COMP 550.002: Fall 2023
Assignment 1

Announced: September 6, 2023

Due Date: September 20, 2023

Problem 1 and Problem 2(a) are individual. The remaining ones are collaborative. You MUST mention the names of your collaborators, and cite any material you took help from (including discussions on canvas by other students) except the textbook.

CLRS refers Cormen et al. textbook.

Problem 1 (10 points)

Consider an array $A = \langle 53, 37, 41, 69, 4, 9, 1\rangle$. Illustrate the steps of INSERTION-SORT on input array A. Use Fig. 2.1 in CLRS as a reference to answer this question.

Problem 2 (10 + 15 + 15 + 10 + 10 = 60 points)

The SELECTION-SORT algorithm sorts an array A of integers. The pseudo-code is given below.

Algorithm 1 SELECTION-SORT(A, n)

Input: Array A of n elements
Output: Sorted array A

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1: for i = 1 to n − 1 do
2:     smallest = i
3:     for j = i + 1 to n do
4:         if A[j] < A[smallest] then
5:             smallest = j
6:     Exchange A[i] and A[smallest]
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(a) Consider the array $A = \langle 53, 37, 41, 69, 4, 9, 1\rangle$ in Problem 1. Illustrate the steps of SELECTION-SORT on input array A. Your illustration should include the array A after each iteration of the **for** loop of lines 1–6.

(b) State and prove a loop invariant for the *for* loop in lines 3–5. Your proof should use the structure of the loop-invariant proof presented in Chapter 2 of CLRS.

(c) State and prove a loop invariant for the *for* loop in lines 1–6. In your proof, you can use the loop invariant and parts of your proof (with proper labelling and referring of those parts) you proved in (a). Use your loop invariant to prove the correctness of SELECTION-SORT in the termination step. Your proof should use the structure of the loop-invariant proof presented in Chapter 2 of CLRS.

(d) ~~Using asymptotic notations, express the worst- and best-case running time of SELECTION-SORT algorithm. What is SELECTION-SORT's running time (in all cases) in asymptotic notations? You running-time expressions should be as precise as possible (e.g., $O(n)$ instead of $O(n^2)$ if possible or using the Θ-notation instead of O-notation). Briefly justify your answer.~~

(d) Express the worst- and best-case running time of SELECTION-SORT algorithm. You may use asymptotic notations to represent the running times. Alternatively, you can express running times as a function of input size as done in class.

(e) Can you improve the best-case running time of SELECTION-SORT by modifying or editing the algorithm? Your modification should be as simple as possible. (i) Write your algorithm in pseudo-code. (ii) What is the best-case running time of your algorithm in asymptotic notations? (iii) Give an example input array of at least 5 elements for which the best-case running time occurs.

Problem 3 (30 Points)

For each of the following pair of functions $f(n)$ and $g(n)$, determine whether $f(n) = O(g(n))$, $f(n) = \Omega(g(n))$, $f(n) = \Theta(n)$, $f(n) = o(g(n))$, or $f(n) = \omega(g(n))$. You should indicate all asymptotic notations that apply.

	$f(n)$	$g(n)$
(a)	$10^{10}n^2 + 5n$	$n^3 + 1$
(b)	3^n	$n \cdot 2^n$
(c)	$n \lg n$	$10n \lg 10n$
(d)	$n^2 \lg n$	$n(\log_{10} n)^2$