COMP 550.001 - Fall 2017
Assignment 6

Part A due: Monday, November 20, 2017 (start of class)

This assignment does not contain a Part B.

You should submit a physical copy of your written homework at the start of class.

Optional Part C due: Monday, November 27, 2017 (start of class)

This assignment includes an optional Part C. Earning half of the points will be worth half of a late day (only integral late days may be used to turn in homework late, but a partial late day can count as partial extra credit at the end of the semester), and earning at least 80% of the points will be worth a full late day. You should submit your code in a .zip or .tar.gz file to Sakai, and the analysis in a physical copy.

Part A: Due Monday, November 20, 2017

Be sure to include a collaboration statement with your assignment, even if you worked alone.

[10 points] Problem 1: CLRS Exercise 22.3-11

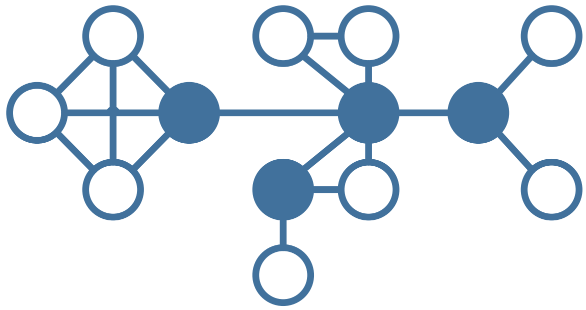
Explain how a vertex u of a directed graph can end up in a depth-first tree containing only u, even though u has both incoming and outgoing edges in G. Assume that u does not have any self-loops, and that $|V| > 1$.

[8 points] Problem 2: CLRS Exercise 22.3-8

Give a counterexample to the conjecture that if a directed graph G contains a path from u to v, and if $u.d < v.d$ in a depth-first search of G, then v is a descendant of u in the depth-first forest produced.

[30 points] Problem 3: subset of CLRS Problem 22-2

Let $G = (V, E)$ be a connected, undirected graph. An ***articulation point*** of G is a vertex whose removal disconnects G. For example, articulation points are shown as the filled-in vertices in the figure below.



We can determine articulation points using depth-first search. Let $G_\pi = (V, E_\pi)$ be a depth-first tree of G.

a) Prove that the root of G_π is an articulation point of G if and only if it has at least two children in G_π.

b) Let v be a nonroot vertex of G_π. Prove that v is an articulation point of G if and only if v has a child s such that there is no back edge from s or any descendant of s to a proper ancestor of v.

c) Let

$$v.low = \min \begin{cases} v.d, \\ w.d : (u, w) \text{ is a back edge for some descendant } u \text{ of } v. \end{cases}$$

Show how to compute $v.low$ for all vertices $v \in V$ in $O(E)$ time.

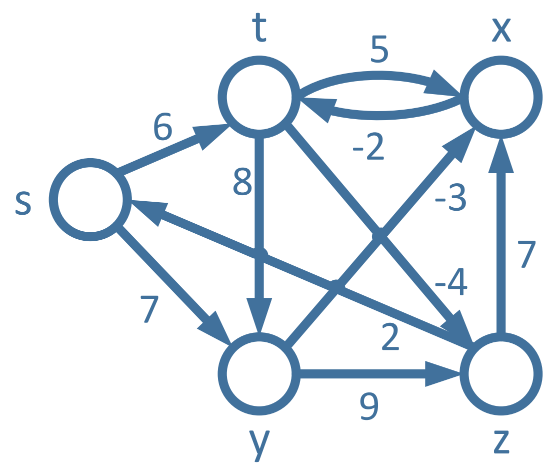
d) Show how to compute all articulation points in $O(E)$ time.

[10 points] Problem 4: CLRS Exercise 23.1-8

Let T be a minimum spanning tree of a graph G, and let L be the sorted list of the edge weights in T. Show that for any other minimum spanning tree T' of G, the list L is also the sorted list of edge weights of T'.

[10 points] Problem 5: CLRS Exercise 24.1-1

a) Run the Bellman-Ford algorithm on the directed graph shown below, using vertex z as the source. In each pass, relax edges in the same order as in the figure, and show the d and π values *after each pass.*



b) Now, change the weight of edge (z, x) to 4 and run the algorithm again, using s as the source.

[10 points] Problem 6: CLRS Exercise 24.3-5

Professor Newman thinks he has worked out a simpler proof of correctness for Dijkstra's algorithm. He claims that Dijkstra's algorithm relaxes the edges of every shortest path in the graph in the order in which they appear on the path, and therefore the path-relaxation property applies to every vertex reachable from the source. Show that the professor is mistaken by constructing a directed graph for which Dijkstra's algorithm could relax the edges of a shortest path out of order.

[12 points] Problem 7: CLRS Exercise 29.1-2

Give three feasible solutions to the linear program given below. What is the objective value of each one?

$$
\begin{array}{lrcrcrcr}
\text{maximize} & 2x_1 & - & 3x_2 & + & 3x_3 & & \\
\text{subject to} & x_1 & + & x_2 & - & x_3 & \leq & 7 \\
 & -x_1 & - & x_2 & + & x_3 & \leq & -7 \\
 & x_1 & - & 2x_2 & + & 2x_3 & \leq & 4 \\
 & & & x_1, x_2, x_3 & & & \geq & 0
\end{array}
$$

[10 points] Problem 8: CLRS Exercise 29.1-5

Convert the following linear program into slack form:

$$
\begin{array}{lrcrcrcr}
\text{maximize} & 2x_1 & & & - & 6x_3 & & \\
\text{subject to} & x_1 & + & x_2 & - & x_3 & \leq & 7 \\
 & 3x_1 & - & x_2 & & & \geq & 8 \\
 & -x_1 & + & 2x_2 & + & 2x_3 & \geq & 0 \\
 & & & x_1, x_2, x_3 & & & \geq & 0.
\end{array}
$$

What are the basic and nonbasic variables?