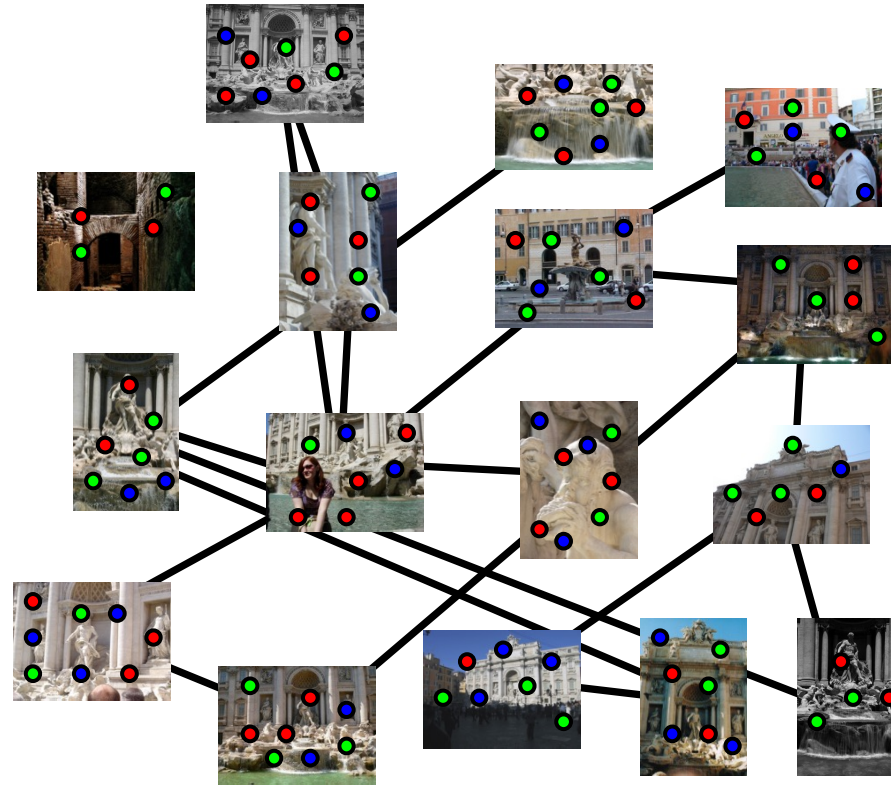
Feature detection

Detect features using SIFT [Lowe, IJCV 2004]



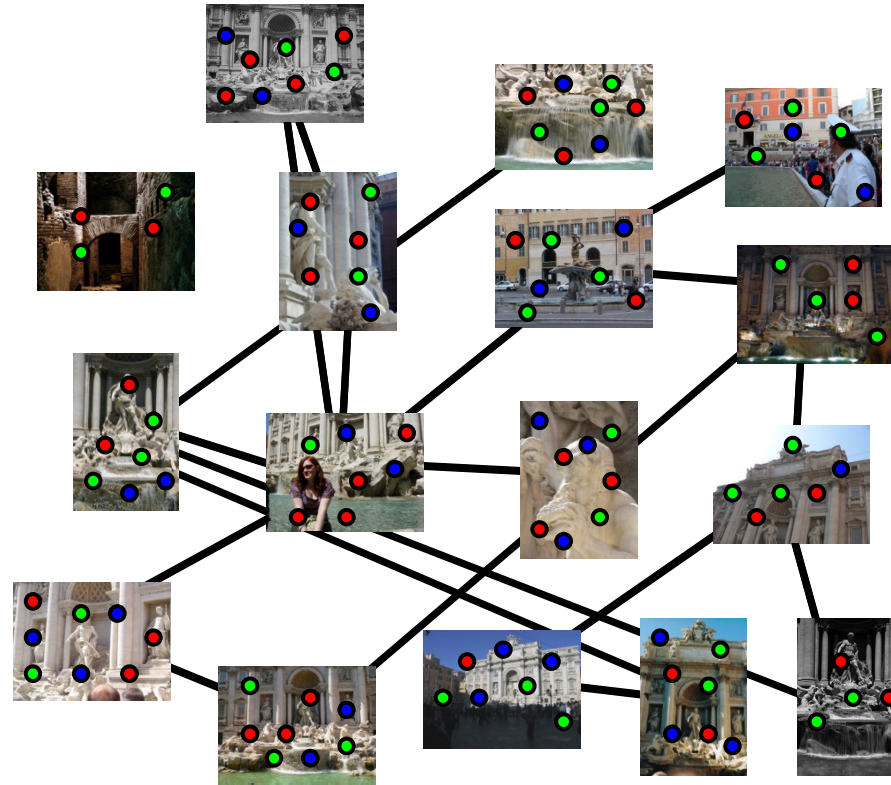
Feature matching

Match features between each pair of images



Feature matching

Refine matching using RANSAC to estimate fundamental matrix between each pair



Correspondence estimation

- Link up pairwise matches to form connected components of matches across several images

