The University of North Carolina at Chapel Hill
Department of Computer Science

SCAAT Tracking: Numerical Error and Stability Concerns

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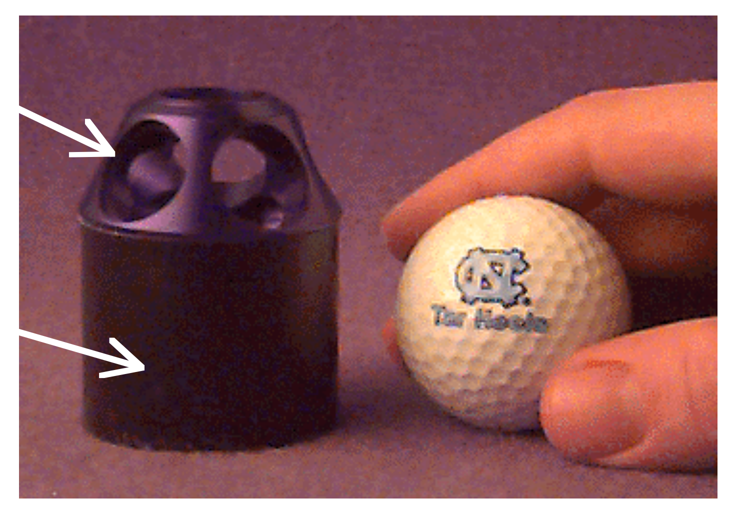
SCAAT Tracking

The Problem

camera fixture

electronics



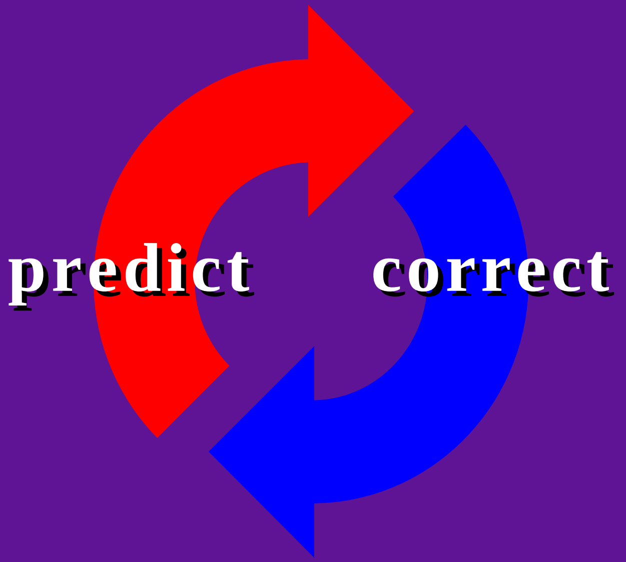
Estimation in General

- The scientific aspect
  - Control and estimation theory
  - Man-made and natural systems
  - Systematic methods
- The numerical aspect
  - Stochastic approach
  - Uses a computer numerically
  - KF *developed for a computer*
  - Affects of design and numerical error

The Kalman Filter

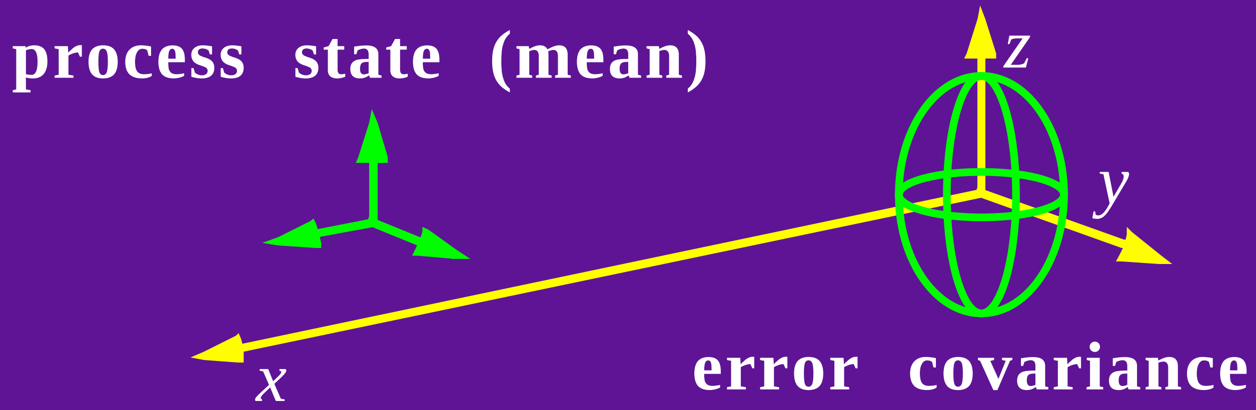
Predictor-Corrector

- Seminal paper by R.E. Kalman, 1960
- Set of mathematical equations
- Optimal estimator (min. mean-square error)
- Estimation, filtering, prediction, fusion
- Predictor-corrector
- Recursive