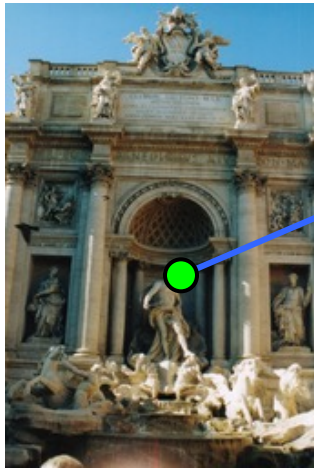


Image 1



Image 2

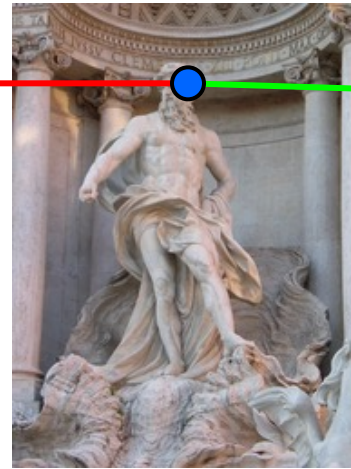


Image 3

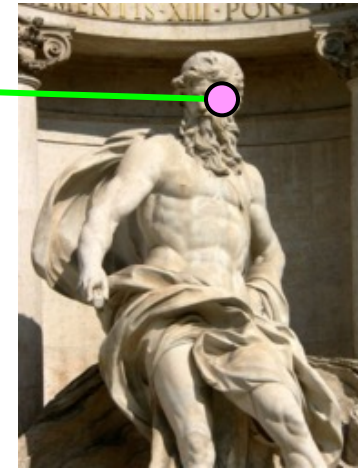
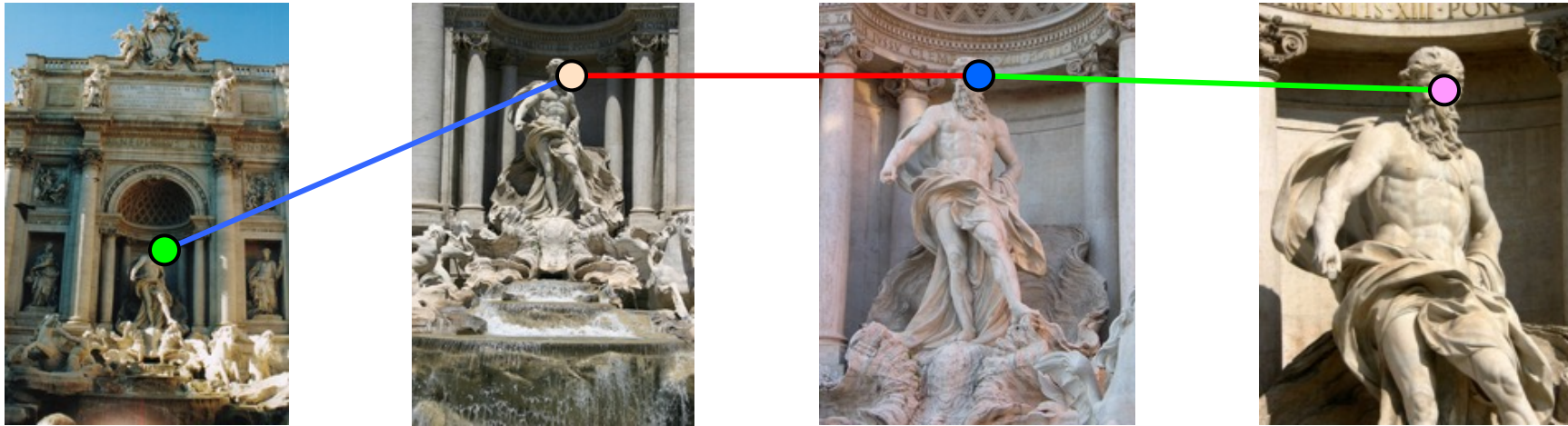


Image 4

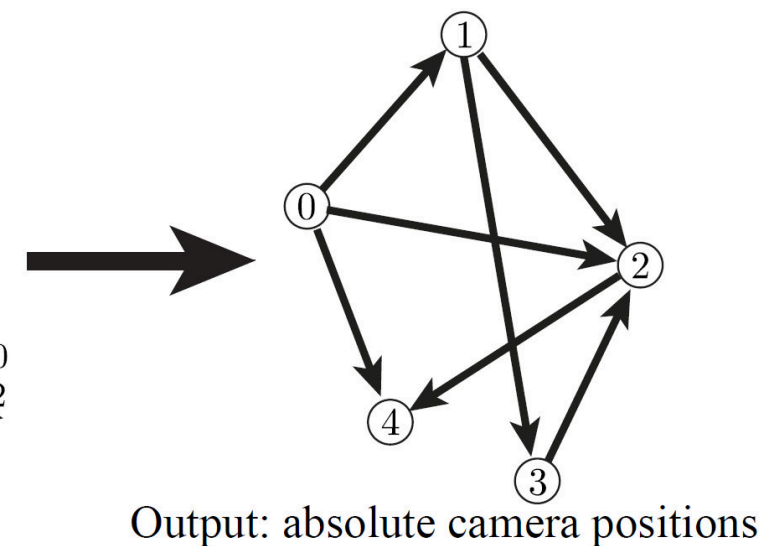
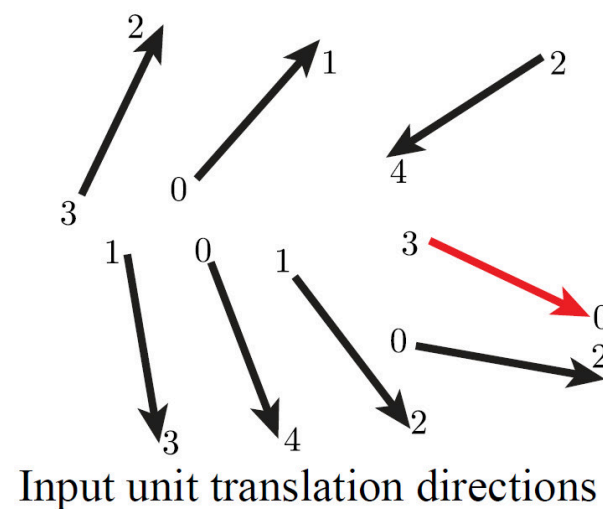
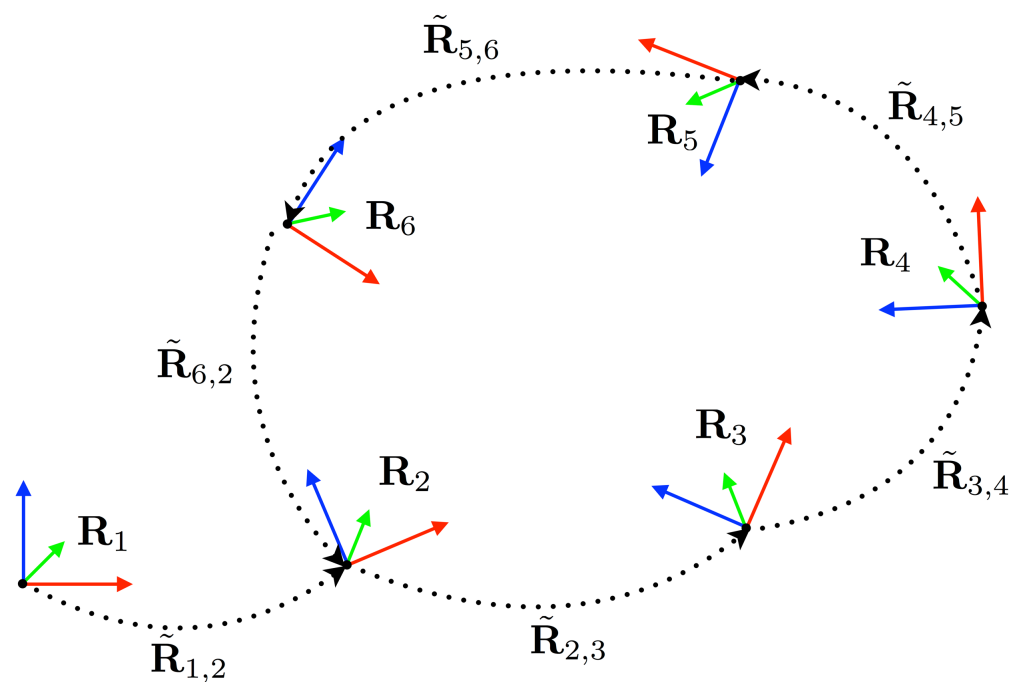


Global SfM

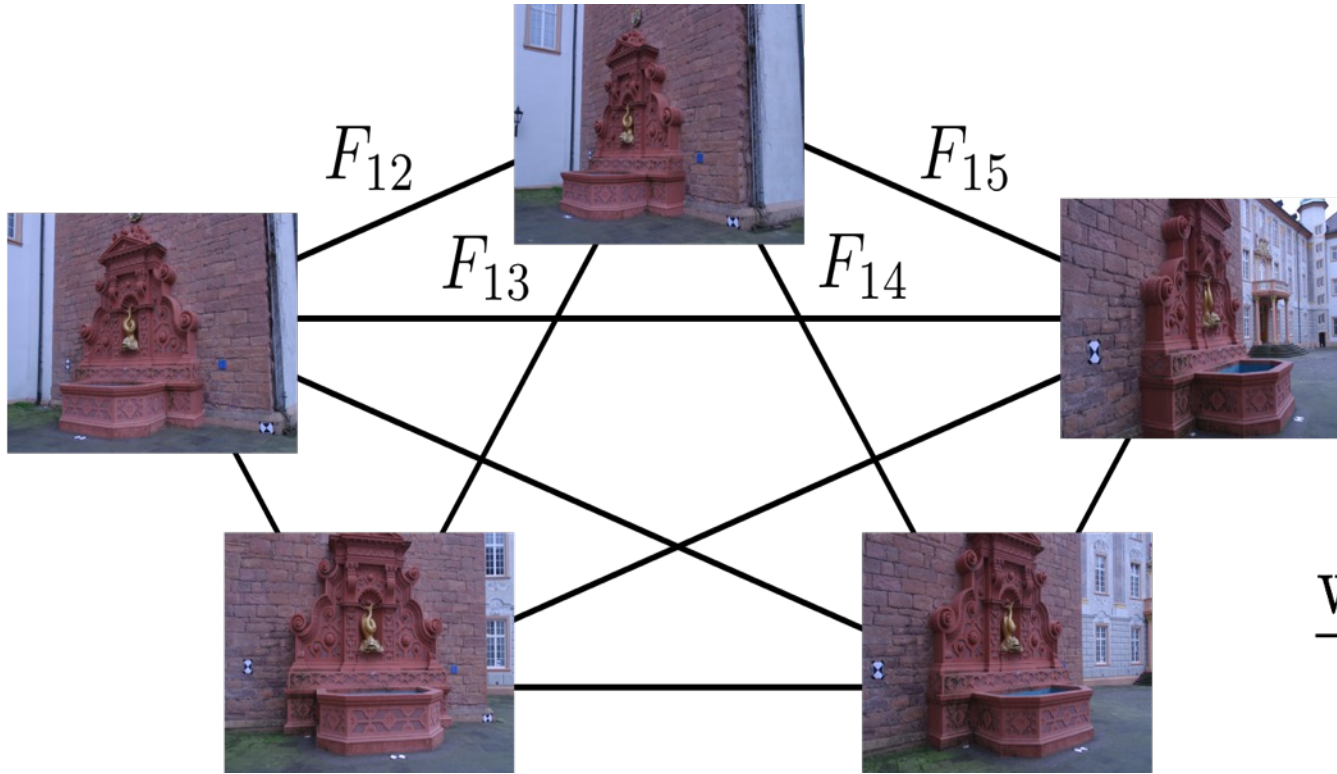
- Given N images, there are NC_2 pairs. Many of these pairs will have no overlaps in views and/or Fundamental/Essential matrix between them can not be reliably estimated using RANSAC.
 - Consider we have N_0 pairs of images with fundamental matrix estimated
- For each N_0 pairs of images decompose essential matrix into relative rotation and translation between two cameras: R_{ij} and t_{ij} .
- Can we solve for global (world coordinate) rotation and translation of the cameras, given pairwise measurements, i.e.
 - Given R_{ij} and t_{ij} for N_0 pairs, find R_k & T_k for N cameras.
- Once we have the cameras we can better initialize the Bundle Adjustment problem.

Rotation & Translation Averaging

Given R_{ij} and t_{ij} for $N(N-1)/2$ pairs, find R_k & T_k for N cameras



Camera Pose estimation as matrix completion over Fundamental matrices



$$F = \begin{bmatrix} 0 & F_{12} & F_{13} & F_{14} & F_{15} \\ F_{21} & 0 & F_{23} & F_{24} & F_{25} \\ F_{31} & F_{32} & 0 & F_{34} & F_{35} \\ F_{41} & F_{42} & F_{43} & 0 & F_{45} \\ F_{51} & F_{52} & F_{53} & F_{54} & 0 \end{bmatrix}$$

with $F = A + A^T$ and $rank(A) = 3$.

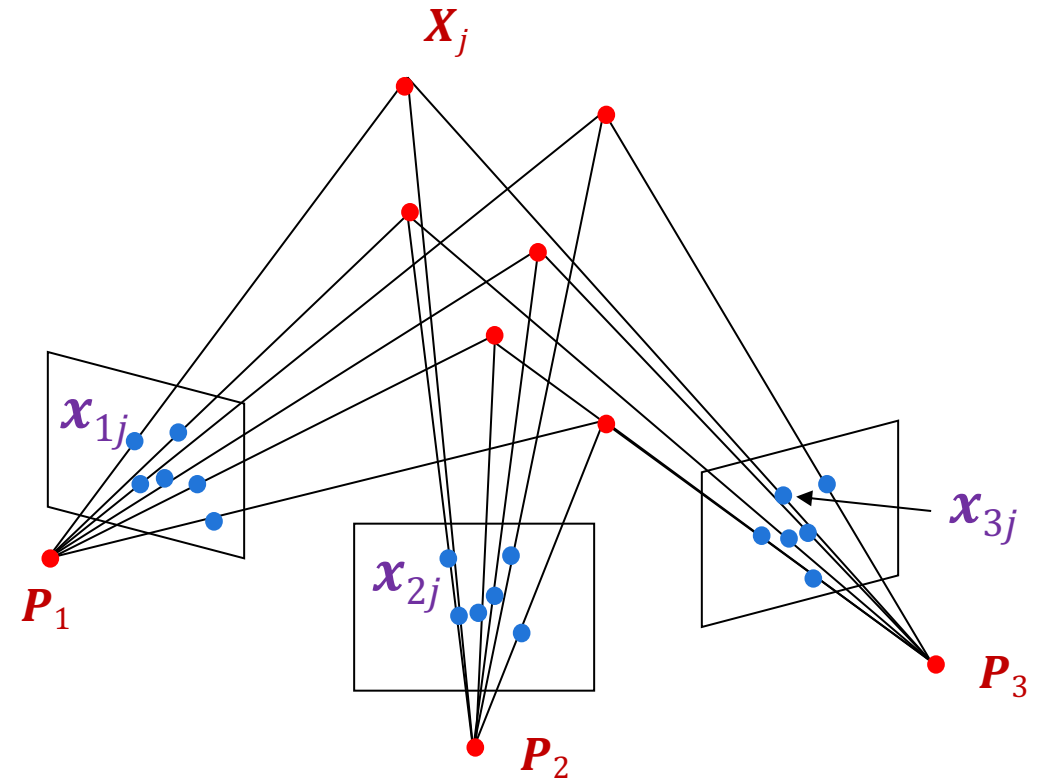
- Proves a low-rank property of all the cameras capturing different images of a scene.
- Solves a low-rank camera pose recovery algorithm from Structure from Motion.