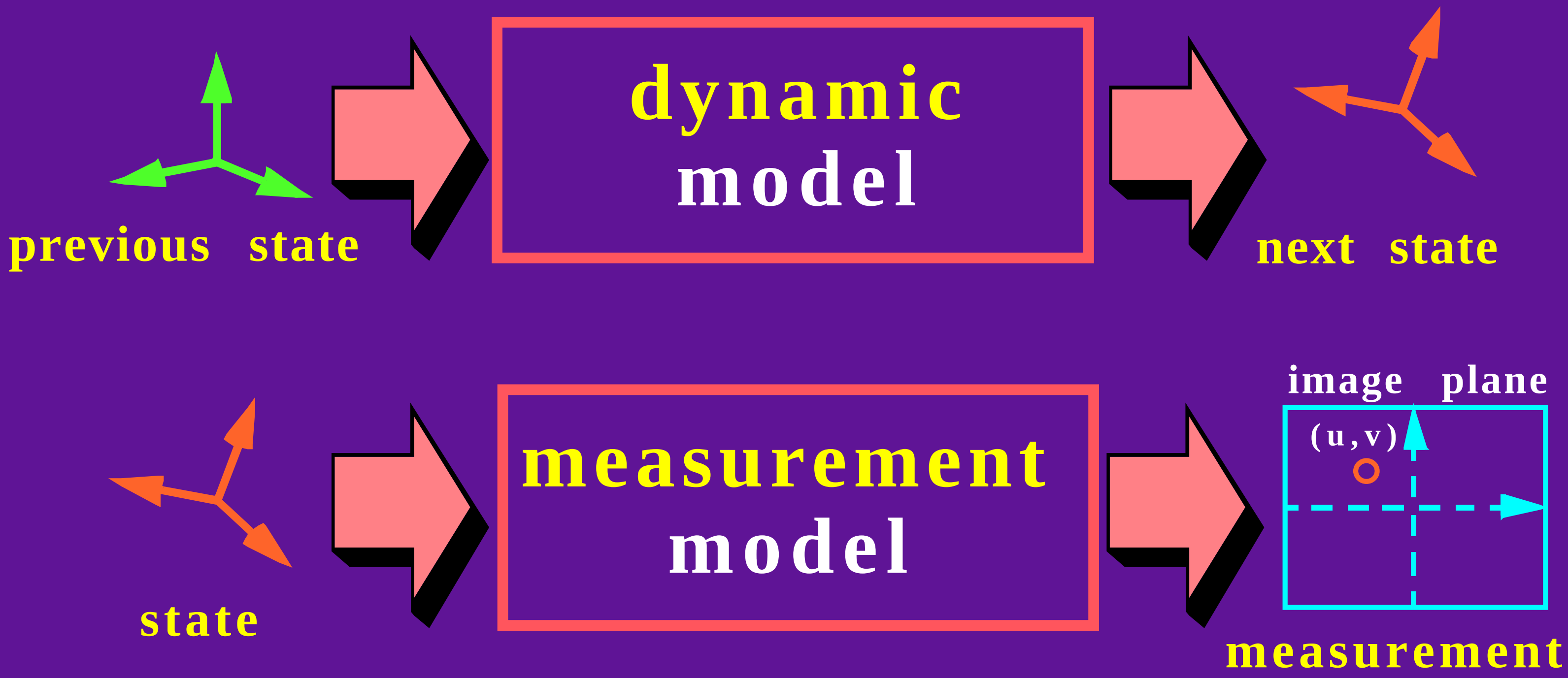
The Kalman Filter

Maintains First Two Moments



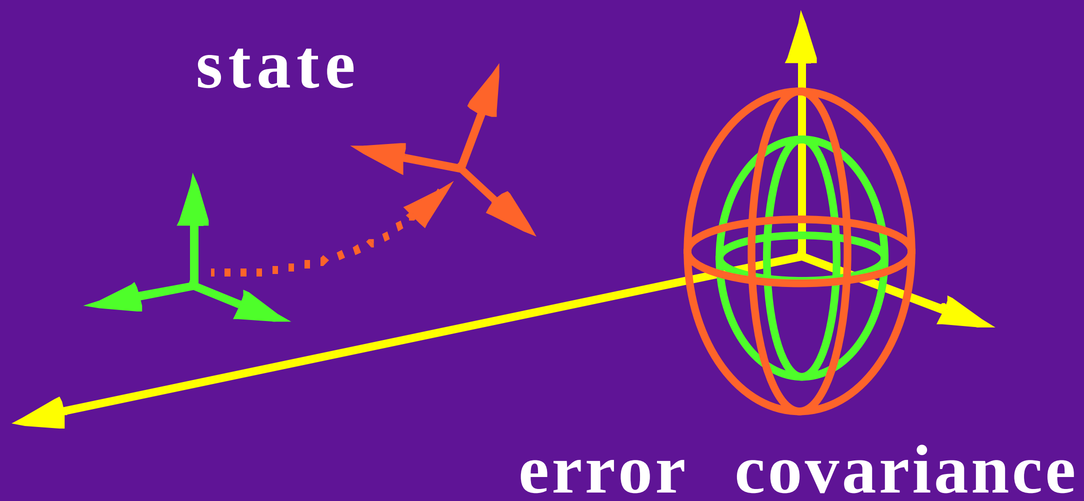
The Kalman Filter

Needs Two Models



Predict

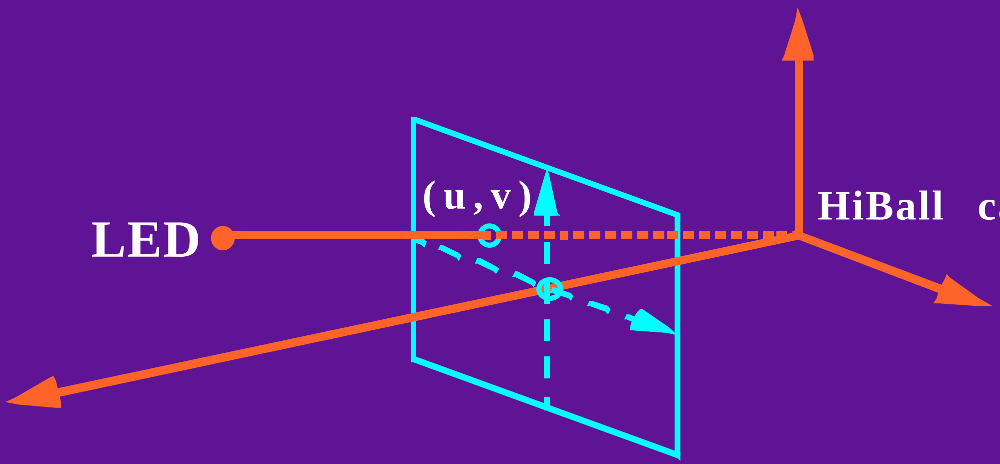
Use the Dynamic Model



Correct (1 of 3)

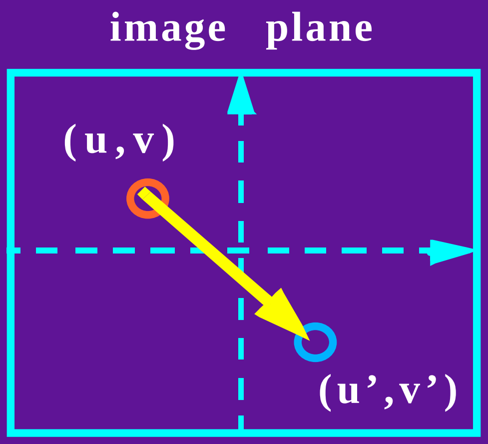
Predict the Measurement

(with measurement model and predicted state)



Correct (2 of 3)