Projective structure from motion

- **Given:** m images of n fixed 3D points such that (ignoring visibility):

$$\bullet \mathbf{x}_{ij} \cong \mathbf{P}_i \mathbf{X}_j, \quad i = 1, \dots, m, \quad j = 1, \dots, n$$

- **Problem:** estimate m projection matrices \mathbf{P}_i and n 3D points \mathbf{X}_j from the mn correspondences \mathbf{x}_{ij}



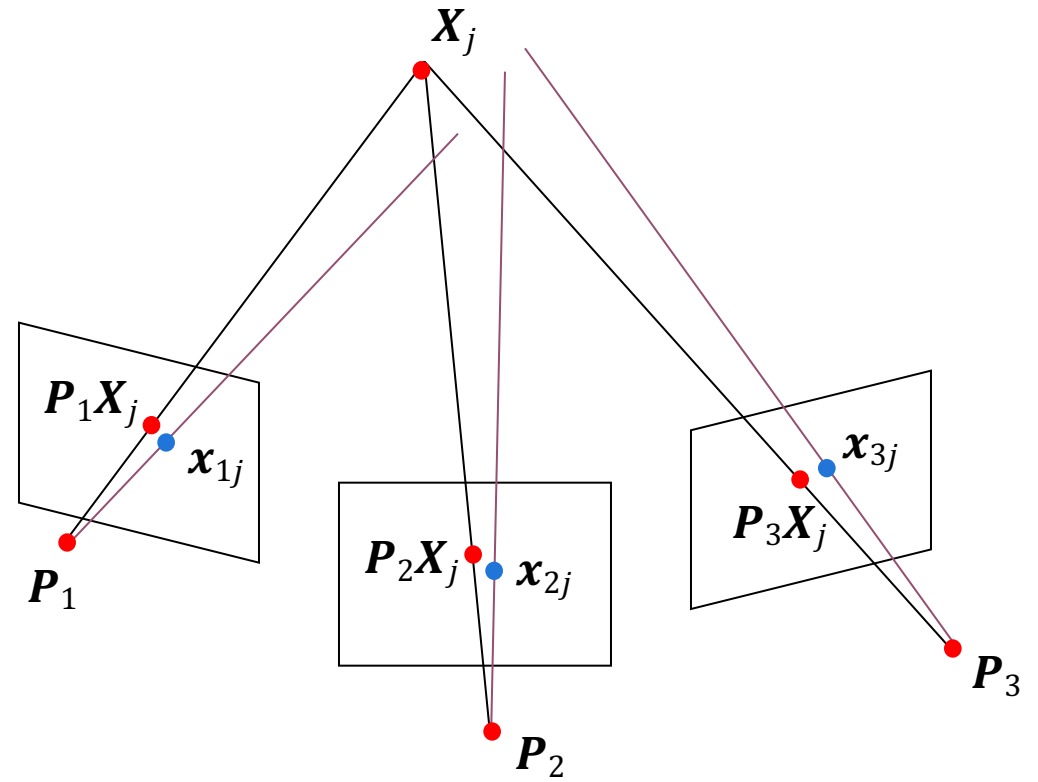
Bundle adjustment

- Non-linear method for refining structure and motion
- Minimize reprojection error (with lots of bells and whistles):
-

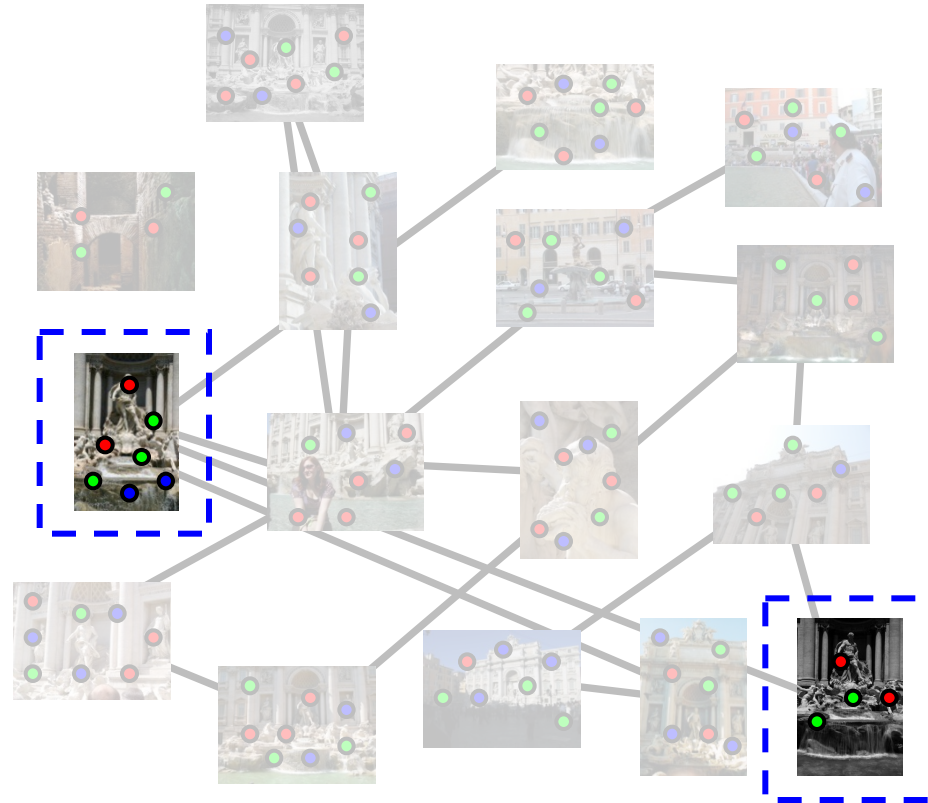
$$\bullet \sum_{i=1}^m \sum_{j=1}^n w_{ij} d \left(\mathbf{x}_{ij} - \text{proj}(\mathbf{P}_i \mathbf{X}_j) \right)^2$$

↑
visibility flag: is
point j visible in
view i ?

- **Initialize \mathbf{P}_i 's by solving global SfM**
 - **Rotation Averaging**
 - **Translation Averaging**



Incremental SfM



- Automatically select an initial pair of images

1. Picking the initial pair

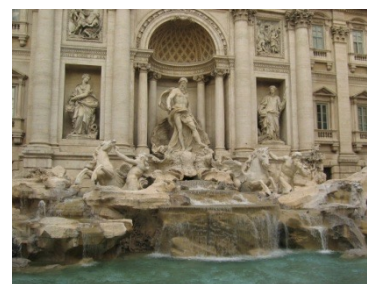
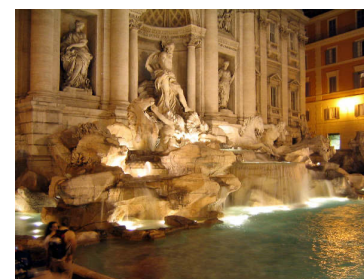
- We want a pair with many matches, but which has as large a baseline as possible



✅ lots of matches
❌ small baseline



✅ large baseline
❌ very few matches



✅ large baseline
✅ lots of matches

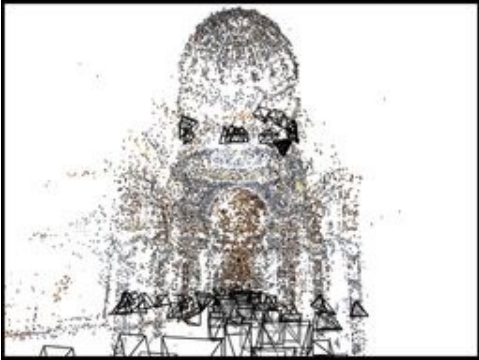
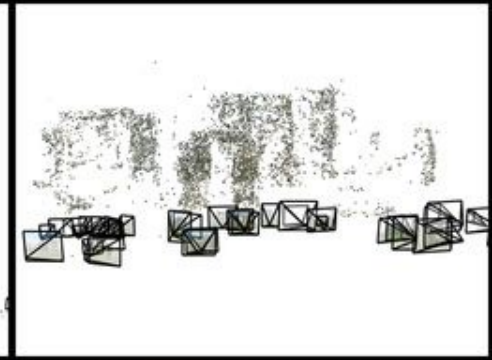
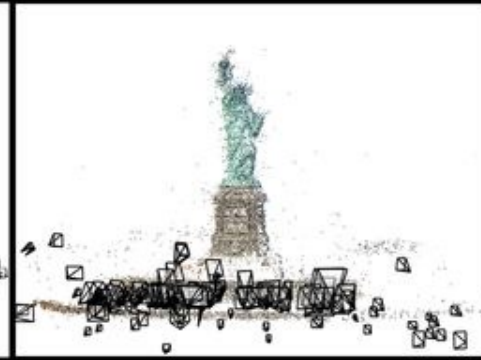
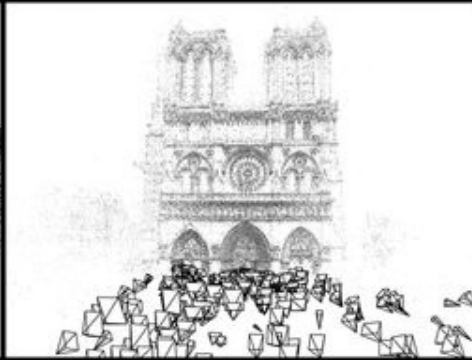


Incremental SFM

- Pick a pair of images with lots of inliers (and preferably, good EXIF data)
 - Initialize intrinsic parameters (focal length, principal point) from EXIF
 - Estimate extrinsic parameters (R and t) using [five-point algorithm](#)
 - Use triangulation to initialize model points
- While remaining images exist
 - Find an image with many feature matches with images in the model
 - Run RANSAC on feature matches to register new image to model
 - Triangulate new points
 - Perform bundle adjustment to re-optimize everything
 - Optionally, align with GPS from EXIF data or ground control points

Next Best View Problem

- Choice of next view impacts reconstruction quality
 - almost identical view => high uncertainty in triangulation
 - very different view => low overlap and high camera uncertainty
 - single bad choice may impact the whole reconstruction
- Popular next best view methods:
 - choose view with seeing the most triangulated points
 - minimize reconstruction uncertainty
 - depends on number of observations
 - distribution in the image



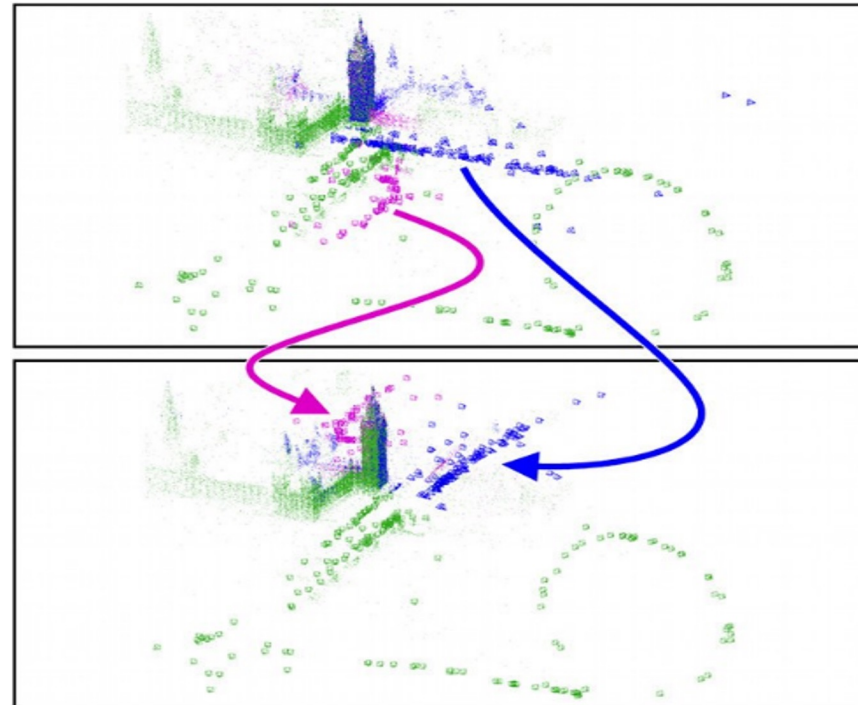
Today's Class

- Ambiguities in SfM
- Affine SfM
- Projective SfM
 - Global SfM
 - Incremental SfM
- Challenges and Applications

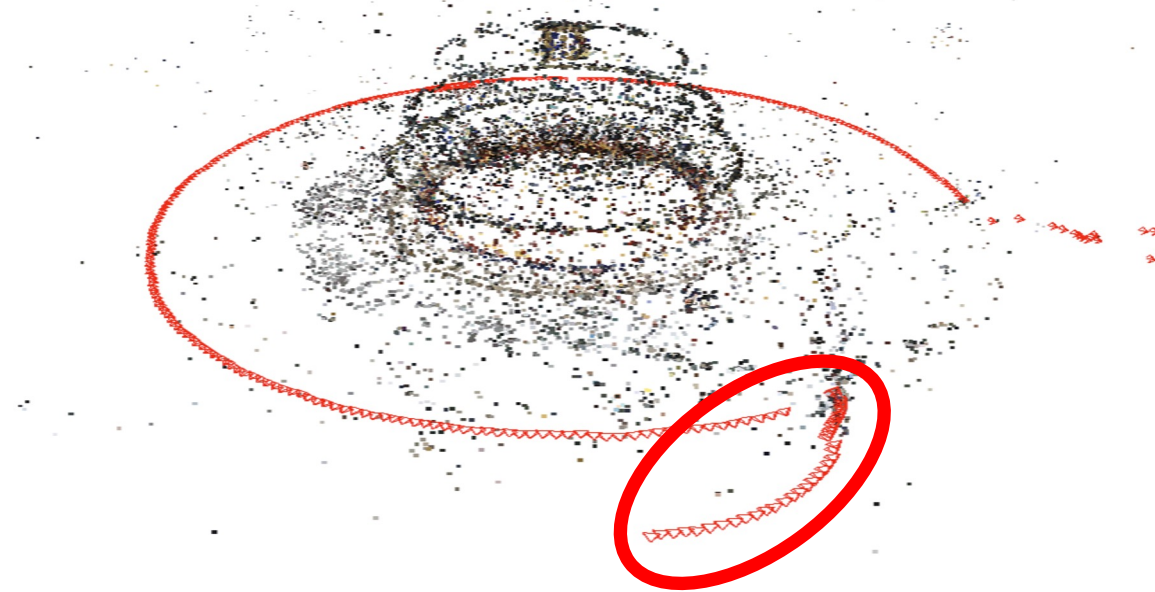
The devil is in the details

- Handling degenerate configurations (e.g., homographies)
- Filtering out incorrect matches
- Dealing with repetitions and symmetries