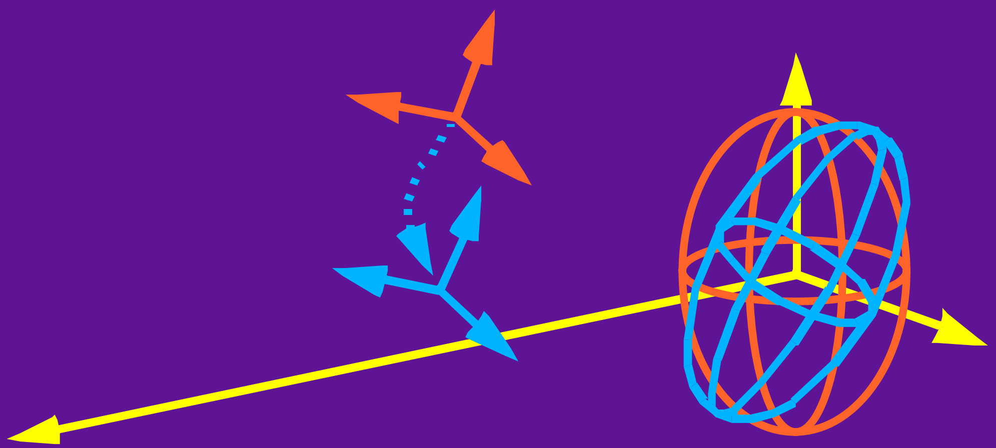
Compute Measurement Residual



Correct (3 of 3)

Correct State and Covariance

state = predicted + Kalman gain x residual



The Kalman Gain

state = predicted + Kalman gain x residual

$$\text{Kalman gain} \cong \frac{P}{P + R}$$

Where

P = estimate uncertainty

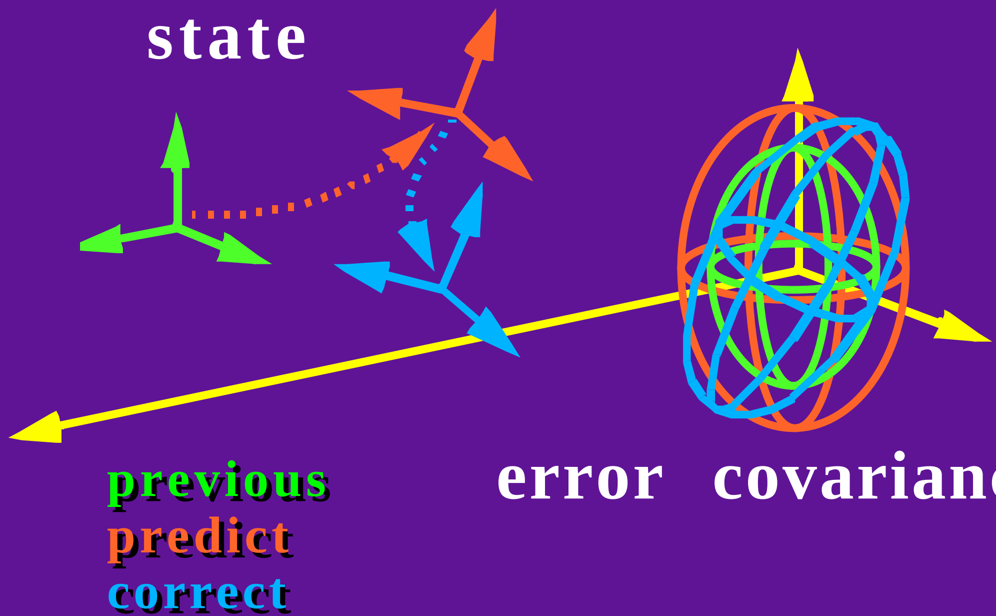
R = measurement uncertainty

Incorporates a notion of

direction of measurement information

Intra-estimate Summary

State & Covariance Changes



The Kalman Filter

- www.cs.unc.edu/~welch
  - Dissertation (appendix)
  - Kalman filter web page

SCAAT Video

The SCAAT Kalman Filter

- Constraints == Equations
  - Single constraint == single equation
- Compare with Gauss-Seidel
  - Standard vs. SCAAT
  - Jacobi vs. Gauss-Seidel
  - Successive vs. simultaneous displacment
  - But stochastic not deterministic!

Predict

System Dynamics

- Use dynamic model
- Predict state
- Predict error covariance

Kalman Gain

Geometric Interpretation

- Project state unertainty into measurement space
  - uses Jacobian
  - error magnification viewpoint
- Combine with measurement uncertainty
  - added uncertainty
- Re-project normalized term
- Ratio (weight)

Correct

With Actual Measurement

- Predict measurement
  - lower dimension than state
  - incomplete information
- Measure
- Compute residual
- Correct state
- Correct error covariance

SCAAT Error and Stability

(† denotes special concern)

Error

Some Sources & Effects

- Measurement error
  - improper model structure
  - non-white, non-normal
  - derivation depends on it!
- Dynamic error
  - improper model structure
  - improper parameters
- Linearization error
  - an extension to the KF

Divergence

Roundoff (Steady State)

- **Problem**
  - P must remain positive definite
  - No driving noise...semidefinite...
- **Solutions**
  - High-precision operations
  - Avoid completely deterministic systems
  - Factor P (square-root, U-D)

Divergence

Modeling Error

- Problem
  - You model a random constant
  - Actually a random ramp
- Solutions
  - *System identification* is hard!
  - Analyze residuals
  - A priori system knowledge
  - Minimize estimate interval

Divergence

Observability †

- Observability test
- (Controllability test)
- Local vs. global observability

Stability

General Conditions

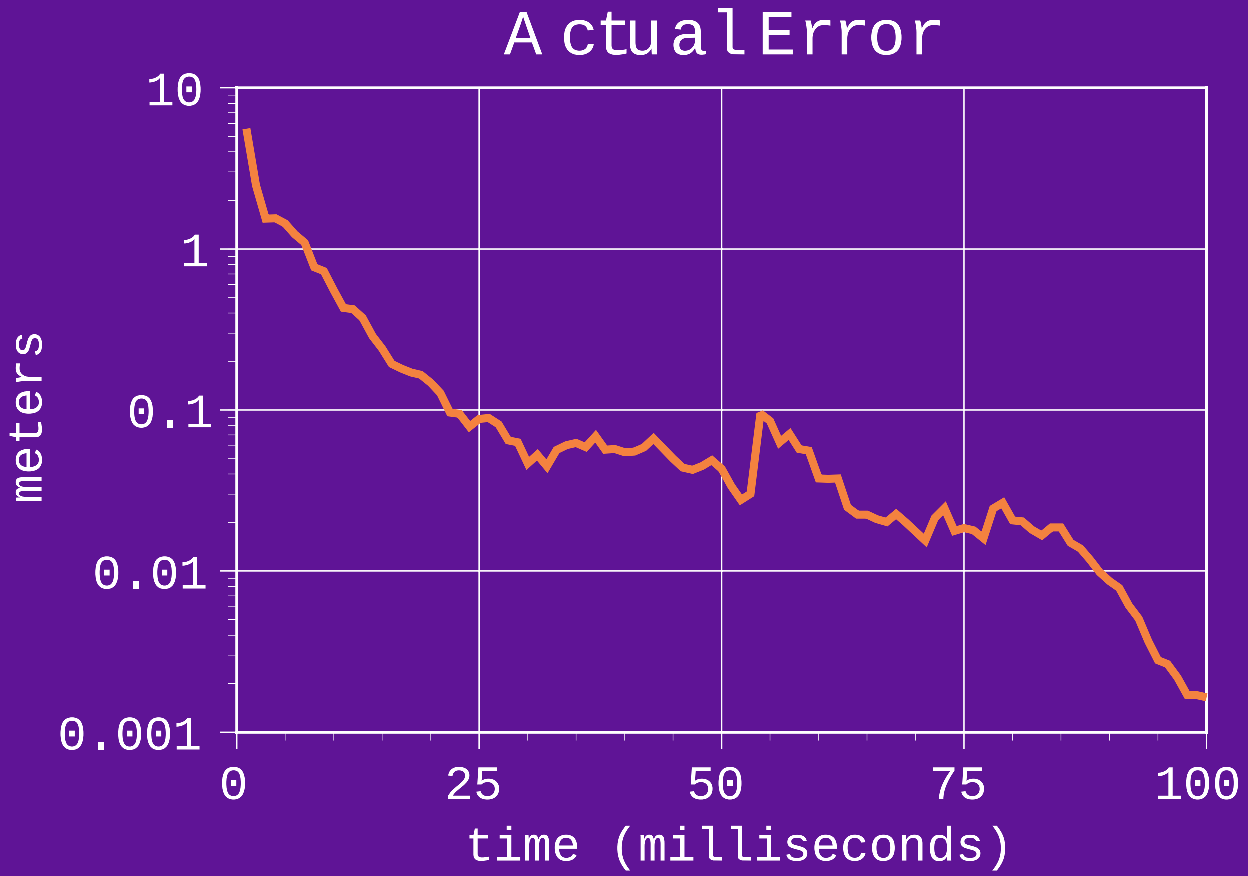
- Uniformly completely observable †
- Bounded dynamic & measurement noise models
- Bounded dynamic behavior