Repetitive structures cause catastrophic failures



Repetitive structures cause catastrophic failures



The devil is in the details

- Handling degenerate configurations (e.g., homographies)
- Filtering out incorrect matches
- Dealing with repetitions and symmetries
- Reducing error accumulation and closing loops

Loop Detection/Closure

- Problem:
 - Structure from motion is an incremental process
 - Drift accumulates
- Mitigation:
 - Retrieval of long range connections

Reducing error accumulation and closing loops



seattle1

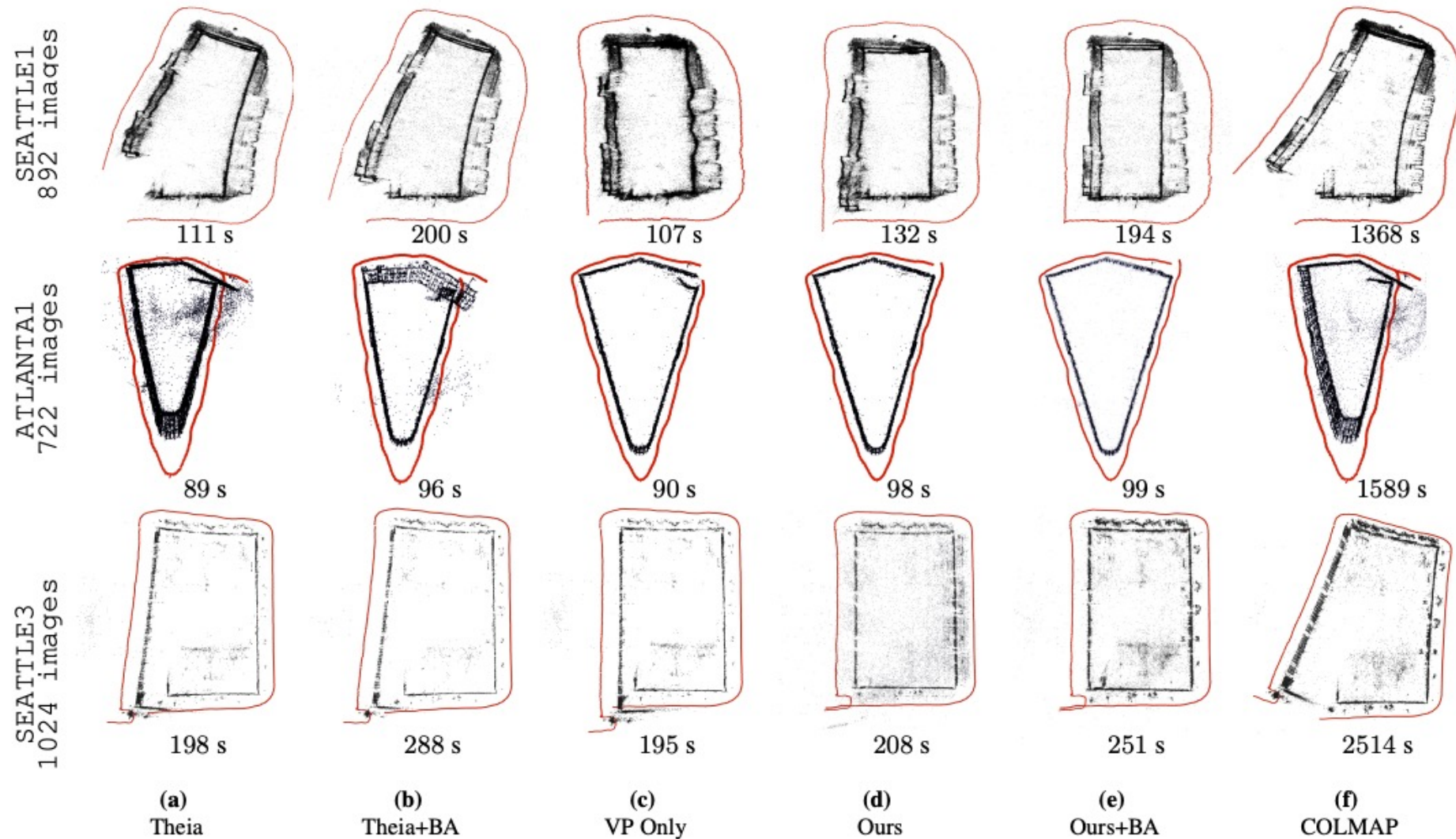
more_half

seattle2

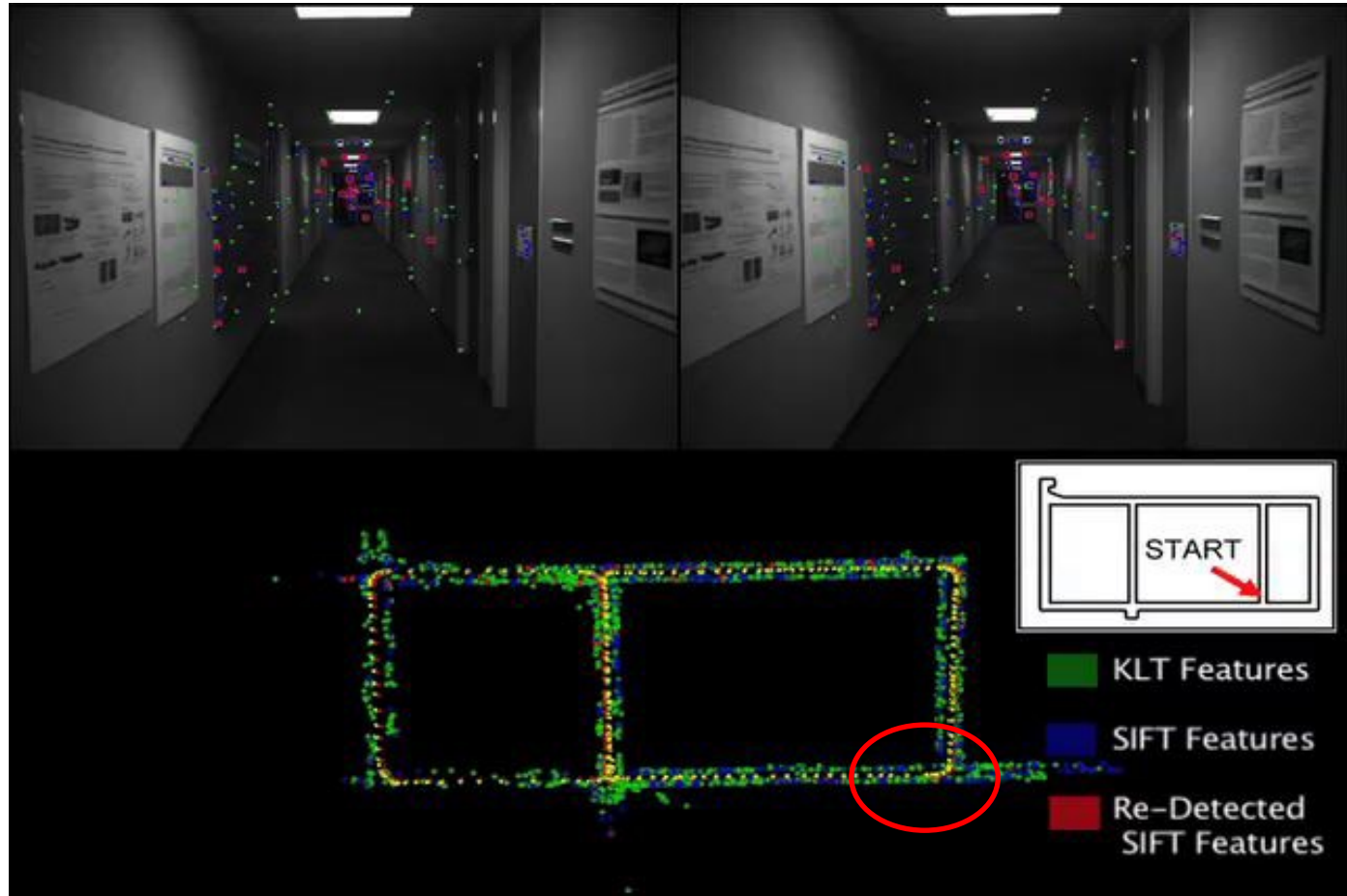
atlanta1

seattle3

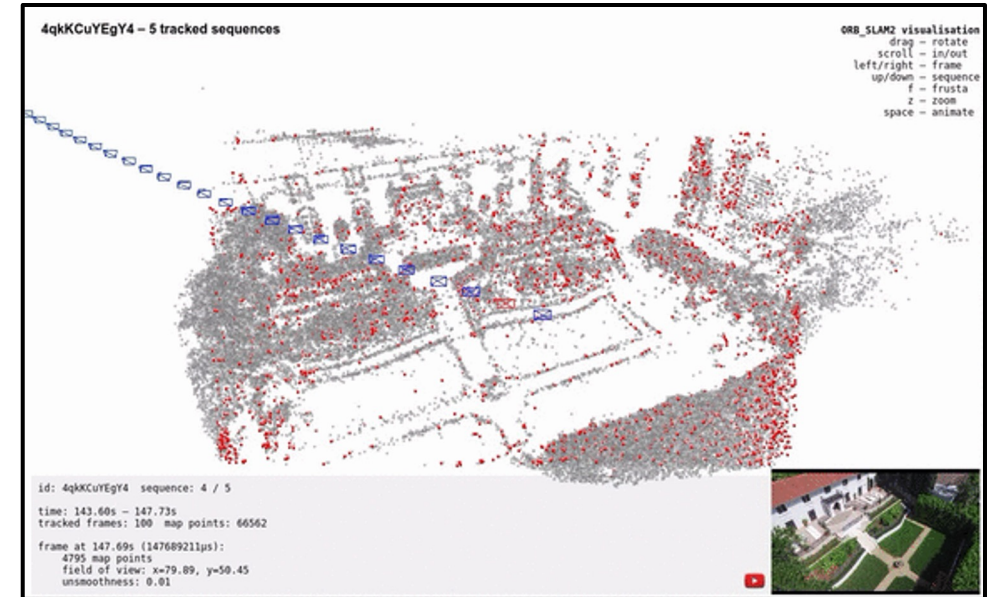
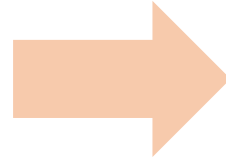
Reducing error accumulation and closing loops



Loop Closure

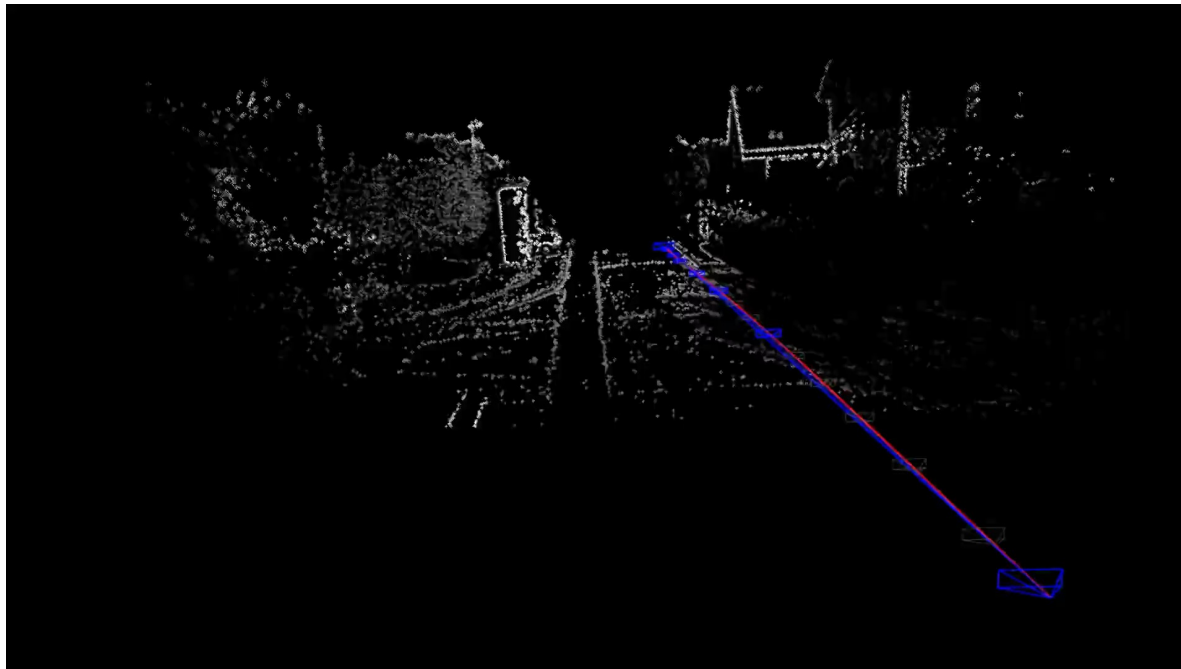


Can also compute camera poses from video (often called Visual SLAM)



Visual Simultaneous Localization and Mapping (V-SLAM)

- Main differences with SfM:
 - Continuous visual input from sensor(s) over time
 - Gives rise to problems such as loop closure
 - Often the goal is to be online / real-time



Video from Daniel Cremer's Lab

SFM software

- [Bundler](#)
- [OpenSfM](#)
- [OpenMVG](#)
- [VisualSFM](#)
- [COLMAP](#) ([Structure-from-motion revisited](#), JL Schonberger, JM Frahm, CVPR 2016, from UNC!)
- See also [Wikipedia's list of toolboxes](#)

SfM applications

- 3D modeling
- Surveying
- Robot navigation and mapmaking
- Virtual and augmented reality
- Visual effects ("Match moving")
 - https://www.youtube.com/watch?v=RdYWp70P_kY

Applications: Match Moving

Or Motion tracking, solving for camera trajectory
Integral for visual effects (VFX)

Why?