Stability

Complete Conditions

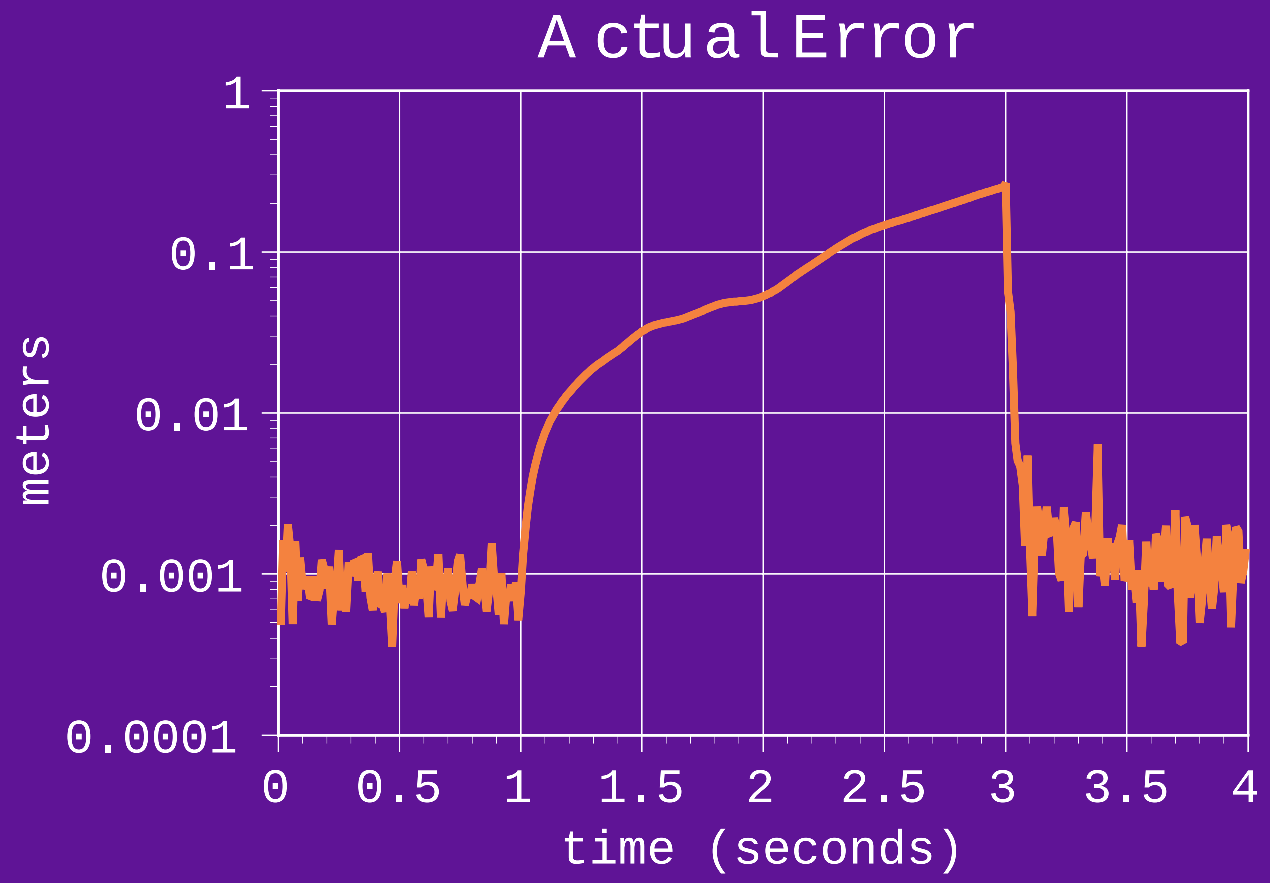
- See equations 5.2 and 5.3
- Bounded dynamics over time
  - Q finite but !=0
- Bounded measurement noise
  - Reasonable for systems of interest
- Sufficient constraints over time †
  - met by design or at run-time
  - Ironically SCAAT helps (fast)
- Sufficient sample rate †

Cold Start



Actual Error
10
1
0.1
0.01
0.001
meters
0
25
50
75
100
time (milliseconds)

Blocked Cameras



Steady-State Stability

- Steady-state not always reached
- KF is a linear operator
  - transforms inputs into outputs
  - transfer function
- Characteristic function
  - denominator of transfer function
  - roots provide information about stability
    - discrete: within unit circle in z plane
    - continuous: left half of s plane

General Application

SCAAT and Systems of Equations

- Similar to Gauss-Seidel
  - successive vs. simultaneous
  - always use latest estimate
- Trade-off accuracy for work
  - Single constraints until “certain”
  - P matrix indicates certainty
- Small noise for stability

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