Applications: Visual Reality & Augmented Reality



Oculus

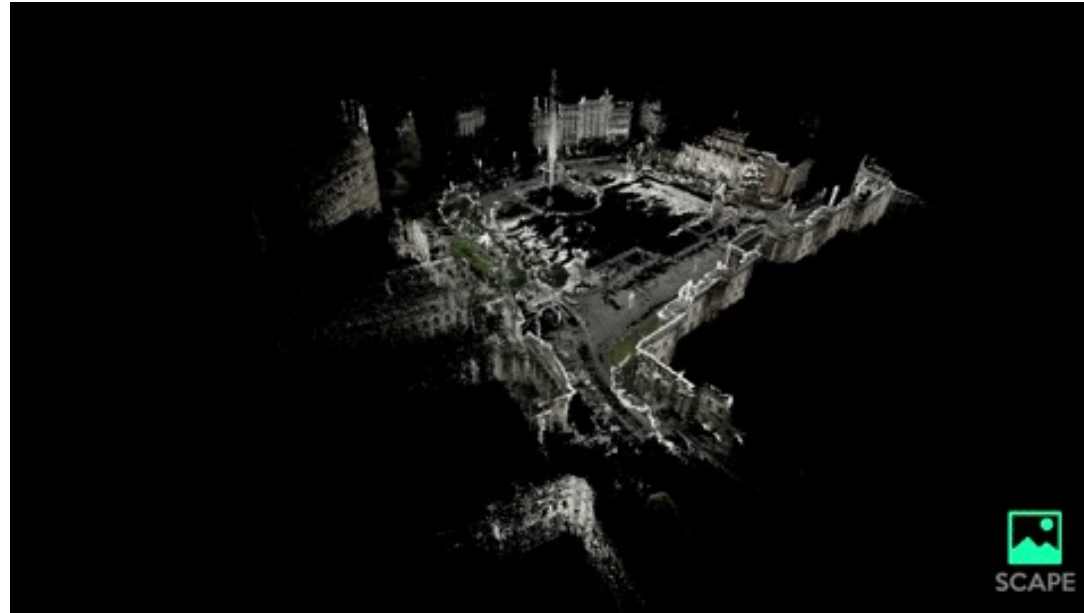
<https://www.youtube.com/watch?v=KOG7yTz1iTA>



Hololens

<https://www.youtube.com/watch?v=FMtvrTGnP04>

Applications: Visual Reality & Augmented Reality



Scape: Building the 'AR Cloud': Part Three —3D Maps, the Digital Scaffolding of the 21st Century

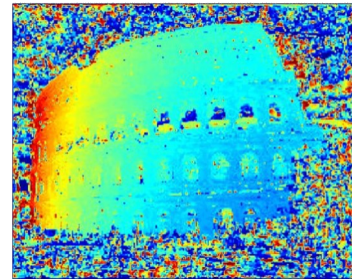
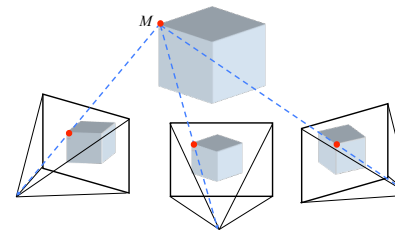
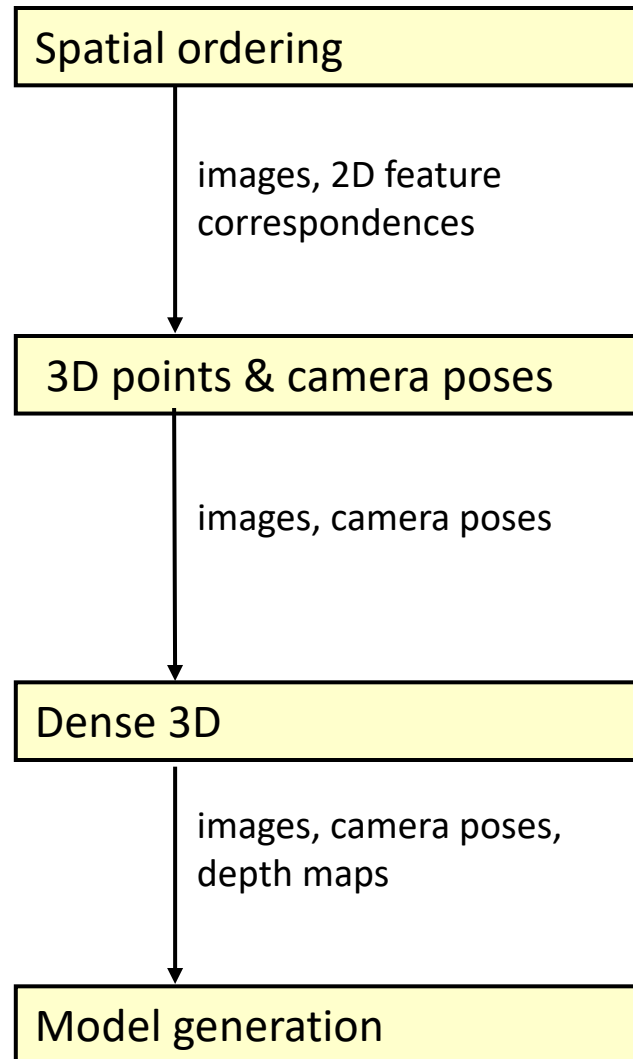
<https://medium.com/scape-technologies/building-the-ar-cloud-part-three-3d-maps-the-digital-scaffolding-of-the-21st-century-465fa55782dd>

Application: AR walking directions



<https://www.theverge.com/2019/8/8/20776247/google-maps-live-view-ar-walking-directions-ios-android-feature>

3D model from video



Summary: 3D geometric vision

- Fundamentals:
 - Camera Models: Intrinsic & Extrinsic
 - 3D to 2D projections, perspective distortions
 - Vanishing Points & Lines
 - Epipolar Geometry
 - Essential & Fundamental Matrices
- Core problems:
 - Camera calibration: single camera + two camera (estimate E/F matrix)
 - Stereo: depth from two calibrated cameras
- Reconstruction Techniques:
 - Active Stereo
 - Multi-view Stereo
 - Structure from Motion
 - Photometric Stereo (next class)

Slide Credits

- [CS5670, Introduction to Computer Vision](#), **Cornell Tech**, by Noah Snavely.
- [CS 194-26/294-26: Intro to Computer Vision and Computational Photography](#), **UC Berkeley**, by Angjoo Kanazawa.
- **CS 543** [Computer Vision](#), by Stevlana Lazebnik, UIUC.
- **COMP 776**, by Jan-Michael Frahm, UNC